

(3 Hours)

[ Total Marks : 80

- N.B :** (1) Question No. 1 is compulsory.  
 (2) Solve any **three** questions out of remaining **three** questions.  
 (3) All questions carry **equal** marks as indicated by **figures** to the **right**.  
 (4) Assume appropriate data whenever required. State all assumptions clearly.

1. (a) Prove by mathematical induction  $x^n - y^n$  is divisible by  $x - y$ . 5  
 (b) How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2. 5  
 (c) Show that a relation is reflexive and circular if and only if it is an equivalence relation. 5  
 (d) Prove that the set  $G = \{1, 2, 3, 4, 5, 6\}$  is an abelian group under multiplication modulo 7. 5
2. (a) Is it possible to draw a tree with five vertices having degrees 1, 1, 2, 2, 4? 4  
 (b) Find how many integers between 1 and 60 are 8  
     (i) not divisible by 2 nor by 3 and nor by 5.  
     (ii) Divisible by 2 but not by 3 and nor by 5.  
 (c) Solve the recurrence relation  $a_{r+2} - a_{r+1} - 6a_r = 4$  8
3. (a) Show that  $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$  4  
 (b) State and explain Pigeonhole principle, extended Pigeonhole principle. How many numbers must be selected from the set  $\{1, 2, 3, 4, 5, 6\}$  to guarantee that at least one pair of these numbers add up to 7? 8  
 (c) Let  $R$  be a relation on set  $A = \{1, 2, 3, 4\}$ , given as 8  
 $R = \{(1, 1), (1, 4), (2, 2), (2, 3), (3, 2), (3, 3), (4, 1), (4, 4)\}$ .  
 Find transitive closure using Warshall's Algorithm.
4. (a) Find the generating function for the following sequence 4  
     (i) 1, 2, 3, 4, 5, 6, .....  
     (ii) 3, 3, 3, 3, 3, .....  
 (b) Show that the (2, 5) encoding function  $e: B^2 \rightarrow B^5$  defined by 8  
 $e(00) = 00000$      $e(01) = 01110$   
 $e(10) = 10101$      $e(11) = 11011$  is a group code.  
 How many errors will it detect and correct.  
 (c) Draw Hasse Diagram of  $D_{42}$ . Find the complement of each element in  $D_{42}$ . 8

5. (a) Define Distributive Lattice along with one appropriate example. 4

(b) Let the functions  $f, g,$  and  $h$  defined as follows : 8

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 3$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = 3x + 4$$

$$h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = 4x$$

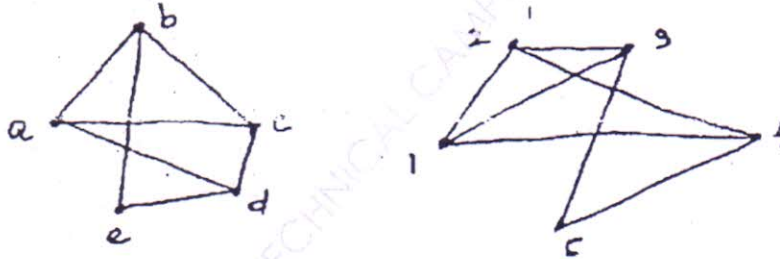
Find  $\text{gof}, \text{fog}, \text{foh}, \text{hof}, \text{gofoh}$

(c) let  $H = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$  8

Be a parity check matrix. Determine the group code  $e_{\text{if}}: B^3 \rightarrow B^6$

6. (a) Determine if  $[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$  is a tautology. 4

(b) Define isomorphic graphs. Show that following graphs are isomorphic. 8



(c)  $R$  be a relation on set of integers  $Z$  defined by 8

$$R = \{(x, y) \mid x - y \text{ is divisible by } 3\}$$

Show that  $R$  is an equivalence relation and describe the equivalence classes.

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