

Duration: 3 Hrs

Maximum marks: 70

Note: All Questions are compulsory

Use of simple calculator is allowed

Figure at right indicate maximum marks

Q1. (a) Attempt any 7 [ 2 marks each]:

[14]

(i) If  $A = \begin{bmatrix} -1 & 2 \\ -3 & 7 \end{bmatrix}$   $B = \begin{bmatrix} 3 & 6 \\ -4 & -13 \end{bmatrix}$  then  $2A + B^T$  is :

- (a)  $\begin{bmatrix} 4 & 7 \\ -19 & 10 \end{bmatrix}$  (b)  $\begin{bmatrix} -6 & -9 \\ -3 & 20 \end{bmatrix}$  (c)  $\begin{bmatrix} 1 & 10 \\ -10 & 1 \end{bmatrix}$  (d)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(ii) If  $\begin{vmatrix} 2 & x \\ 1 & 4 \end{vmatrix} = \begin{vmatrix} x & 6 \\ 2 & 4 \end{vmatrix}$  then the value of x is:

- (a) 1 (b) -2 (c) 4 (d) 2

(iii) The  $N^{th}$  derivative of  $f(x) = \sin^2 x$  is :

- (a)  $-2^{n-1} \cos(2x+n\pi/2)$  (b)  $2 \sin x \cos x$  (c)  $2^{n-1} \cos(2x+n\pi/2)$  (d)  $2^{n-1} \sin(2x+n\pi/2)$

(iv) With respect to Rolle's theorem the value of 'c' corresponding to  $f(x) = x^2 - 4x + 3$  is:

- (a) 1 (b) 2 (c) 3 (d) 4

(v) If  $y = 2x^2$ , then  $\Delta y$  by taking  $h = 1$  is:

- (a)  $2x+1$  (b)  $4x+2$  (c)  $2x^2 - 2x$  (d)  $2x^2 - 1$

(vi) The  $N^{th}$  derivative of  $f(x) = \log(2x+1)$  is:

- (a)  $Y_n = \frac{1}{2(2x+1)}$  (b)  $Y_n = \frac{(-1)^{n-1} (n-1)! 2^n}{(2x+1)^n}$  (c)  $Y_n = \frac{(-1)^n (n)! 2^n}{(2x+1)^n}$   
 (d)  $Y_n = \frac{(-1)^n (n-1)! 2^n}{(2x+1)^n}$

(vii) General solution for the differential equation  $(D^2 - 5D + 6)y = 0$  is:

- (a)  $c_1 e^{-3x} + c_2 e^{2x}$  (b)  $c_1 e^{2x} + c_2 e^{-3x}$  (c)  $c_1 e^{-3x} + c_2 e^{-2x}$  (d)  $c_1 e^{-x} + c_2 e^{-2x}$

(viii) The value of  $\int_{-2}^2 x^3 dx$  is: (a)  $16/3$  (b)  $8/3$  (c) 0 (d)  $3/16$

(ix) The partial derivative of  $Z = 3x^2 + 2xy + xy^2$  with respect to x is:

- (a)  $6x + 2y + 2xy$  (b)  $6x + 2y + y^2$  (c)  $3x + 2y + y^2$  (d)  $2x + xy + xy^2$

(b) Attempt any 1:

[1]

(x) If  $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \\ t & 2 \end{bmatrix}$  is a singular matrix, then the value of t is:

- (a) 1 (b) 2 (c) 4 (d) 6

(xi) The  $N^{th}$  derivative of  $f(x) = \log(2x+1)$  is:

- (a)  $Y_n = \frac{1}{2(2x+1)}$  (b)  $Y_n = \frac{(-1)^{n-1} (n-1)! 2^n}{(2x+1)^n}$  (c)  $Y_n = \frac{(-1)^n (n)! 2^n}{(2x+1)^n}$   
 (d)  $Y_n = \frac{(-1)^n (n-1)! 2^n}{(2x+1)^n}$

[TURN OVER]

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**Q2. (a) Attempt any two ( 4 marks each) [8]**

(i) If  $y = \sin^{-1}x$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

(ii) Using Maclaurin's series, give the expansion of  $f(x) = e^{\sin x}$

(iii) Examine the function  $f(x, y) = x^3 + y^3 - 15xy$  for maxima or minima.

**(b) Attempt any one (3 marks) [3]**

(i) State Lagrange's Mean Value Theorem. Use it to verify for  $f(x) = x^2 - 5x + 6$  in  $[2, 4]$

(ii) Find the  $N^{\text{th}}$  derivative of  $y = e^x \cdot \cos x \cdot \sin 3x$

**Q3. (a) Attempt any two ( 4 marks each) [8]**

(i) Obtain the reduction formula for  $\int_0^{\frac{\pi}{2}} \sin^n x dx$ , hence evaluate  $\int_0^{\frac{\pi}{2}} \sin^9 x dx$

(ii) Find out the area common to two parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$

(iii) Evaluate  $\int \sin^{-1} x dx$

**(b) Attempt any one (3 marks) [3]**

(i) Find the length of the curve  $x = 3t^2$ ,  $y = a(t-3t^2)$  at  $t=0$  to  $t=1$ .

(ii) By using the properties of Definite Integral Evaluate  $I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\tan^3 \theta} d\theta$

**Q4. (a) Attempt any two ( 4 marks each) [8]**

(i) By using the Adjoint method, find the inverse of the matrix  $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

(ii) Find the Eigen values of the matrix  $A = \begin{bmatrix} 4 & 1 & -1 \\ 6 & 3 & -4 \\ 6 & 2 & -3 \end{bmatrix}$

(iii) Verify Cayley Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

**(b) Attempt any one (3 marks) [3]**

(i) Solve  $\begin{vmatrix} 1 & -6 & -x \\ 2 & -3 & x-3 \\ -3 & 2 & x+2 \end{vmatrix} = 0$

**[TURN OVER]**

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Q4.

- (ii) Find the Rank of the matrix  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -3 & 2 & -4 \\ 1 & 2 & -3 & 4 \\ -2 & -1 & -4 & -3 \end{bmatrix}$

Q5. (a) Attempt any two ( 4 marks each)

[8]

- (i) Solve the differential equation  $(x^2 - y^2) \frac{dy}{dx} = 2xy$ , given that  $y=1$  when  $x=1$ .

- (ii) Form the Differential Equation of  $y = A \cos(x^2) + B \sin(x^2)$ .

- (iii) Solve  $(D^2 + 2D + 1)y = 2x + x^2$ .

(b) Attempt any one (3 marks)

[3]

- (i) Form the Differential Equation of  $y = A e^x + B e^{-x}$ .

- (ii) Solve the differential equation:  $x dy - y dx = 0$

Q6. (a) Attempt any two ( 4 marks each)

[8]

- (i) By using Newton's Forward Interpolation formula estimate  $f(3)$  from

$$\begin{array}{cccccc} X: & 2 & 4 & 6 & 8 \\ f(x) : & 13 & 21 & 29 & 37 \end{array}$$

- (ii) By using Simpsons 1/3 rule, calculate the approximate value of  $\int_0^6 \frac{1}{1+x^2} dx$ , by taking 7 equidistant ordinates.

- (iii) Estimate the missing term by using  $E$  and  $\Delta$  from the following:

$$\begin{array}{ccccc} x: & 0 & 1 & 2 & 3 & 4 \\ y: & 1 & 3 & 9 & -- & 81 \end{array}$$

(b) Attempt any one (3 marks)

[3]

(i) Evaluate  $\left(\frac{\Delta^2}{E}\right) \sin x$ .

(ii) Evaluate  $\left(\frac{\Delta^2}{E}\right) e^x$  take  $h=1$ .