EXTC - III (old)

A·M·III

14/5/15

(OLD COURSE)

Q.P. Code : **4677**

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions from question no. 2 to 7.
(3) All sub questions of any question must be answered togethter.

1. (a) Find L [Sin t Sin 3t sin 5t] 5

(b) Find z transformation of $\frac{a^k}{k}$, $k \geq 1$ 5

(c) Show that every square matrix A can be uniquely expressed sum of hermitian matrix and skew-hermitian matrix. 5

(c) Find the fourier series of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0 \leq x \leq 2\pi$ 5

2. (a) Show that $\int_0^\infty \frac{(\sin 2t+\sin 3t)}{te^t} dt = \frac{3\pi}{4}$ 6

(b) Show that $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ +i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary hence find A^{-1}. 6

(c) Find the Fourier Expansion for $f(x)=\sqrt{1-\cos x}$ in (0, 2 π), hence deduce 8

$$\sum_1^\infty \frac{1}{4n^2-1} = \frac{1}{2}$$

3. (a) Solve $(D^2 + 2D + 5)\, y = e^{-t} \sin t$ given $y(0) = 0$, $y'(0) = 1$ 6

(b) Find the Fourier series of 6

$f(x) = \cos x \quad -\pi < x < 0$
$\quad = \sin x \quad 0 < x < \pi$

(c) Solve the equations by Gauss seidel method 8

$23x + 4y - z = 32$
$2x + 17y + 4z = 35$
$x + 3y + 10z = 24$

[TURN OVER

4. (a) P.T. $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = \frac{3x^2 - 1}{2}$ are orthogonal over $[-1, 1]$ 6

(b) Find the non-sigular matrices P and Q such that PAQ is normal. Where A is 6

given by $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$

(c) Find the inverse Laplace Transformation 8

(i) $L^{-1}\left[\log\left(\frac{s^2+16}{s^2+25}\right)\right]$ (ii) $L^{-1}\left[\frac{S+4}{(S+1)^2\,(S-1)}\right]$

5. (a) Find the inverse Z - Transformation $f(z) = \frac{1}{(z-3)(z-2)}$ 6

(b) Find the fourier series 6

$f(x) = 2x - x^2 \quad 0 \le x \le 3$

(c) Investigate for what values of λ, μ the equation $x + y + z = 6$, $x + 2y + 3z = 10$ 8

$x + 2y + \lambda z = \mu$ have

(i) no solution (ii) unique solution (iii) infinite number of solution

6. (a) Find z transformation of Z [a cos kα + bsin kα] $k \ge 0$ 6

(b) Find the complex form of Fourier series $f(x) = \cosh ax + \sinh ax$ in $[-\pi, \pi]$ 6

(c) Find the Laplace Transformation of 8

(i) $L\left[\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)\right]$ (ii) $L[t \sin^3 t]$

7. (a) Find the Laplace transformation of 6

$f(t) = E \quad 0 \le t \le a$

$= -E \quad 0 \le t \le 2a$, $f(t) = f(t + 2a)$

(b) Obtain half range cos series 6

$f(x) = x(\pi - x) \quad 0 \le x \le \pi$ and hence deduce $\sum_{1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$