SE-Sem-III - OLD - EXTC

AM-III

20/11/15

QP Code : **1078**

(3 Hours) [ Total Marks : 100

N.B. : (1) Question no. **1** is **compulsory**

(2) Attempt **any four** questions out of the remaining **six** questions.

(3) **Figures** to the **right** indicate **full** marks.

1. (a) Show that every square matrix can be uniquely expressed as the sum of Hermitian and Skew- Hermitian matrix. 5

(b) If $\{f(k)\} = 4^k$ fork <0 ; $\{f(k)\} = 3^k$ if $K \geq 0$ then find $Z\{f(k)\}$ 5

(c) Obtain complex form of fourier series for $f(x) = \cosh 3x + \sinh 3x$ in ( -3, 3) 5

(d) Find the Laplace Transformation of the function $\sqrt{1+\sin 2t}$ 5

2. (a) Find Laplace transform of $\dfrac{\cos at - \cos bt}{t}$ 6

(b) Reduce $A=\begin{bmatrix} 1 & -1 & -2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ in to normal form hence find rank of A 6

(c) Find the Fourier expansion for $f(x) = x\sin(x)$ in $(0, 2\pi)$ 8

3. (a) Test for consistency and solve 6
$2x - y + z = 9,\ 3x - y + z = 6,\ 4x - y + 2z = 6,\ -x + y - z = 4$

(b) Find the Half Range Sine series for $f(x) = x \sin x$ in $(0, \pi)$ 6

(c) Find inverse z-transform of $f(z) = \dfrac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)},\ 2 < z < 4$ 8

4. (a) Find z- transform of $2^k\cos(3k+2)$ , $k \geq 0$ 6

(b) Find the Fourier expansion for $f(x) = \sqrt{1-\cos(x)}$ in $(0, 2\pi)$ 6

(c) Solve Using Laplace transform $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 2y = 20 \sin 2t$ where 8
$y(0) = 1, y'(0) = 2$

[ TURN OVER ]

5. (a) Find fourier integral representation for 6

$$f(x) = x\ ,\ 0 < x < a$$
$$= 0, x > a$$

(b) Find the two non-singular matrices P and Q such that PAQ 6

is in normal form where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ and also find its rank.

(c) Obtain fourier series for $f(x) = x(\pi - x)$ $0 < x < \pi$ as a half range cosine series and 8

hence show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

6. (a) Using Laplace transform evaluate $\int_0^{\infty} (1 + 2t - 3t^2 + 4t^3) H(t-2)\,dt$ 6

(b) Show that the set of functions $\sin(2n+1)x$, $n = 0, 1, 2, \ldots$ is orthogonal 6

Over $\left[0, \frac{\pi}{2}\right]$ Hence construct orthonormal set of functions.

(c) Find inverse Laplace transfornl of the following 8

$$\text{(i)}\ \frac{1}{s}\log\left(1 + \frac{1}{s^2}\right) \qquad \text{(ii)}\ \frac{e^{4-3s}}{(s+4)^{5/2}}$$

7. (a) Find inverse Laplace transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ by convolution theorem 6

(b) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ then show that $(I - N)(I + N)^{-1}$ is a unitary matrix. 6

(c) Obtain fourier series for $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & -0 \le x \le \pi \end{cases}$ 8

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

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