SE. Sem IV CBGS (ET & EE) (247)

AM - IV 21/5/15

QP Code : 3488

(3 Hours) Total Marks: 80

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

Q1. (a) Evaluate $\int_c |z|\,dz$, where c is the left half of unit circle $|z|=1$ from $z=-i$ to $z=i$ 5

(b) If λ is an Eigen value of the matrix A with corresponding Eigen vector X, prove that λ^n is an Eigen value of A^n with corresponding Eigen vector X. 5

(c) Find the extremal of $\int_{x_1}^{x_2} \frac{\sqrt{1+y'^2}}{x}dx$ 5

(d) Find the unit vector orthogonal to both $[1,1,0]$ & $[0,1,1]$ 5

Q2. (a) Find the curve on which the functional $\int_0^1 [y'^2+12xy]dx$ with $y(0)=0$ & $y(1)=1$ can be Extremised. 6

(b) Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$ 6

(c) Obtain two distinct Laurent's series expansions of $f(z)=\frac{2z-3}{z^2-4z+3}$ in powers of $(z-4)$ indicating the region of convergence in each case 8

Q3. (a) If $A=\begin{bmatrix}2&1\\1&2\end{bmatrix}$, find A^{50} 6

(b) Evaluate $\int_c \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}dz$, where c is the circle $|z|=3$ 6

(c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of the functional $I(y)=\int_0^1 (y'^2-2y-2xy)dx$ subject to $y(0)=2,\ y(1)=1$. 8

Q4. (a) Find the vector orthogonal to both $[-6,4,2]$ & $[3,1,5]$ 6

(b) Show that the matrix $A=\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory and find its minimal polynomial. 6

(c) Reduce the matrix of the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2+4x_1x_3-2x_2x_3$ to canonical form through congruent transformation and find its rank, signature, and value class. 8

Q5. (a) Find the extremal of $\int_{x_0}^{x_1}\left(2xy-y''^2\right)dx$ 6

(b) Show that the set $W=\{[x,y,z] \mid y=x+z\}$ is a subspace of $\mathbf{R^n}$ under the usual addition and scalar multiplication. 6

(c) Show that the following matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is diagonalisable. Also find the diagonal form and a diagonalising matrix. 8

Q6. (a) If $f(a)=\int_c \frac{3z^2+7z+1}{z-a}dz$, where c is a circle $|z|=2$, find the values of i) $f(-3)$, ii) $f(i)$, iii) $f'(1-i)$ 6

(b) Evaluate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$ 6

(c) Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4.

Where $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ 8
