SE - CO Sem - IV 15/5/15

Am - IV

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(OLD COURSE) QP Code : **4047**

(3 Hours) [Total Marks : 100

N.B.: (1) Question No. 1 is **compulsory**.

(2) Attempt any **four** questions out of the **remaining six** questions.

(3) **Figures** to the **right** indicate **full** marks.

1\. (a) Find the eigen values of $A^3 - 3A^2 + A$, where 5

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}.$$

(b) Determine the constants a, b, c and d if 5

$f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic function.

(c) Evaluate $\int_0^{1+i} (x - y + ix^2)\, dz$ along the parabola $y^2 = x$. 5

(d) Obtain the dual of the following L.P.P. 5

Maximise $z = x_1 - 2x_2 + 3x_3$

subject to $-2x_1 + x_2 + 3x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0$

2\. (a) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. 6

(b) Find an analytic function f(z) whose imaginary part is $e^{-x}(y \sin y + x \cos y)$. 6

(c) Using duality solve the following L. P. P. 8

Minimise $z = 4x_1 + 14x_2 + 3x_3$

subject to $-x_1 + 3x_2 + x_3 \geq 3$

$2x_1 + 2x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

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3. (a) Evaluate $\int_c \frac{ze^z}{(z-a)^3}dz$, where c is $|z| = b, (a < b)$. 6

(b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, find A^{50}. 6

(c) Solve the following L.P.P. by Simplex method 8

maximise $z = 5x_1 + 4x_2$

subject to constraint $6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

$x_2 \leq 2$

$x_1, x_2 \geq 0$

4. (a) Show that $u = \left(r + \frac{a^2}{r}\right)\cos\theta$ is a harmonic functions and find it's harmonic conjugate. 6

(b) Use the dual simplex method to solve the following L.P.P. 6

Minimise $z = 6x_1 + x_2$

subject to $2x_1 + x_2 \geq 3$

$x_1 - x_2 \geq 0$

$x_1 x_2 \geq 0$

(c) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ hence find A^{-1} and A^4. 8

5. (a) Show that $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix. 6

(b) Obtain Taylor's and Laurent's series for $f(z) = \frac{z-1}{z^2 - 2z - 3}$ indicating the region of convergence. 6

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(c) Solve the following N.L.P.P. by Lagranges's Multiplier's Method. 8

Optimise $z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

subject to $x_1 + x_2 + x_3 = 20$

$x_1, x_2, x_3 \geq 0$

6. (a) Show that matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is decogatory. 6

(b) Find the bilinear transformation which maps th e points $1, -i, 2$ of z plane onto $0, 2, -i$ of w plane respectively. 6

(c) Evaluate $\int_0^{\pi} \frac{d\theta}{3+2\cos\theta}$ using Cauchy Residue theorem. 8

7. (a) Evaluate $\int_C \frac{z+4}{z^2 + 2z + 5} dz$, where C is 6

(i) $|z+1-i| = 2$

(ii) $|z| = 1$

(b) Find the image of the circle $(x-3)^2 + y^2 = 2$ under the transformation $w = \frac{1}{z}$. 6

(c) Use Kuhn-Tucker conditions to solve the following N.L.P.P. 8

Maximise $z = 8x_1 + 10x_2 - x_1^2 - x_2^2$

subject to c $3x_1 + 2x_2 < 6$

$x_1, x_2 > 0$
