

QP Code : 30797

(3 Hours)

[Total Marks : 80

N.B.:

1. Question no.1 is compulsory
2. Attempt any three questions out of the remaining five.
3. Assume suitable data wherever necessary.

Q 1] Answer the following

[20]

a) Determine if the following system is memoryless, causal, linear, time invariant

$$y(t) = x^2(t-t_0) + 2$$

b) Explain in brief ROC (Region of Convergence) conditions of Laplace transform.

c) Consider two LTI systems connected in series. Their impulse responses are  $h_1[n]$  and  $h_2[n]$  respectively. Find the output of the systems if  $x[n]$  is the input being applied to one of the systems.

$$x[n] = \{1, 2\} \quad h_1[n] = \{1, 0, -1\} \quad h_2[n] = \{2, 1, -1\}$$

d) State and prove time reversal property of Continuous time Fourier Series.

e) Find energy of a causal exponential pulse  $x(t) = e^{-\alpha t} u(t)$   $\alpha > 0$

Q 2] a) A DT signal is given by the following expression. Find its Z transform

[10]

$$x[n] = n \left(-\frac{1}{2}\right)^n u[n] * \left(\frac{1}{4}\right)^{-n} u[-n]$$

b) A CT signal  $x(t)$  is applied to the input of a CT LTI systems with unit impulse response  $h(t)$ . Find out  $y(t)$  using Convolution integral.

[10]

$$x(t) = e^{-at} u(t) \quad a > 0$$

$$h(t) = u(t)$$

Q3] a) Consider a causal LTI system with  $H(j\omega) = \frac{1}{j\omega + 2}$ . For a particular input  $x(t)$ , this system

produces output  $y(t) = e^{-2t} u(t) - e^{-3t} u(t)$ . Find out  $x(t)$  using Fourier Transform. [10]

b) Obtain Inverse Laplace Transform of the function  $X(s) = \frac{3s + 7}{s^2 - s - 12}$  for following ROCs.

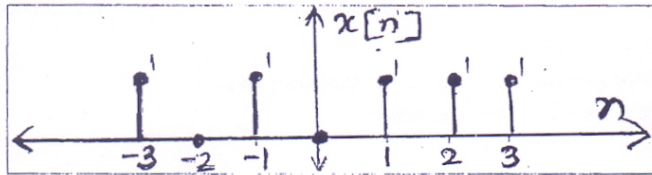
Also comment on the stability and causality of the system for each of the ROC conditions. Support your answer with appropriate sketches of ROCs. [10]

i)  $\text{Re}(s) > 4$

ii)  $\text{Re}(s) < -3$

iii)  $-3 < \text{Re}(s) < 4$

Q. 4] a) A DT signal has been shown. Sketch the following signals. [08]



i)  $x[n-4]$

ii)  $x[4-n]$

iii)  $x[-2n+2]$

iv)  $x[n]u[3-n]$

b) Find out DTFT of the following [06]

i)  $x[n] = \{1, -1, 2, 2\}$

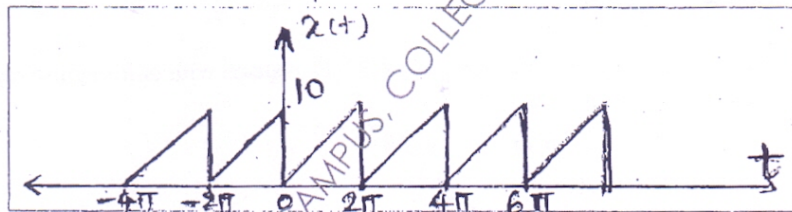
ii)  $x[n] = \sin\left[\frac{\pi n}{2}\right]u[n]$

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c) Determine inverse Z Transform of [06]

$$X(Z) = \frac{3}{(1-Z^{-1})(1+Z^{-1})(1-0.5Z^{-1})(1-0.2Z^{-1})}$$

Q5) a) Find the trigonometric Fourier Series for the waveform shown in the following figure. [10]



b) Determine impulse response of  $h[n]$  for the system described by the second order difference equation. [10]

$$y[n] - 4y[n-1] + 4y[n-2] = x[n] + x[n-1] \text{ when } y[-1] = y[-2] = 0$$

Q6) a) A LTI system has the following transfer function [10]

$$H(Z) = \frac{Z}{(Z - \frac{1}{4})(Z + \frac{1}{4})(Z - \frac{1}{2})}$$

i) Give all possible ROC conditions

ii) Show pole-zero diagram of a system

iii) Find impulse response of system

iv) Comment on the system stability and causality for all possible ROCs

b) Answer any two.

[10]

1) The Impulse response of DT system is given by  $h[n]=\{1,2,3\}$  and the output response is given by  $y[n]=\{1,1,2,-1,3\}$ . Using Z transform, determine  $x[n]$  by long division method.

2) Determine the autocorrelation of the CT signal given by  $x(t)=A \text{ rect}\left(\frac{t}{2}\right)$ .

3) For the following discrete time signals with a fundamental period of  $N=6$ , determine the Fourier Series Coefficients.

a)  $x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right)$

b)  $y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)$

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