

QP Code : 1002

(REVISED COURSE)

(3 Hours)

[Total Marks : 80

- N. B. : (1) Question No. 1 is compulsory.
(2) Answer any **three** questions from remaining.
(3) Assume suitable data if necessary.

1. (a) If $\tan \frac{x}{2} = \tanh \frac{u}{2}$ then S.T.

$$u = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

(b) If $u = x^y$ find $\frac{\partial^3 u}{\partial x \partial y \partial x}$

(c) If $ux = yz, vy = zx, wz = xy$

$$\text{find } J \begin{pmatrix} u, v, w \\ x, y, z \end{pmatrix}$$

(d) If $y = (x-1)^n$ then P.T. $y + \frac{y_1}{1!} + \frac{y_2}{2!} + \frac{y_3}{3!} + \dots + \frac{y_n}{n!} = x^n$

(e) P.T. $\sinh x = X + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$

(f) Express the matrix A as sum of Hermitian and skew Hermitian matrix where

$$A = \begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

2. (a) Solve $x^7 + x^4 + i(x^3 + 1) = 0$

(b) Reduce the matrix A to normal form and hence find its rank where

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 4 & 3 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

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6. (a) Determine linear dependence or independence of vectors
 $x_1 = [1, 3, 4, 2]$ $x_2 = [3, -5, 2, 6]$
 $x_3 = [2, -1, 3, 4]$ and if dependent find the relation between them. 6
(b) If $u = x^2 - y^2, v = 2xy$ and $z = f(u, v)$ prove that 6

$$\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 = 4\sqrt{u^2 + v^2} \left[\left(\frac{\partial z}{\partial u} \right)^2 + \left(\frac{\partial z}{\partial v} \right)^2 \right]$$

(c) (i) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x \cdot \sin^{-1} x - x^2}{x^6}$ 4

(ii) Fit straight line to the following data 4
 $(x, y) = (-1, -5), (1, 1), (2, 4), (3, 7), (4, 10)$
Estimate y when $x = 7$

- (c) State and prove Euler's theorem for three variables and hence find 8

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \text{ where}$$

$$u = \frac{x^3 y^3 z^3}{x^3 + y^3 + z^3}$$

3. (a) Solve the following system of equations 6

$$2x - 2y - 5z = 0$$

$$4x - y + z = 0$$

$$3z - 2y + 3z = 0$$

$$x - 3y + 7z = 0$$

- (b) Find the maximum and minimum values 6

$$\text{of } x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$$

- (c) Separate into real and imaginary parts of $\tanh^{-1}(x + iy)$ 8

4. (a) If $u = 2xy$, $v = x^2 - y^2$ and $x = r \cos \theta$, $y = r \sin \theta$ then find 6

$$\frac{\partial(u, v)}{\partial(\theta)}$$

- (b) If $i^{i^{\infty}} = A + iB$, prove that 6

$$\left(\frac{\pi A}{2}\right) = \frac{B}{A} \text{ and } A^2 + B^2 = e^{-\pi B}$$

- (c) Solve by crouts method the system of equations 8

$$3x + 2y + 7z = 4$$

$$2x + 3y + z = 5$$

$$3x + 4y + z = 7$$

5. (a) By using De Moivre's thm 6

$$\text{Express } \frac{\sin 7\theta}{\sin \theta} \text{ in powers of } \sin \theta \text{ only.}$$

- (b) By using Taylor's series expand $\tan^{-1} x$ in positive powers of $(x-1)$ upto first four non-zero terms. 6

- (c) If $y = \sin[\log(x^2 + 2x + 1)]$ prove that 8

$$(x+1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2 + 4) y_n = 0$$

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Course: F.E. (REV.) (ALL BRANCHES) (CBSGS) (SEM-I)(Prog-569)

Q.P Code: 1002

Correction:

Q. No.(4)(a)

Read AS:

$$\frac{\partial(u, v)}{\partial(r, \theta)}$$

Instead of:

$$\frac{\partial(u, v)}{\partial(\theta)}$$

Q. no. (3)(A) Equation no. 3

Read AS:

$$3x - 2y + 3z = 0$$

Instead of:

$$3z - 2y + 3z = 0$$

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Note : Take printouts and submit them to all concerned students.

Course: F.E. (REV.) (ALL BRANCHES) (CBSGS) (SEM-I)(Prog-569)

Q.P Code: 1002

Correction:

Q. No.(2)(c)

Read AS:

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$$

Instead of:

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial x}$$

Q. No.(4)(b)

Read AS

$$\tan\left(\frac{\pi A}{2}\right) = \frac{B}{A}$$

Instead of:

$$\left(\frac{\pi A}{2}\right) = \frac{B}{A}$$

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