13/05/16

SE (SEM-IV) old

EXTC - AM-IV

QP Code :544700

5(a) Expand $f(z)=\frac{1}{z^2(z-1)(z+2)}$ about z=0 when i) $|z|<1$ ii) $1<|z|<2$ iii) $|z|>2$ [8]

(b) Using Cayley Hamilton theorem find $A^6-6A^5+9A^4+4A^3-12A^2+2A-I$

where $A=\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix}$ [6]

(c)Find the bilinear transformation which maps the points z= 1,i,-1 from the Z plane on to the points 0,1,∞ in W plane [6]

6(a) By using Stoke's theorem evaluate $\int_C [(x^2+y^2)\hat{i}+(x^2-y^2)\hat{j}].dr$ where C is the boundry of the region enclosed by circles $x^2+y^2=4,\ x^2+y^2=16$. [8]

(b) Show that the matrix A=$\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 3 & 4 & 5\end{bmatrix}$ is non derogatory. [6]

(c)Show that the following function

$$f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}} \quad z\neq 0$$

$$=0 \quad z=0$$ is not analytic at the origin although Cauchy Riemann equations are satisfied. [6]

7(a) Evaluate $\iint_S \bar{F}.d\bar{s}$ using Gauss Divergence theorem, where $\bar{F}=4x\bar{i}-2y^2\hat{j}+z^2\hat{k}$ and S is the region bounded by $y^2=4x, x=1, z=0, z=3$ [8]

(b) Find the image of a circle |z|=2 under the transformation w= z+3+2i.Also draw the figure [6]

(c)Expand f(x)=1 in (0<x<1) in a series as $1=\sum \frac{2}{\lambda_n J_1(\lambda_n)} J_0(\lambda_n x)$ where $\lambda_1,\ldots\lambda_n\ldots$ are positive roots of $J_0(x)=0$ [6]

QP Code :544700

[OLD COURSE]

(3 Hours) [Total Marks:100]

N.B. (1) Question No. **1 is compulsory.**

(2)Attempt **Any FOUR** questions out of the remaining **SIX questions.**

(3) Figures to the right indicate full marks.

1(a) Prove that eigen values of Hermitian matrix are real. [5]

(b) Construct an analytic function whose real part is $x^4 - 6x^2y^2 + y^4$ [5]

(c) A vector field is given by $\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\hat{j} + (xy\cos z + y^2)\hat{k}$.

Show that $\bar{F}$ is irrotational and hence find its scalar potential. [5]

(d) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. [5]

2(a) Verify Green's theorem in plane for $\int_C (xy + y^2)dx + x^2dy$ where C is the close curve of the region bounded by $y = x$ and $y = x^2$. [8]

(b) If A=$\begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that 3 tanA=A tan3 [6]

(c) Find the image of the region bounded by x=0,x=2,y=0,y=2 in the Z plane under the transformation w=(1+i)z [6]

3(a) Show that the matrix A=$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. [8]

(b) Evaluate $\int z\,dz$ along $x = t^2, y = t$ from O(0,0) to B(4,2) [6]

(c) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$ where C is circle |z|=3. [6]

4(a) Reduce the given quadratic form $2x^2 + y^2 - 3z^2 + 12xy - 4xz - 8yz$ to canonical form and find rank and signature. [8]

(b) Evaluate by Residue theorem,

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 + 4\cos\theta}d\theta$$ [6]

(c) Prove that $J_{\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3 - x^2}{x^2}\sin x - \frac{3}{x}\cos x\right\}$ [6]

[TURN OVER]