SE - Sem - III - Old - Mechanical.

20/11/15

AM - III

QP Code : 1054

(3 Hours) **[Total Marks : 100**

N.B. 1) Question No. 1 is compulsory.

2) Attempt any four questions out of the remaining six questions.

3) Figures to right indicate full marks.

Q 1. a) Find the Laplace transform of $\frac{\cos 2t \cdot \sin t}{e^t}$ 5

b) Prove that $f(z) = x^2 - y^2 + 2ixy$ is analytic and find $f'(z)$. 5

c) Evaluate $\int_0^{1+2i} z^2\, dz$, along the curve $2x^2 = y$ 5

d) Is the following matrix orthogonal? If not, can it be converted into an orthogonal matrix? If yes how.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$$ 5

Q 2. a) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$ 6

b) Find Non-singular matrices P & Q such that PAQ is in normal form. Also find rank of A where A is

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$ 6

c) Find the Laplace Transform of

i) $e^{2t}(1+t)^2$ ii) $\int_0^t u e^{-3u} \sin 4u\, du$ 8

Q 3. a) Find the values of k for which the equations $x+y+z=1$, $x+2y+3z=k$, $x+5y+9z=k^2$ have solution. Solve them for these values of k. 6

b) Find the analytic function whose real part is $u = x^4 - 6x^2y^2 + y^4$ 6

c) Find inverse Laplace transform of following

i) $\log\left(\frac{s+a}{s+b}\right)$ ii) $\frac{s}{(s+3)(s-4)}$ 8

Q 4. a) Find the image of $|z-2| = 3$ under the transformation $w = 1/z$. 6

b) Determine Eigen values and Eigen vectors for the matrix. $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. 6

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c) Evaluate i) $\int_c \frac{z+2}{(z-3)(z-4)}\,dz$, where c is the circle $|z|=1$ 8

ii) $\int_c \frac{dz}{(z+3)}$,,where c is the circle $|z|=2$

Q 5. a) Express the following matrix as the sum of symmetric and skew-symmetric matrix

$$A=\begin{bmatrix} 1 & 5 & 7 \\ -1 & -2 & -4 \\ 8 & 2 & 13 \end{bmatrix}$$ 6

b) Evaluate $\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt$ 6

c) Obtain Taylor's and Laurent's expansions of $f(z)=\frac{z-1}{z^2-2z-3}$ indicating ROC 8

Q 6. a) Find the bilinear transformation, which maps $z=\infty, i, 0$ on to the points $w=0, i, \infty$. 6

b) Evaluate the following using Residues theorem

$$\int_c \frac{z-1}{(z+1)^2(z-2)}\,dz$$ where c is the circle $|z-i|=2$ 6

c) Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$ using convolution theorem. 8

Q 7. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ 6

b) Verify that $A\,(adj\,A)=|A|I$ for $A=\begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 1 \\ 4 & -5 & 2 \end{bmatrix}$ 6

c) Using Laplace Transform solve the following differential equation with given conditions 8

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0\text{, where } y(0)=0,\ y'(0)=4$$