

QP Code : 28674

Duration (03) hours

Marks 100

N.B. (i) Question no. ONE is compulsory.

(ii) Attempt any FOUR questions out of the remaining six questions.

(iii) Figures to right indicate full marks.

Q.1(a) Find Laplace transform of $\cos^5(t)$ (b) Find z-transform of $\{a^{|k|}\}$ (05)(c) Obtain complex form of fourier series for $f(x) = \cosh(ax)$ in $(-l, l)$ (05)

(d) Show that every square matrix can be uniquely expressed as the sum of a Hermitian and skew-Hermitian matrix. (05)

Q.2(a) Find Laplace transform of $(\int_0^t e^{-4u} \cos^2(u) du)$ (06)(b) If $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal, find l, m, n , also find A^{-1} . (06)(c) Find the Fourier expansion for $f(x) = \sqrt{1 - \cos(x)}$ in $(0, 2\pi)$ (08)

Q.3(a) Test for consistency and solve

$$2x - 3y + 5z = 1, \quad 3x + y - z = 2, \quad x + 4y - 6z = 1 \quad (06)$$

(b) Find the Fourier expansion for $f(x) = 9 - x^2$ in $(-3, 3)$ (06)(c) Find inverse z-transform of $f(z) = \frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$, $3 < z < 4$ (08)Q.4(a) Solve Using Laplace transform $\frac{d^2y}{dt^2} + 9y = 18t$ where (06)

$$y(0) = 0, y(\pi/2) = 0$$

(b) Find the Fourier expansion for $f(x) = 2x - x^2$ in $(0, 3)$ (06)(c) Find z-transform of $c^k \cosh(ak)$, $k \geq 0$ (08)

[TURN OVER]

Q.5 (a) Find fourier integral representation for (06)

$$f(x) = f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$

(b) Find the two non-singular matrices P and Q such that (06)

PAQ is in normal form where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ and also find its rank.

(c) Obtain fourier series for $f(x) = lx - x^2$ $0 < x < l$ as a half range cosine series and sine series. (08)

Q.6(a) Using Laplace transform evaluate (06)

$$\int_0^{\infty} e^{-t}(1 + 2t - t^2 + t^3)H(t - 1)dt$$

(b) Find inverse Laplace transform of $\cot^{-1}\left(\frac{2}{s^2}\right)$ (06)

(c) Find inverse Laplace transform of the following (08)

(i) $\frac{1}{s} \tan^{-1}\left(\frac{a}{s}\right)$ (ii) $\frac{se^{-\pi s}}{s^2 + 2s + 2}$

Q.7 (a) Find inverse Laplace transform of $\frac{1}{(s^2 + 4s + 13)^2}$ by convolution theorem (06)

(b) Show that the matrix $A = \frac{1}{2} \begin{bmatrix} \sqrt{2} & i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is a unitary matrix and

Hence find A^{-1}

(c) Obtain fourier series for $f(x) = x \sin(x)$ in $(-\pi, \pi)$ (08)

Hence deduce that $\frac{\pi-2}{4} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$