

(Revised course)

Time : 3 hours

Total marks : 80

- N.B : (1) Question No.1 is compulsory.
 (2) Answer any three questions from remaining.
 (3) Assume suitable data if necessary.

Evaluate

1. (a) $\int_0^{\infty} e^{-t} \left(\frac{\cos 3t - \cos 2t}{t} \right) dt$ 05

(b) Obtain the Fourier Series expression for $f(x) = 2x - 1$ in $(0, 3)$ 05

(c) Find the value of 'p' such that the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \left(\frac{py}{x} \right)$ is analytic. 05

(d) If $\vec{F} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$. Show that \vec{F} is irrotational. Also find its scalar potential. 05

2. (a) Solve the differential equation using Laplace Transform 06

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = 3te^{-t}, \text{ given } y(0) = 4 \text{ and } y'(0) = 2$$

(b) Prove that 06

$$J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x} \right) J_1(x) - \left(\frac{24}{x^2} - 1 \right) J_0(x)$$

(c) i) In what direction is the directional derivative of $\phi = x^2 y^2 z^4$ at $(3, -1, -2)$ maximum. Find its magnitude. 08

ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Prove that $\nabla r^n = nr^{n-2} \vec{r}$

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3. (a) Obtain the Fourier Series expansion for the function

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \leq x \leq 0 \quad 06$$

$$= 1 - \frac{2x}{\pi}, 0 \leq x \leq \pi$$

- (b) Find an analytic function $f(z) = u + iv$ where.

$$u - v = \frac{x - y}{x^2 + 4xy + y^2} \quad 06$$

- (c) Find Laplace transform of

i) $\cosh t \int_0^t e^u \sinh u$ 08

ii) $t\sqrt{1 + \sin t}$

4. (a) Obtain the complex form of Fourier series for

$$f(x) = e^{ax} \text{ in } (-L, L) \quad 06$$

- (b) Prove that

$$\int x^4 J_1(x) dx = x^4 J_1(x) - 2x^3 J_3(x) + c \quad 06$$

- (c) Find

i) $L^{-1} \left[\frac{2s-1}{s^2+4s+29} \right]$ 08

ii) $L^{-1} \left[\cot^{-1} \left(\frac{s+3}{2} \right) \right]$

5. (a) Find the Bi-linear Transformation which maps the points 1, i, -1 of z plane onto 0, 1, ∞ of w-plane 06

- (b) Using Convolution theorem find 06

$$L^{-1} \left[\frac{s^2}{(s^2+4)^2} \right]$$

(c) Verify Green's Theorem for $\int_C \bar{F} \cdot d\bar{r}$ where 08

$\bar{F} = (x^2 - y^2)\hat{i} + (x + y)\hat{j}$ and C is the triangle with vertices (0,0), (1,1) and (2,1)

6. (a) Obtain half range sine series for 06
 $f(x) = x, 0 \leq x \leq 2$

$$= 4 - x, 2 \leq x \leq 4$$

(b) Prove that the transformation 06

$w = \frac{1}{z+i}$ transforms the real axis of the z-plane into a circle in the w-plane.

(c) i) Use Stoke's Theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ where 08

$\bar{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ and C is the rectangle in the plane $z=0$, bounded by $x=0, y=0, x=a$ and $y=b$.

ii) Use Gauss Divergence Theorem to evaluate
 $\iiint_S \bar{F} \cdot \hat{n} ds$ where $\bar{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ and S is the surface
 bounded by $x=0, y=0, z=0$ and $2x+2y+z=4$

Course: S.E. (SEM - III) (REV.-2012) (CBSGS) 2015

QP Code: 5106

Correction:

Corrections in Question paper code :5106

Q.1 a) Evaluate $\int_0^{\infty} e^{-t} \left(\frac{\cos 3t - \cos 2t}{t} \right) dt$

Q.2.(a) $\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + y = 3te^{-t}$

Q.3 c i) $\cosh t \int_0^t e^u \sinh u du$

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