

Time: 03 hours

Max. Marks: 100

N.B. 1. Question No.1 is compulsory and attempts any four questions from Q.No.2 to 7

2. Figures to the right indicate full marks.

Q.1 a) Determine value of a such that $\vec{F} = (x+3y)\mathbf{i} + (y-2z)\mathbf{j} + (az+x)\mathbf{k}$ is Solenoidal. 05

b) Determine l, m, n and find inverse of matrix A if $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal. 05

c) X is a continuous random variable with probability distribution $f(x) = \frac{x}{6} + k$ when x belongs to $[0,3]$ and is zero elsewhere. Find k . 05

d) Two lines of regression are given by $3x+2y=26$ and $6x+y=31$. Find mean values of x and y and coefficient of correlation between x and y . 05

Q.2 a) Prove that $\vec{F} = (y^2 \cos x + z^3)\mathbf{i} + (2y \sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}$ is Irrotational. Find scalar potential of \vec{F} and work done in moving an object in this field from $(0,1,-1)$ to $(\frac{\pi}{2}, -1, 2)$. 06

b) Find Adj A if $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ 06

c) Seven dice are thrown 729 times. How many times do you expect at least four dice to show number 3 or 5? 08

Q.3 a) A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson variable with mean = 1.5. Calculate proportion of days on which i) neither car is used ii) some demand is refused. 06

b) Show that $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is nondegenerate 06

c) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ find A^{50} . 08

- Q.4 a) 06
 If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ find matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

- b) 06
 Verify Stoke's theorem for $\vec{F} = (x+y)i + (y+z)j - xk$ over surface S of the plane $2x+y+z=2$ in first octant.
- c) 08
 Find values of a and b so that the system $x+y+z=6$; $x+2y+3z=10$; $x+2y+az=b$ have i) no solution ii) unique solution iii) infinite no. of solutions.

- Q.5 a) 06
 Reduce to normal form and find rank if $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$

- b) 06
 Marks scored by students in a college are normally distributed with mean = 65 and variance = 25. If 3 students are selected at random what is the probability that at least one of them has scored more than 75 marks.

- c) 06
 Verify Green's theorem for $\int_C \frac{dx}{y} + \frac{dy}{x}$ where C is the boundary of the region given by $x=1, x=4, y=1, y=\sqrt{x}$.

- Q.6 a) 06
 Evaluate $\iint_S (4xi - 2y^2j + z^2k) \cdot dS$ where S is the region bounded by

$$z=0, z=3, x=1, y^2=4x.$$

- b) 06
 Find eigen values and eigen vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

- c) 08
 Show that matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable. Find the transforming matrix.

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- Q.7 a) Reduce the following quadratic form to normal form using congruent transformations and find rank, signature and value class. 06

$$Q = 2x^2 + y^2 - 3z^2 - 8yz - 4zx + 12xy$$

- b) Calculate coefficient of correlation from the following data 06

X : 12 17 22 27 32

Y : 113 119 117 115 121

- c) Prove that every square matrix can be uniquely expressed as sum of a symmetric and a skew symmetric matrix. 08