Comp III (old)
A·M·III
3 pg. 14/5/15

(OLD COURSE) QP Code : **4596**

(3 Hours) [Total Marks : 100

N.B. (1) Question no. **1** is **compulsory.**
(2) Attempt any **four** questions from remaining **six** questions
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find A^{-1} where 5

$$A=\begin{bmatrix}1 & 2 & 2\\ 2 & 3 & 1\\ 1 & 2 & 1\end{bmatrix}$$ Also verify that $A(\text{adj} A) = |A|.I$

(b) Find $L\{te^{-4t}\sin 3t\}$ 5

(c) Find the Fourier series of $f(x) = 1-x^2$ on the interval (-1, 1) 5

(d) Find z-transform of $f(x)=\dfrac{\alpha^k}{k}$, $k \geq 1$ 5

2. (a) Find $L[\sinh^5 t]$ 6

(b) Find the Fourier series for 6

$$f(x)=\begin{cases}-\pi & -\pi \leq x < 0\\ x & 0 \leq x \leq \pi\end{cases}$$

(c) Find the rank of 8

$$A=\begin{bmatrix}2 & 1 & 3 & 4\\ 3 & -1 & 2 & 2\\ 4 & 1 & 0 & -1\\ 9 & 1 & 5 & 6\end{bmatrix}$$

by reducing it to normal form.

3. (a) Find $L^{-1}\left\{\dfrac{(s^2+2s+3)}{(s^2+2s+2)(s^2+2s+5)}\right\}$ 6

by convolution theroem.

(b) Prove that every Hermitian matrix A can be written as P+ iQ where P is real symmetric and Q is real skew-symmetric matrix 6

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(c) Find half rage cosine series for $f(x)=(x-1)^2$ where $0<x<1$ 8

Hence find $\sum_{n=1}^{\infty}\frac{1}{n^2}$ and $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$

4. (a) Solve $\frac{d^2y}{dt^2}+\frac{2dy}{dt}+y=3te^{-t}$ 6

Given $y(0)=4$, $y'(0)=2$ using Laplaces transformation

(b) Prove that $f_1(x)=1$, $f_2(x)=x$, $f_3(x)=\left(\frac{3x^2-1}{2}\right)$ 6

are arthogonal over (-1, 1)

(c) Find λ and μ such that the equations $x+2y+\lambda z=1, x+2\lambda y+z=\mu, \lambda x+2y+z=1$ 8

have (i) no solution (ii) only one solution (iii) infinite many solutions

5. (a) Find $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\}$ 6

(b) Prove that $A=\frac{1}{3}\begin{bmatrix}-2 & 1 & 2\\ 2 & 2 & 1\\ 1 & -2 & 2\end{bmatrix}$ is orthogonal and hence find A^{-1} 6

(c) Find the Fourier Sine Transform of f(x) if 8

$$f(x)=\begin{cases}0 & 0<x<a\\ x & a\le x\le b\\ 0 & x>b\end{cases}$$

6. (a) Prove that if A is a skew-symmetric matrix of odd order then A is singular 6

(b) Obtain complex form of Fourier series of $f(x)=e^{ax}$ in $(-\pi,\pi)$, where a is not an integer. 6

(c) Find inverse z-transform of $F(z)=\frac{1}{(z-3)(z-2)}$ 8

if ROC is (i) $|z|<2$ (ii) $2<|z|<3$ (iii) $|z|>3$

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7. (a) Find $L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{5/2}}\right\}$ 6

(b) If $f(k) = 4^k U(k)$ and $g(k) = 5^k U(k)$ then find the z - transform of $f(k) * g(k)$ 6

(c) Obtain Fourier series for 8

$$f(x) = x+\frac{\pi}{2} \qquad -\pi < x < 0$$

$$= \frac{\pi}{2} - x \qquad 0 < x < \pi$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.........$

Also deduce that $\frac{\pi^4}{96} = \frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.........$
