SE-ExTC-Sem IV - OLD

AM IV

23/11/15

QP Code : 1331

[OLD COURSE]

(3 Hours) **[Total Marks:100]**

N.B. (1) Question No. **1 is compulsory.**

(2)Attempt **Any FOUR** questions out of the remaining **SIX questions.**

(3) Figures to the right indicate full marks.

1(a) Prove that eigen values of Hermitian matrix are real. [5]

(b) Construct an analytic function whose real part is $x^4 - 6x^2y^2 + y^4$ [5]

(c) A vector field is given by $\overline{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Show that $\overline{F}$ is irrotational and find its scalar potential. [5]

(d) Prove that $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. [5]

2(a) Verify Green's theorem in plane for $\int (xy + y^2)dx + x^2dy$ where C is the close curve of the region bounded by $y = x$ and $y = x^2$. [8]

(b) If A= $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then find A^{50} [6]

(c) Find the image of a circle |z|=2 under the transformation w= z+3+2i.Also draw the figure [6]

3(a) Show that the matrix A= $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalizable . Find the transforming matrix and the diagonal matrix. [8]

(b) Evaluate $\int_A^B (y^2dx + xydy)$ along $x = t^2, y = 2t$ from (1,-2) to B(0,0) [6]

(c) Evaluate $\int \frac{3z^2 + z}{z^2 - 1}dz$ where C is circle |z|=2. [6]

4(a) Reduce the given quadratic form $2x^2 + y^2 - 3z^2 + 12xy - 4xz - 8yz$ to canonical form and find rank and signature , [8]

(b) Evaluate by Residue theorem ,

$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta}d\theta$$ [6]

(c) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\{\frac{3 - x^2}{x^2}\sin x - \frac{3}{x}\cos x\}$ [6]

P.T.O.

5(a) Expand $f(z)=\dfrac{1}{z(z+1)(z-2)}$ when i) $0<|z|<1$ ii) $1<|z|<2$ iii) $|z|>2$ [8]

(b) Using Cayley Hamilton theorem find $A^6-6A^5+9A^4+4A^3-12A^2+2A-I$

where $A=\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix}$; [6]

(c)Find the bilinear transformation which maps the points z= 1,i,-1 on to the points i,0,-i in W plane [6]

6(a) By using Stoke's theorem evaluate $\int_C [(x^2+y^2)\hat{i}+(x^2-y^2)\hat{j}].d\bar{r}$ where C is the boundry of the region enclosed by circles $x^2+y^2=4,\quad x^2+y^2=16$. [8]

(b) Show that the matrix A= $\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$ is non derogatory. [6]

(c)Show that the following function

$$f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}} \quad z\neq 0$$

$$=0 \qquad z=0$$

is not analytic at the origin although Cauchy Riemann equations are satisfied. [6]

7(a) Evaluate $\iint \bar{F}.\bar{ds}$ using Gauss Divergence theorem, where $\bar{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and S is the region bounded by $y^2=4x, x=1, z=0, z=3$ [8]

(b) Show that the map of real axis of the Z plane is a circle under the transformation,, $w=\dfrac{2}{z+i}$ Find its center and radius. [6]

(c)Expand f(x)=1 in (0<x<1) in a series as $1=\sum \dfrac{2}{\lambda_n J_1(\lambda_n)} J_0(\lambda n(x))$ where $\lambda_1,\ldots\lambda_n\ldots$ are positive roots of $J_0(x)=0$ [6]
