FE/OLD/SEM-I/AM-I

Q.P. Code : **28505**

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** out of remaining **six** questions.

1. (a) If $\sin\psi = i\tan\theta$, prove that $\cos\theta + i\sin\theta = \tan\left(\frac{\psi}{2}+\frac{\pi}{4}\right)$ 5

(b) If $u = \left(1-2xy+y^2\right)^{-1/2}$, prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$ 5

(c) Prove that $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$ and hence find f(r) if $\nabla f(r) = 3r^5\bar{r}$ 5

(d) If f (x) and g(x) are respectively $\sqrt{x}$ and $\frac{1}{\sqrt{x}}$ then prove that c of Cauchy's Mean value Theorem is the Geometric mean between a and b, $a > 0$, $b > 0$. 5

2. (a) Show that $32\sin^4\theta\cos^2\theta = \cos6\theta - 2\cos4\theta - \cos2\theta + 2$. 6

(b) Find the directional derivative of $f(x,y,z) = 4e^{2x-y+z}$ at the point (1, 1, -1) in the direction toward the point (-3, 5, 6) 7

(c) If $u = A\,e^{-gx}\sin(nt-gx)$ satisfies the equation $\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2}$, prove that $n = 2g^2\mu$ 7

3. (a) Find the equation whose roots are $2\cos\frac{\pi}{7}$, $2\cos\frac{3\pi}{7}$, $2\cos\frac{5\pi}{7}$ 6

(b) If $z = f_1(x+ct) + f_2(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{dx^2}$ 7

(c) If a vector field is given by $\bar{F} = \left(x^2+xy^2\right)\bar{i} + \left(y^2+x^2y\right)\bar{j}$. Show that $\bar{F}$ is irrotational and find its scalar potential. 7

4. (a) Test for convergence of the series $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^2+\ldots\ldots\ldots(x>0)$ 6

(b) Find the values of a, b, c so that $\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$ 7

(c) If $y = \frac{\log x}{x}$ prove that $y_5 = \frac{5!}{x^6}\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}-\log x\right]$ 7

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5 (a) Find the stationary values of $3x^2 - y^2 + x^3$ 6

(b) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that $\cos^4\theta = \sin^2\alpha = \sinh^4\phi$ 7

(c) Prove that $\dfrac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \dfrac{b-a}{\sqrt{1-b^2}}$ 7

6. (a) Prove that $\cos\left[i\log\left(\dfrac{a-ib}{a+ib}\right)\right] = \dfrac{a^2-b^2}{a^2+b^2}$ 6

(b) Expand $\log(1+x+x^2+x^3)$ upto x^8 7

(c) If $x = \tan(\log y)$, prove that $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$ 7

7. (a) If $i^{\alpha+i\beta} = \alpha + i\beta$, prove that $\alpha^2+\beta^2 = e^{-(4n+1)\beta\pi}$ where n is any positive integer. 6

(b) If $u = \tan^{-1}\left[\dfrac{x^3+y^3}{2x+3y}\right]$, prove that 7

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$$

(c) If $y^{1/m} - y^{-1/m} = 2x$, prove that 7

$$y = 1 + mx + \frac{m^2}{2!}x^2 + \frac{m^2(m^2+1)}{3!}x^3 + \ldots$$
