EE-III (Old)

AM-III

Engg. Maths-III

(OLD COURSE) **QP Code : 4548**

(3 Hours) Max. Marks:-100

N.B:-i) Q.No.1) is compulsory,

ii) Attempt any FOUR from the remaining,

iii) Figures to right indicate full marks.

Q.No.1) a) Prove that $\int_0^\infty e^{-3t}\, t \sin t \, dt = \frac{3}{50}$ (05)

b) Find the image of $x^2 + y^2 = 2x$ under the transformation $w = \frac{1}{z}$ (05)

c) Evaluate $\oint_C \tan z \, dz$ where $C: |z| = 1$ (05)

d) Find Fourier series of $f(x) = \sin x$ in $(0,2\pi)$ (05)

Q.No.2) a) Prove that $\int_0^\infty \frac{e^{-t} \sin^2 t}{t} dt = \frac{1}{4}\log 5$ (06)

b) Show that $u = e^x(x\cos y - y \sin y)$ is harmonic, Find the harmonic conjugate v and the analytic function $f(z)$ (06)

c) Find Fourier series for $f(x)$ in $(0,2\pi)$

$$f(x) = x \qquad 0 < x \le \pi$$

$$= 2\pi - x \qquad \pi \le x < 2\pi$$

Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ (08)

Q.No.3) a) Find inverse Laplace transform of $\frac{s}{s^4+4}$ (06)

b) Find an analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ (06)

c) Find Fourier series of $f(x) = x^2$ in $(-\pi, +\pi)$

Hence deduce that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$ (08)

Q.No.4) a) Find the bilinear transformation which maps the point

$z = \infty, i, 0$ to $w = 0, i, \infty$ (06)

b) Obtain half range sine series for $f(x) = lx - x^2$ in $(0, l)$

Hence show that $\frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$ (06)

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c) Solve by using Laplace transform $(D^2 + 2D + 5)y = e^{-t}\sin t$, $y(0) = 0$, $y'(0) = 1$ (08)

Q.No.5) a) Obtain half range sine series for $f(x) = \pi x - x^2$ in $(0, \pi)$

Hence show that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\dots$ (06)

b) Find i) Laplace transform of $\sin^4 t$ ii) inverse Laplace transform of $\cot^{-1} s$ (06)

c) Find all possible Laurent's expansion of $f(z) = \frac{z}{(z-1)(z-2)}$ about $z = -2$ indicating the region of convergence (08)

Q.No.6) a) Find complex form of Fourier series for $f(x) = \cosh x + \sinh x$ in $(-\pi, +\pi)$ (06)

b) Evaluate $\oint_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$ where $C: |z| = 1$ (06)

c) If $f(t)$ is periodic function with period "T" then prove that $L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-su} f(u)du$

Hence find the Laplace transform of following periodic function with period "2 "

$$f(t) = 1 \quad 0 < t < 1$$
$$= -1 \quad 1 < t < 2$$
(08)

Q.No.7) a) Evaluate $\oint_C \frac{z^2}{(z-1)^2(z-2)} dz$ where $C: |z| = 2.5$ (06)

b) Express the function $f(x) = 1 \quad |x| < 1$

$= 0 \quad |x| > 1$

as Fourier integral Hence evaluate $\int_0^\infty \frac{\sin\omega \sin\omega x}{\omega} d\omega$ (06)

c) Evaluate i) $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ ii) $\int_{-\infty}^{+\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$ (08)
