

Max. Marks: 100

Duration: 3 Hr.

Instructions:

- (1) Question No.1 is Compulsory.
- (2) Solve any four out of remaining six questions.
- (3) Assume suitable data if necessary.

- Q1(a) State and prove Baye's Theorem. 05
- (b) Suppose X and Y are two random variables, when do we say that X and Y are 05
1) Orthogonal 2) Uncorrelated
- (c) Prove that Poisson process is Markov Process. 05
- (d) Define probability density function. State and prove any two properties of 05
probability density function (p.d.f).
- Q2(a) Box 1 contains 5 white balls and 6 black balls. Box 2 contains 6 white balls and 4 10
black balls. A box is selected at random and then a ball is chosen at random from
the selected box.
- 1) What is the probability that the chosen ball will be a white ball?
 - 2) Given that the ball chosen will be white, what is the probability that it
came from Box1?
- (b) The transmission times X of messages in a communication system obeys the 10
following exponential probability law with parameter K.

$$f(x) = k e^{-\lambda x}, x > 0$$

- 1) Find the value of K.
 - 2) Find the probability density function (p.d.f) of X and cumulative density
function (c.d.f) of X. sketch both functions.
- Q3(a) The joint probability density function of a two dimensional random variable (X,Y) 10
is given by $f_{xy}(x,y) = k e^{-(x+y)}, x > 0, y > 0$
- 1) Find the value of K.
 - 2) Find the marginal probability density functions of X and Y.
 - 3) Check for independence of X and Y.
- (b) If x and y are two independent exponential random variables and $Z = X + Y$, then 10
prove that the probability density function of Z is given by convolution of their
individual density functions.
- Q4(a) Find the moment generating function of Binomial distribution and hence, find its 10
mean and variance.

[TURN OVER

- (b) Let X_1, X_2, \dots be a sequence of random variables. Define
- 1) Convergence almost anywhere
 - 2) Convergence in probability
 - 3) Convergence in mean square sense
 - 4) Convergence in Distribution
- for the above sequence for a random variable X . 10
- Q5(a) State and prove Chapman-Kolmogorov equation. 10
- (b)
- 1) Define Central Limit Theorem and give its significance. 10
 - 2) Define strong law of large numbers.
 - 3) Describe sequence of random variables.
- Q6(a) Explain power spectral density function. State its important properties and prove any one property. 10
- (b) Show that the random process given by
- $$x(t) = A \cos(w_0 t + \theta)$$
- where A and w_0 are constants and θ is uniformly distributed over $(0, 2\pi)$ is Wide Sense stationary (WSS). 10
- Q7(a) Three boys A, B, C play a game of throwing a ball to each other. A always throws the ball to B and B always throw the ball to C, however C is just as likely to throw the ball to B as to A. Find the transition matrix. Show that the process is Markovian. Also classify the states. 10
- (b) Write Short notes on any two : 10
- 1) Ergodic Process
 - 2) Poisson Process
 - 3) Gaussian Process