

13/05/16

S.E. (SEM - IV) Old

EE - EM - IV

(Old Course)

Q.P. Code : 543002

(3 hours)

[ Total Marks : 100

- N.B. 1. Question No. 1 is compulsory. Attempt any FOUR questions from Question No 2 to Question No 7.
2. Figures to the right indicate full marks.
3. Use of statistical tables is permitted.

- Q.1 a) If  $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$  find eigen values and vectors of  $4A^{-1} + 3A + 2I$ . 05
- b) Find values of a,b and c if  $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$  is irrotational. 05
- c) Obtain mean and variance of Binomial distribution. 05
- d) Two lines of regression are given by  $6y = 5x + 90; 15x = 8y + 130$  Find  $\bar{x}, \bar{y}$  and coefficient of correlation. 05

- Q.2 a) Reduce to normal form and find rank of  $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$  06
- b) Prove that vector field  $\vec{F} = (y+z)i + (z+x)j + (x+y)k$  is irrotational and find its scalar potential. 06
- c)  $X$  is a continuous random variable with probability density function  $f(x) = kx(1-x); 0 \leq x \leq 1$ . Find k, mean value and variance. 08

- Q.3 a) Find eigen values and eiger. vectors of  $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ . 06
- b) Using Green's theorem evaluate  $\int_C \frac{1}{y} dx + \frac{1}{x} dy$  where C is the boundary of the region defined by  $x = 1, x = 4, y = 1, y = \sqrt{x}$ . 06
- c) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. Find mean and standard deviation of the distribution. 08

- Q.4 a) If  $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$  prove that  $3 \tan A = A \tan 3$ . 06
- b) In a bombing attack, there is 50% chance that a bomb dropped hits the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to have 99% or more chance of destroying the target? 06

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- c) Using Stoke's theorem evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = yi + zj + xk$  08  
 where C is the boundary of the region given by  $x^2 + y^2 = 1 - z$  and  $z > 0$ .
- Q.5 a) Using congruent transformations reduce 06  
 $Q = 3x^2 + 2y^2 + z^2 + 4xy - 2xz + 6yz$  to diagonal form. Find rank, index, signature and value class of Q.
- b) Test whether the matrix  $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$  is diagonalizable. Justify 06  
 your answer.
- c) What is rank correlation? Obtain Spearman's rank correlation coefficient for the following data. 08  
 X: 10 12 18 18 15 40  
 Y: 12 18 25 25 50 25
- Q.6 a) Test whether  $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$  is derogatory or not. Justify your 06  
 answer.
- b) Examine the following system for consistency and solve if it is consistent: 06  
 $2x - 3y + 7z = 5; 3x + y - 3z = 13; 2x + 19y - 47z = 32$ .
- c) Use Gauss-divergence theorem to evaluate  $\iiint_S \vec{F} \cdot \vec{N} ds$  where 08  
 $\vec{F} = 4xi + 3yj - 2zk$  and S is the surface bounded by  $x = 0, y = 0, z = 0, 2x + 2y + z = 4$ .
- Q.7 a) Prove that every square matrix A can be uniquely expressed as sum of a hermitian and a skew hermitian matrix. 06
- b) Find work done in moving a particle from A(1,0,1) to B(2,1,2) along straight line AB in the force field  $\vec{F} = x^2i + (x - y)j + (y + z)k$ . 06
- c) When do we use Poisson distribution? Fit a Poisson distribution to the following data. 08  
 X : 0 1 2 3 4  
 f : 123 59 14 3 1