13/05/16

SE (SEM - IV) old

CO - AM - IV

Q.P. Code : 540102

(Old Course)

Duration: 3Hrs.

Total marks :100

Note :

1. Question No.1 is compulsory.
2. Answer any four from the remaining six questions.

1. a) Verify that the eigen values of an orthogonal matrix are of unit modulus for

$$A = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$$ [5]

b) If $u = -r^3 \sin 3\theta$ find the analytic function f(z) whose real part is u [5]

c) Evaluate $\int f(z)\,dz$ along the parabola $y=2x^2$ from $z=0$ and $z=3+18i$

where $f(z) = x^2 - 2ixy$. [5]

d) Determine all basic solutions for the problem

Maxmize $z = x_1 + 3x_2 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$ [5]

2. a) Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and evaluate $2A^4 - 5A^3 - 7A + 6I$ [6]

b) Determine the analytic function whose imaginary part is $\tan^{-1}\left(\frac{y}{x}\right)$. [6]

c) Solve the L.P.P using duality

Minimise $z = 4x_1 + 3x_2 + 6x_3$

Subject to $x_1 + x_2 \geq 2$

$x_2 + x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$. [8]

3. a) If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find e^{At} [6]

b) Solve the L.P.P by Simplex Method

Maximise $z = 100x_1 + 50x_2 + 50x_3$

Subject to $4x_1 + 3x_2 + 2x_3 \leq 10$

$3x_1 + 8x_2 + x_3 \leq 8$

$4x_1 + 2x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$ [6]

c) Evaluate $\int_0^{2\pi} \frac{d\theta}{25 - 16\cos^2\theta}$ [8]

4. a) Show that $w = \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}$ is an analytic function and find $\frac{dw}{dz}$ in terms of z. [6]

b) Obtain two distinct Laurent's series expansion of $f(z) = \frac{2z-3}{z^2-4z+3}$ in powers of $(z-4)$ indicating the region of convergence [6]

c) Find eigen values and eigen vectors of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ [8]

5. a) Show that $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ is derogatory [6]

b) Find the image of the circle $|z| = 2$ under the transformation $w = z+3+2i$. Sketch the region. [6]

c) Using the method of Lagrange's multipliers solve the N.L.P.P.

Optimize $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

Subject to $x_1 + x_2 + x_3 = 15$,

$2x_1 - x_2 + 2x_3 = 20$

$x_1, x_2, x_3 \geq 0$ [8]

6. a) Find the orthogonal trajectory of the family of curves $2x - x^3 + 3xy^2 = a$ [6]

b) Solve the N.L.P.P. using Kuhn-Tucker conditions

Maximise $z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

Subject to $2x_1 + 5x_2 \leq 98$,

$x_1, x_2 \geq 0$ [6]

c) Evaluate using Residue Theorem

(i) $\int_C \frac{dz}{4z^2+1}$; C is the circle $|z| = 1$

(ii) $\int_C \frac{(z+4)^2}{z^4+5z^3+6z^2}\,dz$ C is the circle $|z| = 1$ [8]

7. a) Find the Bilinear Transformation that maps the points $z = 2, 1, 0$ onto $w = 1, 0, i$ [6]

b) Evaluate $\int_C \frac{dz}{(z^3-1)^2}$ where C is $|z-1| = 1$ [6]

c) Use the dual simplex method to solve the following L.P.P

Minimise $z = 3x_1 + 2x_2 + x_3 + 4x_4$

subject to $2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$

$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$

$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$

$x_1, x_2, x_3, x_4 \geq 0$ [8]
