4. (a) Show that the set of funcitons $\text{Sin}\frac{\pi x}{2L}$,$\text{Sin}\frac{3\pi x}{2L}$,$\text{Sin}\frac{5\pi x}{2L}$................ 6
is orthogonal over (0, L). Construct set of orthonormal functions.

(b) Using Laplace Transform method, solve the differential equation 6
$y'' + 2y' + 5y = e^{-t}\sin t$ given $y(0) = 0$, $y'(0) = 1$

(c) Test for consistency and if consistent solve the equations 8
$x + y + z = 6$
$x - y + 2z = 5$
$3x + y + z = 8$
$2x - 2y + 3z = 7$

5. (a) Using convolution theorem find $L^{-1}\left[\frac{1}{(s-2)^2(s+3)}\right]$ 6

(b) Obtain half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$ and hence find 6
$$\sum\frac{(-1)^n}{(2n+1)^3}$$

(c) Find Fourier Sine Integral of $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x & 1<x<2 \\ 0, & x<2 \end{cases}$ 8

6 (a) Find $L^{1}\left[\frac{s\ e^{-2s}}{s^2+2s+2}\right]$ 6

(b) Prove that the matrix 6
$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$$ is unitary

[TURN OVER

(c) Obtain the fourier series of $f(x)=1+\frac{2x}{\pi}$, $-\pi \le x \le 0$ 8

$=1-\frac{2x}{\pi}$, $0 \le x \le \pi$

Hence deduce that

$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots\ldots$

7. (a) Find Z transform of $f(k) = 2^{|k|}$ 6

(b) Find $L^{-1}\left[\frac{3s+1}{(s+1)(s^2+2)}\right]$ 6

(c) Find the inverse z transform of $f(z)=\frac{z}{(z-2)(z-3)}$ for $|z|>3$ 8

SE-sem-III - OLD - Computer

AM-III

20/11/15

Q.P. Code : **1066**

OLD COURSE

(3 Hours)

[Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions out of the remaining **six** questions.

(3) **Figures** to the **right** indicate **full** marks.

1. (a) If $A = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ Find A^{-1} 5

(b) Find $L\left[\frac{\cos 2t \ \sin t}{e^t}\right]$ 5

(c) Find the z transform of $f(k) = b^k$, $K < 0$ 5

(d) Find the complex form of Fourier series for $f(x) = e^x$ for $-\pi < x < \pi$ 5

2. (a) Find Laplace transform of $f(t) = t\sqrt{1+\sin t}$ 6

(b) If $f(x) = 9-x^2$ for $-3<x<3$. Obtain the fourier series of f(x). 6

(c) Find the rank of the matrix $A = \begin{bmatrix} 25 & 31 & 17 & 43 \\ 75 & 94 & 53 & 132 \\ 75 & 94 & 54 & 134 \\ 25 & 32 & 20 & 48 \end{bmatrix}$ 8

by reducing it to the normal form.

3. (a) Prove that $\int_0^\infty e^{-3t} t \ \sin dt = \frac{3}{50}$ 6

(b) Examine whether the vectors 6

$x_1=[3, 1, 1]$, $x_2=[2, 0, -1]$, $x_3=[4, 2, 1]$ are linearly independent

(c) Find the Fourier series of the function $f(x) = \sqrt{1-\cos x}$ in $[0, 2\pi]$ and hence 8

show that $\sum \frac{1}{(4n^2-1)} = \frac{1}{2}$

[TURN OVER