SE - CO - Sem IV - OLD

(4)

Am IV

23/11/15

QP Code : 1316

Old course(R-2007)

Duration: 3Hrs. **Total marks :100**

Note : **1. Question No.1 is compulsory.**
2. Answer any four from the remaining six questions.

1. a) Verify that the eigen values of a unitary matrix are of unit modulus for

$$A=\frac{1}{6}\begin{bmatrix} -4 & -2-4i \\ 2-4i & -4 \end{bmatrix}$$ [5]

b) State Laplace's equation in polar form and verify it for $u=\left(r+\frac{a^2}{r}\right)\cos\theta$ [5]

c) Evaluate $\int \bar{z}\,dz$ from $z=0$ to $z=4+2i$ along the curve $z=t^2+it$ [5]

d) Determine all basic solutions for the problem

Maxmize $z=x_1-2x_2+4x_3$

Subject to $x_1+2x_2+3x_3=7$

$3x_1+4x_2+6x_3=15$ [5]

2. a) Verify Cayley-Hamilton Theorem for $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find A^{-1} [6]

b) Determine the analytic function whose imaginary part is $e^{-x}(y\sin y+x\cos y)$ [6]

c) Solve the L.P.P using duality

Maximise $z=2x_1+x_2$

Subject to $2x_1-x_2\le 2$

$x_1+x_2\le 4$

$x_1\le 3$

$x_1,x_2\ge 0$ [8]

3. a) If $A=\begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$, prove that $A^{50}=\begin{bmatrix} -149 & -150 \\ 150 & 151 \end{bmatrix}$ [6]

b) Solve the L.P.P by Simplex Method

Maximise $z=4x_1+2x_2+5x_3$

Subject to $12x_1+7x_2+9x_3\le 1260$

$22x_1+18x_2+16x_3\le 19008$

$2x_1+4x_2+3x_3\le 396$

$x_1,x_2,x_3\ge 0$ [6]

c) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ [8]

4. a) If $f(z) = (x^2 + 2axy + by^2) + i\,(cx^2 + 2dxy + y^2)$ is analytic, find a, b, c and d. [6]

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions $1 < |z-3| < 2$ and $|z| < 1$ [6]

c) Find eigen values and eigen vectors of $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ [8]

5. a) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory [6]

b) Find the image of the rectangle bounded by x = 0, y = 0, x = 1, y = 2 under the transformation w = (1 + i)z+(2 - i). Sketch the region. [6]

c) Using the method of Lagrange's multipliers solve the N.L.P.P.

Optimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to $x_1 + x_2 + 3x_3 = 2$;

$5x_1 + 2x_2 + x_3 = 5$

$x_1, x_2, x_3 \geq 0$ [8]

6. a) Find the orthogonal trajectory of the family of curves $e^{-x}\cos y + xy = k$ [6]

b) Solve the N.L.P.P.

Optimize $z = -4x_1 - 2x_2 + x_1^3$

Subject to $x_1 + x_2 \leq 1$, $x_1, x_2 \geq 0$ [6]

c) Evaluate $\int_C \frac{z+6}{z^2-4} dz$; C is the (i) circle $|z| = 1$

(ii) circle $|z-2| = 1$ (iii) circle $|z+2| = 1$ [8]

7. a) Find the Bilinear Transformation that maps the points z = -2, i, 2 into w = 0, i, -i [6]

b) Using residue theorem evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is $|z| = 4$ [6]

c) Use the dual simplex method to solve the following L.P.P

Minimise $z = -3x_1 - 2x_2$

subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$ $x_1, x_2 \geq 0$ [8]