T. E - SEM - V - CBGS EXTC - RSA 13/05/15

06

Pg 1/1

Q.P. Code : **3374**

(3 Hours) [ Total Marks : 80

N.B.:

1) Question Number 1 is Compulsory
2) Attempt any Three questions from the remaining Five questions
3) Assumptions made should be clearly stated.
4) Use of normal table is permitted

**1 Answer the following** **20**

**a)** State and prove Bayes' s theorem.

**b)** A certain test for a particular cancer is known to be 95% accurate. A person submits to the test and the results are positive. Suppose that the person comes from a population of 100,000 where 2000 people suffer from that disease. What can we conclude about the probability that the person under test has that particular cancer?

**c)** Let $X$ and $Y$ be independent, uniform r.v.'s in (-1, 1). Compute the pdf of $V = (X + Y)^2$.

**d)** If the spectral density of a WSS process is given by

$$S(w) = b(a-|w|)/a, \quad |w| \le a$$
$$= 0 \quad , \quad |w| > a$$

Find the autocorrelation function of the process.

**2a)** State and prove Chapman-Kolmogorov equation. **10**

**b)** The joint density function of two continuous r.v.'s $X$ and $Y$ is **10**

$$f(x, y) = cxy \quad 0 < x < 4, \ 1 < y < 5$$
$$= 0 \quad \text{otherwise.}$$

i) Find the value of constant c.

ii) Find $P(X \ge 3, Y \le 2)$

iii) Find marginal distribution function of $X$.

**3a)** Explain strong law of large numbers and weak law of large numbers. **05**

**b)** Explain the central limit theorem. **05**

**c)** A distribution with unknown mean $\mu$ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. **10**

[TURN OVER

4a) Given a r.v. $Y$ with characteristic function 10

$$\Phi(w) = E\{e^{jwY}\}$$

and a random process defined by $X(t) = \cos(\lambda t + Y)$, show that $X(t)$ is stationary in wide sense if

$$\Phi(1) = \Phi(2) = 0.$$

b) Define an ergodic process. Determine whether the stochastic process 10

$X(t) = A\sin(t) + B\cos(t)$ is ergodic. Here $A$ & $B$ are normally distributed independent r.v.'s with zero mean and equal standard deviation.

5a) The joint probability function of two discrete r.v.'s $X$ and $Y$ is given by $f(x, y) = c(2x + y)$, where $x$ and $y$ can assume all integers such that $0 \le x \le 2$, $0 \le y \le 3$ and $f(x, y) = 0$ otherwise. Find $E(X)$, $E(Y)$, $E(XY)$, $E(X^2)$, $E(Y^2)$, var($X$), var($Y$), cov($X$, $Y$), and $\rho$. 10

b) State and explain various properties of autocorrelation function and power spectral density function. 10

6a) The transition probability matrix of Markov Chain is 10

$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

Find the limiting probabilities.

b) Write notes on any two of the following: 10

i) Markov chains

ii) Little's formula

iv)LTI systems with stochastic input

v) M/G/1 queuing system

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