15/5/15

SE - RT Sem IV

Pg. 1/3

Am - IV

QP Code : 4134

(OLD COURSE)

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No.1 is compulsory.
(2) Attempt any FOUR questions out of the remaining six questions.
(3) Figures to the right indicate full marks.

1. (a) Show that the map of real axis of the Z plaine is a circle under the transformation $W=\frac{2}{Z+i}$ Find its centre and the radius. 5

(b) Verify cayley Hamilton theorem and hence find A^{-1} for $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ 5

(c) Prove that $J_{\frac{-1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\cos x$ 5

(d) Evaluate $\int \bar{F}.d\bar{r}$ where $\bar{F}=2x\hat{i}+(xz-y)\hat{j}+2z\hat{k}$ from 0(0,0,0) to P(3,1,2) along the line OP. 5

2. (a) Find an analytic function f(z)=u+iv where $u+v=e^{x}(\cos y+\sin y)$ 6

(b) If $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ then prove that $A^{100}=\begin{bmatrix}201 & -400\\ 100 & -199\end{bmatrix}$ 6

(c) Prove that $\bar{F}=(y^2\cos x+z^3)\hat{i}+(2y\sin x-4)\hat{j}+(3xz^2+2)\hat{k}$ is conservative field and also find 8

(i) scalar potential for $\bar{F}$

(ii) the work done in moving an object in this field from (0,1,-1) to $\left(\frac{\pi}{2},-1,2\right)$.

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3\. (a) Show that the given function $f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}}, z\neq 0$ **6**

$= 0 \qquad z = 0$

is not analytic at the origin although Cauchy Riemann's equations are satisfied

(b) Evaluate $\int_c \frac{\sin\pi z^2+\cos\pi z^2}{(z-2)(z-3)}dz$ where 'c' is the circle $|z|=4$ **6**

(c) Show that matrix $A=\begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 0 & 0 & 1\end{bmatrix}$ is diagonalisable. **8**

4\. (a) Reduce the quadratic form $5x^2+26y^2+10z^2+6xy+4yz+14zx$ to normal form through congruent transformation **6**

(b) Find the bilinear transformation which maps the points 1, -i, 2 on z plane onto 0, 2, -i respectively on W plane. **6**

(c) Expand $f(z)=\frac{1}{z^2(z-1)(z+2)}$ about $z=0$ for **8**

(i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

5\. (a) Use Gauss divergence theorem to evaluate $\iint_S \overline{N}.\overline{F}\, ds$ where $\overline{F}=4x\hat{i}+3y\hat{j}-2z\hat{k}$ and S is the surface bounded by x=0, y=0, z=0 and 2x+2y+z=4 **6**

(b) Expand f(x)=1 in (0<x<1) in a series as $1=\sum\frac{2}{\lambda n J_1(\lambda n)}J_0(\lambda n x)$ where $\lambda_1, \lambda_2, \ldots\lambda n\ldots$ are positive roots of Jo(x)=0 **6**

(c) Evaluate $\int_0^{2\pi}\frac{\cos3\theta}{5-4\cos\theta}d\theta$ **8**

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6\. (a) Find the characteristic equation of matrix 'A' given below and hence find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ where 6

$$A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$$

(b) Prove that $u=x^2-y^2$ $v=\frac{-y}{x^2+y^2}$ both u and v satisfy Laplace's equation but that u+iv is not an analytic function of z. 6

(c) Show that $\overline{F}=\left(ye^{xy}\cos z\right)\hat{i}+\left(xe^{xy}\cos z\right)\hat{j}-\left(e^{xy}\sin z\right)\hat{k}$ is irrotational and find the scalar potential for $\overline{F}$ and evaluate $\int \overline{F}.dr$ along the curve joining the points (0,0,0) and $(-1,2,\pi)$ 8

7\. (a) Show that the matrix $A=\begin{bmatrix}5 & -6 & -6\\ -1 & 4 & 2\\ 3 & -6 & -4\end{bmatrix}$ is derogatory 6

(b) Evaluate $\int_c \frac{z^2}{(z-1)^2(z+1)}dz$ where 'c' is (1) $|z|=\frac{1}{2}$.(2) $|z|=2$ 6

(c) Find the imaginary part of the analytic function whose real part is e^{2x} (x cos2y-y sin2y). Also verify that v is harmonic function where v is imaginary part of analytic function. 8
