QP Code : **1001**

(3 Hours) [Total Marks : 100

N.B. : (1) Question **No.1** is compulsory.
(2) Answer any **four** out of reamaining **six** questions.
(3) Assume any suitable data wherever necessary.

1. a) Express $\frac{(\sqrt{3}-i)^7}{(1+i)^{10}}$ in the form of P+i Q. 3

b) Find y_n if $y = \sin^2\theta \cos^3\theta$ 3

c) Prove that $\bar{a}x(\bar{b} \times \bar{c}) + \bar{b}x(\bar{c} \times \bar{a}) + \bar{c}x(\bar{a} \times \bar{b}) = 0$ 3

d) Prove that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\ldots$ 3

e) If $u = (1-2xy+y^2)^{-1/2}$ prove that 4

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$$

f) Find the point upon the plane $ax + by + cz = P$ at which the function $f = x^2+y^2+z^2$ has a minimum value and find this minimum f. 4

2. a) Find all the roots of $x^{12} - 1 = 0$ and identify the roots which also are the roots of $x^4 - x^2 + 1 = 0$ 6

b) if $x - \frac{1}{x} = 2i\sin\theta$; $y - \frac{1}{y} = 2i\sin\phi$ 6

& $z - \frac{1}{z} = 2i\sin\psi$, prove that $xyz + \frac{1}{xyz} = 2\cos(\theta + \phi + \psi)$

c) If $z = f(u)$ ishomogeneous function in two variables x and y. Prove that 8

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

Hence verify the theorem for the function $u = \log\left(\frac{x^2 + y^2}{xy}\right)$

3. a) Find if LMVT is applicable to the function 6

$$f(x) = x + \frac{1}{x} \text{ on } [1,3]$$

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b) Verify the result 6

$$\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{a} \times \frac{d\vec{b}}{dt} + \frac{d\vec{a}}{dt} \times \vec{b}$$

for $\vec{a} = 5t^2\,\hat{i} + t\,\hat{j} - t^3\hat{k}$

$\vec{b} = \sin t\,\hat{i} - \cos t\,\hat{j} - 0\,k$

c) Prove that $e^{\cos^{-1}x} = e^{\pi/2}\left[1 - x + \frac{x^2}{2} - \frac{x3}{3} + \ldots\ldots\right]$ 8

4. a) If $\sin h(\theta + i\varphi) = \cos\alpha + i\sin\alpha$ prove that $\sin h^4\theta = \cos^2\alpha = \cos^4\varphi$ 6

b) Test the convergence of $\sum_{n=1}^{\infty} \frac{3^n + 4^n}{4^n + 5^n}$ 6

c) If $y = e^{a\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ 8
find $y_n(0)$

5. a) If $y = x\log(x+1)$ prove that $y_n = \frac{(-1)^{n-2}(n-2)(x+n)}{(x+1)^n}$ 6

b) Using L 'Hospitals' rule evaluate $\lim_{x \to 0} \frac{1}{x}(1 - x\cot x)$ 6

c) Find the directional derivative of $f = \frac{1}{(x^2+y^2+z^2)^{1/2}}$ at P(1, 1, 1) in the direcion of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ 8

6. a) If $u = \sin^{-1}(x/y) + \tan^{-1}(y/x)$ 6
Find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

e) A fluid motion is given by $\bar{v} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ 6
show that the motion is irrotational

c) Prove that $\tan^{-1}(e^{i\theta}) = \frac{n\pi}{2} + \pi/4$ 8

$$= \log\tan\left(\pi/4 - \theta/2\right)$$

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7. a) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. 6

   b) Find $[(3.82)^2 + 2(2.1)^3]^{1/5}$ approximately by using the theory of approxiroation. 6

   c) (i) Separate $\left(\sqrt{i}\right)^i$ into real and imaginary parts. 4

   (ii) Find the general value of $\text{Log}\left(1+i\sqrt{3}\right)+\text{Log}\left(1-i\sqrt{3}\right)$ 4
