

QP Code : 30557

(3 Hours)

[Total Marks : 80

- 1) Question No. 1 is compulsory.
- 2) Attempt any **THREE** of the remaining.
- 3) **Figures** to the **right** indicate **full** marks.

Q 1.A) If  $\int_0^{\infty} e^{-2t} \sin(t + \alpha) \cos(t - \alpha) dt = \frac{1}{4}$ , find  $\alpha$  (5)

B) Find half range Fourier cosine series for  $f(x) = x$ ,  $0 < x < 2$  (5)

C) If  $u(x,y)$  is a harmonic function then prove that  $f(z) = u_x - iu_y$  is an analytic function. (5)

D) Prove that  $\nabla f(r) = f'(r) \frac{\vec{r}}{r}$  (5)

Q.2) A) If  $v = e^x \sin y$ , prove that  $v$  is a harmonic function. Also find the corresponding analytic function. (6)

B) Find Z-transform of  $f(k) = b^k$ ,  $k \geq 0$  (6)

C) Obtain Fourier series for  $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$  in  $(0, 2\pi)$ ,

where  $f(x+2\pi) = f(x)$ . Hence deduce that  $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  (8)

Q.3) A) Find inverse Laplace of  $\frac{(s+3)^2}{(s^2+6s+5)^2}$  using Convolution theorem (6)

B) Show that the set of functions  $\{\sin x, \sin 3x, \sin 5x, \dots\}$  is orthogonal over  $[0, \pi/2]$ . Hence construct orthonormal set of functions (6)

C) Verify Green's theorem for  $\int_C \frac{1}{y} dx + \frac{1}{x} dy$  where  $C$  is the boundary of region defined by  $x = 1$ ,  $x = 4$ ,  $y = 1$  and  $y = \sqrt{x}$  (8)

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- 2 -

Q.4) Find  $Z\{k^2 a^{k-1} U(k-1)\}$  (6)

B) Show that the map of the real axis of the  $z$ -plane is a circle under the transformation  $w = \frac{z}{z+i}$ . Find its centre and the radius. (6)

C) Express the function  $f(x) = \begin{cases} \sin x & |x| < \pi \\ 0 & |x| > \pi \end{cases}$  as Fourier sine Integral. (8)

Q.5) A) Using Gauss Divergence theorem evaluate  $\iint_S \bar{N} \cdot \bar{F} ds$

where  $\bar{F} = x^2 \mathbf{i} + z \mathbf{j} + yz \mathbf{k}$  and  $S$  is the cube bounded by  $x=0, x=1, y=0, y=1, z=0, z=1$  (6)

B) Find inverse  $Z$ -transform of  $F(z) = \frac{z}{(z-1)(z-2)}$ ,  $|z| > 2$  (6)

C) Solve  $(D^2+3D+2)y = e^{-2t} \sin t$ , with  $y(0)=0$  and  $y'(0)=0$  (8)

Q.6) A) Find Fourier expansion of  $f(x) = 4 - x^2$  in the interval  $(0,2)$  (6)

B) A vector field is given by  $\bar{F} = (x^2 + xy^2) \mathbf{i} + (y^2 + x^2y) \mathbf{j}$ . Show that  $\bar{F}$  is irrotational and find its scalar potential. (6)

C) Find (i)  $L^{-1}\left\{\tan^{-1}\left(\frac{a}{s}\right)\right\}$   
 (ii)  $L^{-1}\left(\frac{e^{-\pi s}}{s^2 - 2s + 2}\right)$  (8)

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Course: S.E. (ALL BRANCH)

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Correction:

6, 14, 15, 4, 13

Q1 b) read as " Find half range Fourier cosine series for  $f(x) = x, 0 < x < 2$  "

Date and Time 12/05/2016 03:45 PM

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