SE - Sem III - OLD - CO

DSDT 16/12/15

Q.P. Code : 1228

(3 Hours) **[Total Marks : 100**

1) Question No.1 is compulsory.
2) Solve any four questions out of remaining six questions.
3) All questions carry equal marks as indicated by figures to the right.
4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Use mathematical induction to show that (05M)

$1+2+3...+n=n(n+1)/2$ for all natural number values of n.

b) Find the generating function for the following finite sequences (05M)

i) 2,2,2,2,2,2 ii) 1,1,1,1,1,1

c) Let the universal set U={1,2,3,...,10}

Let A={2,4,7,9} B={1,4,6,7,10} and C={3,5,7,9}

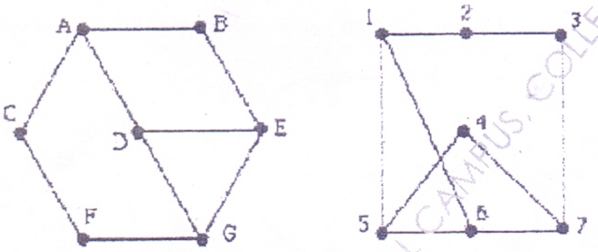
Find 1) AUB 2)A⋂B 3) B⋂C 4) (A⋂B)UC 5)(BUC) ⋂C (05M)

d) Let A = {2, 3, 6, 12, 24, 36, 72}. aRb if and only if a | b

i) Find R ii) Draw Hasse diagram for the same. (05M)

Q.2 a) Let $f : \mathbf{R} \to \mathbf{R}$, where $f(x) = 2x - 1$ and $f^{-1}(x) = (x+1)/2$. (04 M)
Find $(f \circ f^{-1})(x)$

b) Define Isomorphic Graphs. Find if the following two graphs are isomorphic. If yes give their one-to-one correspondence. (08 M)



c) Consider set G = {1,2,3,4,5,6} under multiplication module 7 (08 M)

I. Find the multiplication table of the above.
II. Find the inverse Of 2,3, and 5,6
III. Prove that it is a cyclic group
IV. Find the orders and subgroups generated by {3,4} and {2,3}

Q.3 a) Explain Extended pigeonhole Principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. (04 M)

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b) Show that the (3,6) encoding function $e:B^3 \rightarrow B^6$ defined by (08 M)

e(000)=000000 e(001)=000110

e(010)=010010 e(011)=010100

e(100)=100101 e(101)=100011

e(110)=110111 e(111)=110001 is a group code.

c) Let R be a relation on set S = {a,b,c,d,e} , given as
R = { (a,a) ,(a,d) , (b,b) , (c,d) , (c,e) , (d,a) , (e,b) , (e,e)}
Find transitive closure using Warshall's Algorithm. (08 M)

Q.4 a) Draw Hasse Diagram for the poset A = {1, 2, 3, 6, 12, 24, 36, 72} under the divisibility relation. Is this poset a lattice? Justify (04 M)

b) Consider Z together with binary operations of ⊕ and ⊙ which are defined by (08 M)

$x \oplus y = x + y - 1$

$x \odot y = x + y - xy$

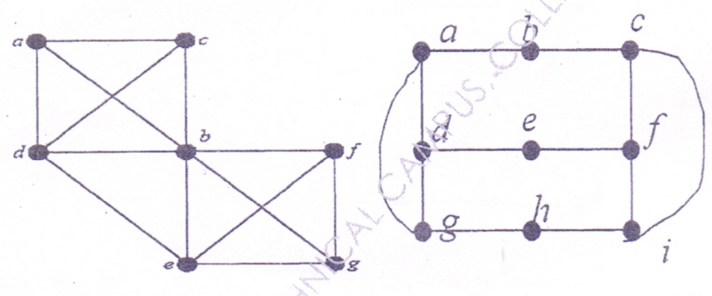
then prove that (Z, O , O) is a ring.

c) Solve $a_r - a_{r-1} - 6a_r = -30$ given $a_0 = 20$, $a_1 = -5$ (08 M)

Q.5 a) Let R be the relation on the set of integers defined by x R y if x-y is divisible by 4. Show that R is an equivalence relation. (04 M)

b) i) Determine Hamiltonian Cycle and path in graph shown in (a)

ii) Determine Euler Cycle and path in graph shown in (b)



(a) (b)

c) A survey of 500 television watchers produced the following information:

285 watch football games, 195 watch hockey games, 115 watch basket ball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch basketball and hockey games. 50 do not watch any three kinds of games. Find: (08 M)

i) How many in the survey watch all 3 kinds of games?

ii) How many watch exactly one of the sports languages?

iii) Draw Venn Diagram showing results of the survey.

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Q.6 a) Prove p∧ (q∨r) and (p∧ q)∨(p∧r) are logically equivalent. (04M)

b) Let H=

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

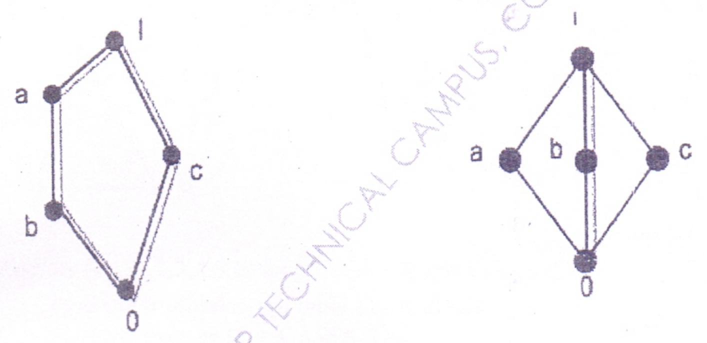
Be a parity check matrix. Determine the group code $e_H: B^3 \rightarrow B^6$ (08 M)

c) Let G be a set of rational numbers other than 1. Let * be an operation on G defined by a*b=a+b-ab for all a,b €G. Prove that (G,*) is a group. (08 M)

Q.7 a) If f:A→B be both one-to-one and onto, then prove that f-1: B→A is also both one-to-one and onto. (04M)

b) Let G be a group. 1) Prove that the identity element e is unique. 2) Prove that each element a in G has only one inverse in G. (08 M)

c) Explain distributive Lattice. Show that following diagrams represent non-distributive lattices. (08M)



(a) (b)

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