SE - Sem - III - CBSGS - Civil/Mech (455)

AM - III 20/11/15

QP Code : **5052**

(3 Hours) [Total Marks :80

N.B. : (1) Question no. **1** is **compulsory.**
(2) Answer any **three** from remaining.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find Laplace transform of $t\sin^3 t$. 5
(b) Find half range sine series in $(0,\pi)$ for $x(\pi - x)$ 5
(c) Find the image of the rectangular region bounded by $x = 0$, $x = 3$, $y = 0$, $y = 2$ under the transformation $\omega = z + (1+i)$ 5
(d) Evaluate $\int f(z)dz$ along the parabola $y = 2x^2$, $z = 0$ to $z = 3 + 18i$ where $f(z) = x^2 - 2iy$ 5

2. (a) Find two Laurent's series of $f(z) = \dfrac{1}{z^2(z-1)(z+2)}$ about $z = 0$ for 8
(i) $|z| < 1$ (ii) $1 < |z| < 2$
(b) Find complex form of Fourier series for $f(x) = \cosh 2x + \sinh 2x$ in $(-2, 2)$ 6
(c) Find bilinear transformation that maps $0, 1, \infty$ of the z plane into $-5, -1, 3$ of ω plane. 6

3. (a) Solve by using Laplace transform 8
$(D^2 + 2D + 5)y = e^{-t} \sin t$ when $y(0) = 0$ and $y'(0) = 1$
(b) Solve $\dfrac{\partial^2 u}{\partial x^2} - 2\dfrac{\partial u}{\partial t} = 0$ by Bender schmidt method given 6
$u(0, t) = 0$, $u(4, t) = 0$, $u(x, 0) = x(4 - x)$
(c) Expand $f(x) = \ell x - x^2$ $0 < x < l$ in a half range cosine series. 6

4. (a) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$ 8
(b) Evaluate $\displaystyle\int_0^{\infty} e^{-2t} \frac{\cos 2t \sin 3t}{t} dt$ 6
(c) Using Crank Nicholoson method solve 6

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$$

$u(0, t) = 0$, $u(4, t) = 0$

$u(x, 0) = \dfrac{x}{3}(16 - x^2)$

Find u_{ij} for $i = 0, 1, 2, 3, 4$ and $j = 0, 1, 2$.

[TURN OVER

5. (a) Find analytic function whose real part is 8

$$\frac{\sin 2x}{\cosh 2y + \cos 2x}$$

(b) Find (i) $L^{-1}\left[\frac{e^{-\pi s}}{s^2 - 2s + 2}\right]$ 6

(ii) $L^{-1}\left[\tan^{-1}\left(\frac{s+a}{b}\right)\right]$

(c) Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ 6

under the boundary conditions $u(0, t) = 0$
$u(l, t) = 0$ and $u(x, 0) = x$
$0 < x < \ell$, ℓ being length of the rod. 8

6. (a) A string is stretched and fastened to two points distance ℓ apart. Motion is started by displacing the string in the form $y = a \sin\left(\pi x / \ell\right)$ which it is released at time $t = 0$. Show that the displacement of a point at a distance x from one end at time t is given by $y_{(x,t)} = a \sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi c t}{\ell}\right)$. 6

6

(b) Find the residue of $\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)^2}$ at its poles.

(c) Find Fourier series of $x\cos x$ in $(-\pi, \pi)$
