(REVISED COURSE)   **QP Code :11932**

(3 Hours)   Total Marks : 80

N. B. : (1) Q. No.1 is **compulsory**.

(2) **Attempt** any **three** questions from question **no.2** to question **no.6**.

(3) **Figures** to the right indicate **full** marks.

1. (a) If $\tanh x = \dfrac{2}{3}$, find the value of x and then $\cosh 2x$ **3**

    (b) If $u = \tan^{-1}\left(\dfrac{y}{x}\right)$, Find these value of $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$ **3**

    (c) if $x = r\cos\theta,\ y = r\sin\theta$ **3**

    Find $\dfrac{\partial(x,y)}{\partial(r,\theta)}$

    (d) Prove that $\log\sec x = \dfrac{1}{2}x^2 + \dfrac{1}{1z}x^4 + \dfrac{1}{45}x^6 \ldots\ldots$ **3**

    (e) Show that every square matrix can be uniquely expressed as the sum of   **4**
    Hermitian martix and a skew Hermitian matrix.

    (f) Find the nth derivative of   **4**
    $y = \sin x \sin 2x \sin 3x$

2. (a) Solve the equation $x^6+1=0$ **6**

    (b) Reduce the matrix to normal form and find its rank, where   **6**

    $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

    (c) State and prove Eulers theorem for a homogeneous function in two variables: **8**
    Hence verify the Eulers theorem for

    $u = \dfrac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$

3. (a) Test the consistency of the following equations and solve them if they are   **6**
    consistent.
    $2x - y + z = 8, \qquad 3x - y + z = 6$
    $4x - y + 2z = 7, \quad -x + y - z = 4$

    (b) Find the stationary values   **6**
    $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$

    (c) Separate into real and imaginary parts of $\sin^{-1}(e^{i\theta})$ **8**

Correction attached

4. (a) If $x = uv$, $y = \frac{u}{v}$ prove that $J.J' = 1$ 6

   (b) Show that for real values of a and b, $e^{2ai\cot^{-1}b}\left[\dfrac{bi-1}{bi+1}\right]^{-a} = 1$ 6

   (c) Solve the following equations by Gauss-seidel method   8
   $27x + 6y - z = 85$
   $6x + 15y + 2z = 72$
   $x + y + 54z = 110$

5. (a) Expond $\cos^7\theta$ in a series of cosines of multiple of θ 6

   (b) If $\lim\limits_{x \to 0} \dfrac{a\sinh x + b\sin x}{x^3} = \dfrac{5}{3}$, find a and b   6

   (c) If $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$ then prove that $(1 - x^2)y_{n+1} - (2n + 1)xy_n - n^2 y_{n-1} = 0$ 8

6. (a) Examine whether the vectors   6
   $x_1 = [3, 1, 1]$ $x_2 = [2, 0, -1]$
   $x_3 = [4, 2, 1]$ are linearly independent.

   (b) If $u = f(x-y, y-z, z-x)$ then show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$ 6

   (c) Fit a straight line for the following data   8

x	1	2	3	4	5	6
y	49	54	60	73	80	86

GN-Con.:10076-14.