02

TE-SEM-V-EXTC-RSA-OLD 13/05/15

QP Code : **3872**

(OLD COURSE)

(3 Hours) [Total Marks : 100

N. B. : (1) Question No. 1 is **compulsory**.
(2) Solve any **four** out of remaining questions.
(3) Assume suitable data, if needed.

1. (a) State and explain Baye's theorem. 5
(b) State properties of probability distribution function and prove any one property. 5
(c) State and prove any two properties of auto corelation function. 5
(d) Define random process by giving an example. 5

2. (a) The transmission time X of messages in a communication system obey the following exponential probability law with parameter 10
$f(x) = k\,e^{-\lambda X}$; $x > 0$
Find
(i) k
(ii) Probability density function of x, $f_X(x)$
(iii) Cumulative distribution function of x, $F_X(x)$. Sketch $f_X(x)$ and $F_X(x)$.
(b) Box 1 contains 5 white balls and 6 black balls. Box 2 contains 6 white balls and 4 black balls. A box is selected at random and then a ball is drawn from it. 10
(i) What is the probability that the ball drawn will be white?
(ii) Given that the ball drawn is white, what is the probability that it came from box 1?

3. (a) Let X be continuous random variable with uniform probability density function in $(0,2\pi)$. Find the probability density function of $y = \sin x$. 10
(b) Suppose X and Y are two random variables. Define covariance and correlation coefficient of X and Y. When do we say that X and Y are 10
(i) Orthogonal
(ii) Independent and
(iii) Uncorrelated
Are uncorrelated variables independent?

[TURN OVER

4. (a) The joint probability density function of (x, y) is given by 10

$f_{xy}(x, y) = 1/2\, x.e^{-y}$; $0 < x < 2, y > 0$

$= 0$; else

(i) Find the joint cumulative distribution function.

(ii) Find the morginal probability density functions of x and y.

(iii) Are x & y independant?

(b) If X and Y are independant random variables and if Z = X + Y, then prove that the probability density function of Z is given by convolution of their individual densities. 10

5. (a) Explain WSS process. A random process is given by $x(t) = A\cos(w_0 t + \phi)$ where A and w_0 are constants and ϕ is random variable uniform in $(0, 2\pi)$. Show that x(t) is WSS. 10

(b) Explain power spectral density function. State its important properties and prove any one property. 10

6. (a) State and prove chapman-kolmogorov equation. 10

(b) The transition probability matrix of a markov chain is 10

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Find the limiting probabilities.

7. (a) State and explain central limit theorem 5

(b) Define characteristic function and find characteristic function of poisson distribution 5

(c) Let X_1, X_2 --- be sequence of random variable. 10

Define

(i) Convergence almost everywhere

(ii) Convergence in probability

(iii) Convergence in mean square sense

(iv) Convergence in distribution

for the above sequence to a random variable X.
