

(3 Hours)

[Total Marks:-100]

Instructions:-

1. Question one is compulsory.
2. Attempt any four questions from remaining six questions.
3. Statistical table is permitted.

- Q.1 a) Evaluate $\int f(z)dz$ along the curve $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$,
where $f(z) = x^2 - 2iy$ 5
- b) A continuous random variable has probability density function
 $f(x) = 6(x - x^2)$, $0 \leq x \leq 1$. Find mean and variance. 5
- c) A random sample of 900 items is found to have a mean of 65.3cm. Can it be
regarded a sample of large population whose mean is 66.2cm, and standard
deviation is 5cm at 5% level of significance 5
- d) Evaluate by Green's Theorem $\int_C (x^2 - y)dx + (2y^2 + x)dy$, where 'C' is
boundary of the region defined by $y = x^2$ and $y = 4$. 5
- Q.2 a) A firm produces articles, 0.1 % of which one defective. It packs them in cases
containing 500 articles. If a wholesaler purchases 100 such cases, how many
cases can be expected i) to be free from defective, ii) to have one defective? 6
- b) Verify Stoke's Theorem $\vec{F} = yz \mathbf{i} + zx \mathbf{j} + xy \mathbf{k}$ and C is the boundary of the
circle $x^2 + y^2 + z^2 = 1, z = 0$ 6
- c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$ 8
- Q.3 a) Find the line of regression for the following data and estimate y
corresponding to $x = 15.5$ 6
- | | | | | | | | |
|---|----|----|----|----|----|----|----|
| X | 10 | 12 | 13 | 16 | 17 | 20 | 25 |
| y | 19 | 22 | 24 | 27 | 29 | 33 | 37 |
- b) Evaluate $\int_C \frac{dz}{z^3(z+4)}$, where C is the circle $|z| = 2$ 6

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c) In an industry 200 workers employed for a job were classified according to their performance and training received to test independence of training received and performance. The data is given as follows. 8

Performance	Good	Not Good	total
Trained	100	50	150
Untrained	20	30	50
total	120	80	200

Used χ^2 (Chi-square) test for independence at 5% level of significance and write your conclusion.

Q.4 a) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6

b) Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five? 6

c) Show that $\vec{F} = (2xy+z^3) \mathbf{i} + x^2 \mathbf{j} + 3z^2 \mathbf{k}$ is a conservative field. Find its scalar potential and also work done in moving a particle from (1,-2,1) to (3,1,4) 8

Q.5 a) Find the sum of the residues at singular point of $f(z) = \frac{z}{(z-1)^2(z^2-1)}$. 6

b) A random variable X has the following probability mass function 6

X	-2	3	1
P(X=x)	1/3	1/2	1/6

Find (i) first four raw moments, (ii) first four central moments.

c) Find the coefficient of correlation between height of father and height of son from the following data... 8

Height of father	65	66	67	67	68	69	71	73
Height of son	67	68	64	68	72	70	69	70

Q.6 a) For a normal distribution 30% items are below 45 and 8% items are above 64. Find the mean and variance of normal distribution. 6

b) Fit a Poisson distribution to the following data. 6

X	0	1	2	3	4	5
F	142	156	69	27	5	1

c) Expand $f(z) = \frac{1}{z(z+1)(z-2)}$, 8

- i) within the unit circle about the origin.
- ii) within the annulus region between the concentric circles about the origin having radii 1 and 2 respectively,
- iii) in the exterior of the circle with centre at the origin and radius 2.

Q.7 a) The regression lines of a sample are $x + 6y = 6$, and $3x + 2y = 10$ 6
Find (i) Sample means \bar{x} and \bar{y} ,
(ii) coefficient of correlation between x and y,
Also estimate y when x = 12

b) Using Gauss's Divergence theorem evaluate $\iint_S (ax^2 + by^2 + cz^2) ds$ 6
over the sphere $x^2 + y^2 + z^2 = 1$

c) Nine items of a sample had the following values 8
45, 47, 50, 52, 48, 47, 49, 53, 51
Does the mean of 9 items differ significantly from the assumed population mean 47.5 ?

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