19 TE-SEM-V- CBGS- EXTC- RSA

11/5/16

QP Code : 31061

(03 Hours) Total Marks: 80

N.B.:

1) **Question Number 1 is Compulsory**
2) Attempt any **Three** questions from the remaining **Five** questions
3) Assumptions made should be clearly stated.
4) Use of normal table is permitted

1	Answer the following	
a)	For an LTI system with stochastic input prove that autocorrelation of output is given by convolution of cross-correlation (between input-output) and LTI system impulse response.	05
b)	Suppose that a pair of fair dice are tossed and let the RV X denote the sum of the points. Obtain probability mass function and cumulative distribution function for X.	05
c)	If $Z = X + Y$ and if X and Y are independent then derive pdf of Z as convolution of pdf of X and Y.	05
d)	Write a note on the Markov chains.	05
2a)	Define and Explain moment generating function in detail.	05
b)	Let $Z = X/Y$. Determine $fz(z)$	05
c)	The joint cdf of a bivariate r.v. (X, Y) is given by	

$$F_{XY}(x,y) = (1 - e^{-\alpha x})(1 - e^{-\beta y}), x \geq 0, y \geq 0, \alpha, \beta > 0$$
$$= 0 \text{ otherwise.}$$

i)	Find the marginal cdf's of X & Y.	02
ii)	Show that X & Y are independent.	02
iii)	Find $P(X \leq 1, Y \leq 1)$, $P(X \leq 1)$, $P(Y > 1)$ & $P(X > x, Y > y)$	06

3a)	Explain strong law of large numbers and weak law of large numbers.	05
b)	Write a note on birth and death queuing models.	05
c)	A distribution with unknown mean μ has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.90 that the sample mean will be within 0.5 of the population mean.	10
4a)	State and prove Chapman-Kolmogorov equation.	05
b)	State and prove Bayes theorem.	05
c)	(i) State any three properties of power spectral density.	03
	(ii) If the spectral density of a WSS process is given by	07

$$S(w) = b(a - |w|)/a, \quad |w| \leq a$$
$$= 0 \quad , \quad |w| > a$$

Find the autocorrelation function of the process.

[Turn Over

5a) The joint probability function of two discrete r.v.'s X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$ and $f(x, y) = 0$ otherwise. Find $E(X)$, $E(Y)$, $E(XY)$, $E(X^2)$, $E(Y^2)$, $var(X)$, $var(Y)$, $cov(X, Y)$, and ρ. 10

b) Prove that if input LTI system is WSS the output is also WSS. What is ergodic process? 10

6a) The transition probability matrix of Markov Chain is 05

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} \end{array}$$

Find the limiting probabilities.

b) An information source generates symbols at random from a four letter alphabet {a, b, c, d} with probabilities $P(a) = 1/2$, $P(b) = 1/4$ and $P(c) = P(d) = 1/8$. A coding scheme encodes these symbols into binary codes as follows: 05

a	0
b	10
c	110
d	111

Let X be the random variable denoting the length of the code, ie, the number of binary symbols.

i) What is the range of X?

ii) Sketch the cdf $F_X(x)$ of X, and specify the type of X.

iii) Find $P(X \leq 1)$, $P(1 < X \leq 2)$, $P(X > 1)$ & $P(1 \leq X \leq 2)$.

c) Write notes on the following: 10

i) Block diagram and explanation of single & multiple server queuing system

ii) M/M/1/∞ queuing system

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