(3 Hours)

[Total Marks: 80

N.B. : (1) Question No. one is compulsory.

(2) Answer any three questions from Q.2 to Q.6

(3) Use of stastical Tables permitted.

(4) Figures to the right indicate full marks

1. (a) Find the Eigen values of $A^2 + 2I$, where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -2 & 0 \\ 3 & 5 & 3 \end{bmatrix}$ and I is the Identity

matrix of order 3.   5

(b) Evaluate the line integral $\int_0^{1+i}(x^2 + iy)dz$ along the path $y = x$ 5

(c) If x is a continuous random variable with the probability density function given by

$$f(x) = \begin{cases} k(x - x^3) & 0 \le x \le 1 \\ 0 & otherwise \end{cases},$$

Find i) k ii) the mean of the distribution.   5

(d) Compute Spearman's rank correlation coefficient from the following data

X	18	20	34	52	12
Y	39	23	35	18	46

5

2. (a) Is the following matrix Derogatory? Justify.

$$\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$$

6

(b) Evaluate $\oint_c \dfrac{e^{zz}}{(z-1)^4} dz$ where c is the circle $|z| = 2$ 6

(c) The marks of 1000 students in an Examination are found to be normally

distributed with mean 70 and standard deviation 5, estimate the number of students

whose marks will be i) between 60 and 75 ii) more than 75.   8

[Turn over

3. (a) Solve the following non-linear programming problem using Kuhn-Tucker

    conditions

        Maximize $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

        Subject to $2x_1 + x_2 \le 5$; and $x_1, x_2 \ge 0$ 6

    (b) Fit a Binomial distribution to the following data

x	0	1	2	3	4	5	6
F	5	18	28	12	7	6	4

                                                          6

    (c) Is the following matrix diagonalizable? If yes, find the transforming matrix and

    the diagonal matrix.

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$
        8

4. (a) Solve the following LPP using Simplex method

            Maximize $z = 4x_1 + x_2 + 3x_3 + 5x_4$

            Subject to $-4x_1 + 6x_2 + 5x_3 + 4x_4 \le 20$

                       $-3x_1 - 2x_2 + 4x_3 + x_4 \le 10$

                       $-8x_1 - 3x_2 + 3x_3 + 2x_4 \le 20$

                             $x_1, x_2, x_3, x_4 \ge 0$ 6

    (b) If a random variable X follows the Poisson distribution such that

      $P(X = 1) = 2P(X = 2)$, find the mean, the variance of the distribution and

      $P(X = 3)$ 6

    (c) Expand $f(z) = \dfrac{1}{z(z-2)(z+1)}$ in the regions

      $i)$ $|z| < 1, ii)$ $1 < |z| < 2, iii)$ $|z| > 2$ 8

5. (a) Evaluate using Cauchy's Residue theorem $\oint_c \frac{2z-1}{z(2z+1)(z+2)} dz$ where c is

$|z| = 1.$

6

(b) A certain stimulus administered to each of 12 patients resulted in the following

change in blood pressure:

$5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6$

Can it be concluded that the stimulus will increase the blood pressure (at 5%

level of significance.)?

6

(c) Solve the following LPP using the Dual Simplex method

Maximise $z = -3x_1 - 2x_2$

Subject to $\quad x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \geq 10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

8

6.(a) Find the equations of lines of regression for the following data

x	5	6	7	8	9	10	11
y	11	14	14	15	12	17	16

6

(c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$ using contour integration.

6

(b) In an experiments on pea breeding, the following frequencies of seeds were

obtained

Round and Yellow	Wrinkled and yellow	Round and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9: 3: 3: 1.

Examine the correspondence between theory and experiment using Chi-square Test

8

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