

## Kinematic and Dynamic Modeling of a Quad copter following a trajectory using MATLAB

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**Abstract** - Our task comprises of plotting different yields of kinematic and element movement concerning time. This yield is acquired by making a quad copter to follow an endorsed direction in the MATLAB ra13 reenactment stage. The focal point of the mass of the quad copter is kept up at the focal point of the quad copter and the accompanying administering kinematic and element mathematical statements are readied. These comparisons inferred are utilized for the beginning of the kinematic and element displaying of the quad copter made to follow the direction. The yields of the kinematic and element demonstrating are made the data for the amendment of blunder info and tuning of the PID control framework. This shut capacity PID control framework is utilized for the adjustment of the quad copter while taking after a given direction and advance the yield of the kinematic and element demonstrating process for tuning reason for the control.

**Index Terms** - List key record terms here. No female horse than 5.

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### I. INTRODUCTION

A helicopter is a flying vehicle which uses rapidly spinning rotors to push air downwards, thus creating a thrust force keeping the helicopter aloft. Conventional helicopters have two rotors. These can be arranged as two coplanar rotors both providing upwards thrust, but spinning in opposite directions (in order to balance the torques exerted upon the body of the helicopter). The two rotors can also be arranged with one main rotor providing thrust and a smaller side rotor oriented laterally and counteracting the torque produced by the main rotor. However, these configurations require complicated machinery to control the direction of motion; a swash plate is used to change the angle of attack on the main rotors. In order to produce a torque the angle of attack is modulated by the location of each rotor in each stroke, such that more thrust is produced on one side of the rotor plane than the other. The complicated design of the rotor and swash plate mechanism presents some problems, increasing construction costs and design complexity. A quad rotor helicopter (quad copter) is a helicopter which has four equally spaced rotors, usually arranged at the corners of a square body. With four independent rotors, the need for a swash plate mechanism is alleviated. The swash plate mechanism was needed to allow the helicopter to utilize more degrees of freedom, but the same level of control can be obtained by adding two more rotors. The development of quad copters has stalled until very recently, because controlling four independent rotors has proven to be incredibly difficult and impossible without electronic assistance. The decreasing cost of modern microprocessors has made electronic and even completely autonomous control of quad copters feasible for commercial,

military, and even hobbyist purposes. Quad copter control is a fundamentally difficult and interesting problem. With six degrees of freedom (three translational and three rotational) and only four independent inputs (rotor speeds), quad copters are severely under actuated. In order to achieve six degrees of freedom, rotational and translational motions are coupled. The resulting dynamics are highly nonlinear, especially after accounting for the complicated aerodynamic effects. Finally, unlike ground vehicles, helicopters have very little friction to prevent their motion, so they must provide their own damping in order to stop moving and remain stable. Together, these factors create a very interesting control problem. We will present a very simplified model of quad copter dynamics and design controllers for our dynamics to follow a designated trajectory. We will then test our controllers with a numerical simulation.

Unmanned air vehicles (UAVs) are self-propelled aerial robots. They can be equipped with various instruments and payloads, making them capable of performing various civilian or military tasks. Among existing small UAVs, we find quad rotors which are Vertical Take-Off and Landing (VTOL) four rotor helicopters. They are controlled simply by changing the rotation speed of the four rotors. The front and rear rotors (2, 4) rotate in a clockwise direction while the left and right rotors (1, 3) rotate in a counter-clockwise direction to balance the torque created by the spinning rotors. The up/down motion is achieved by increasing/decreasing the rotors speed while maintaining an equal individual speed. The forward/backward, left/right motions are achieved through a differential control strategy of rotors speed. Thanks to this configuration, quad rotors are able to hover, takeoff, and land in small areas and

enable them to perform tasks that fixed-wing craft are unable to do.

## II. PROBLEM DEFINITION

- To carry out mathematical analysis of a quad copter following various forms of path
- To derive generalized governing kinematic and dynamic equations of the quad copter assuming its center of gravity to be at the centre of the quad copter
- To carry out body frame analysis on various coordinate scales:-
  - Earth-frame(E-frame) analysis:- in this form of analysis the earth surface is taken as the reference and the x cordite is assumed to be the north pole , the y coordinate is assumed to be the east and the z coordinate is assumed towards the earth. This also known as absolute or generalized coordinates system.
  - Body -frame analysis (B-frame):-in this form of analysis the x, y & z coordinate is set as per the orientation and is more convenient way of rigid body motion analysis.
- To carry rigid body motion analysis using MATLAB ra13 for simulation of yaw motion.
- to carry trajectory analysis of quad copter tracing a path using MATLAB ra13 for four conditions:-
  - hovering
  - pitching
  - rolling
  - yawing

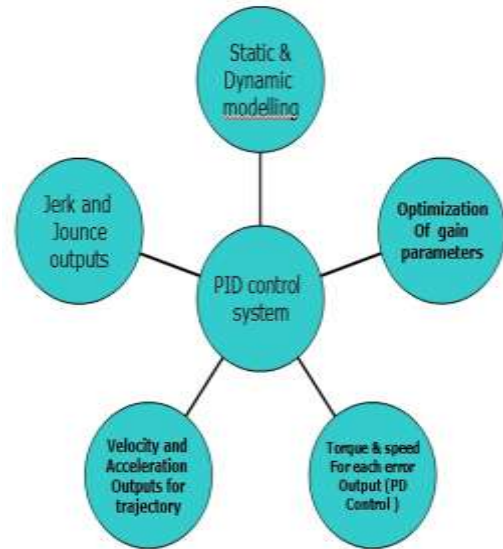
This all orientations are conditional for the simulation of positional error of quad copter in a with respect to a time frame in seconds in 1 dimensional and 3 dimensional mode of results.

- To evaluate the error with respect to time over the output error function from the kinematic and dynamic modeling for a PID control system.
- To set adequate gain parameters for a PID control system so as to optimize stabilization parameter of a quad copter and prevent overshoot of the system by adequate tuning of the PID gain parameters.

## III. OBJECTIVE OF WORK

- design and develop kinematic and dynamic simulation models for a quad copter moving over a trajectory
- study the position and time variables of the motion of quad copter in 1 dimensional and 3 dimensional time variant scale
- study and obtain the change in position , velocity and acceleration of the quad copter motion with respect to the variant time scale
- Study and obtain the results of third and fourth order derivative motions such as jerk and jounce and its error cost function analysis (J).

- Enable PID control system with automatic tuning to avoid steady state error by setting adequate and proper gain parameters viz:  $k_p$  ,  $k_d$  ,  $k_i$  so as to avoid overshoot to large error function value.



## IV. METHODOLOGY

The following steps will be adopted to prepare kinematic and dynamic model and get output results on MATLAB.

- study various thrust equations and propelling conditions for:-
  - hovering
  - rolling
  - pitching
  - yawing
- Study, analyses and evaluate the governing kinematic and dynamic equations for computation of position and time variable of a moving quad copter over a trajectory.
- Computing the above results on MATLAB ra13 simulation platform and obtain respective results of position versus time analysis by writing algorithms and commands on the simulation platform
- Computing the inertia and body frame equations of rigid boggy dynamics on MATLAB ra13 simulation platform.
- Comparing the results and obtain the error function as result of deviation of quad copter displacement with the reference trajectory.
- Optimizing the error function in the form "cost-function analysis" to enable the PID control system with automatic Tuning
- Decide gain parameters by testing against the steady state error for automatic tuning of PID Control system.

- Take the output of the kinematic and dynamic modeling and make it as input for evaluating the error function with respect to time to nullify the steady state error.
- Develop and build SIMULINK\* model of kinematic and dynamic model and control inputs to the PID control system on MATLAB ra13 simulation platform.

#### V. RESULTS OBTAINED

- 1D and 3D time frame analysis of variation of position of quad copter with the reference trajectory for:-
  - hovering
  - rolling
  - pitching
  - yawing
- Rigid body analysis for yawing motion of quad copter
- Thrust calculations for respective motor torque each for hovering , rolling , pitching ,and yawing motions of quad copter moving over a trajectory.
- Adequate input coupling of modeling data for the optimization of gain parameters for proper automatic tuning of PID control system to nullify steady state error in the input data.
- Full-fledged SIMULINK\* model of the kinematic and dynamic modeling and simulation , control inputs and PID control for input data and its automatic tuning for gain parameters on the platform of MATLAB ra13 simulation.

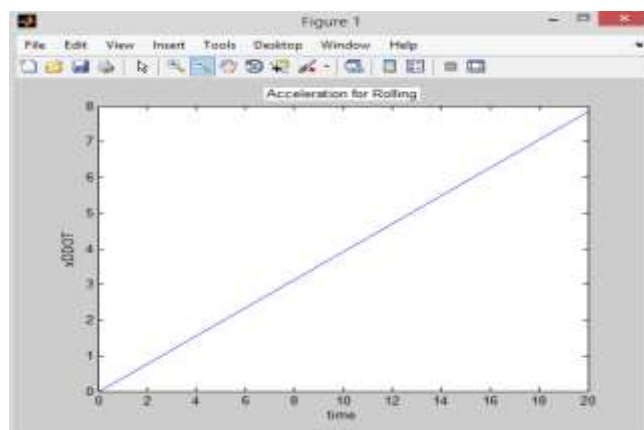
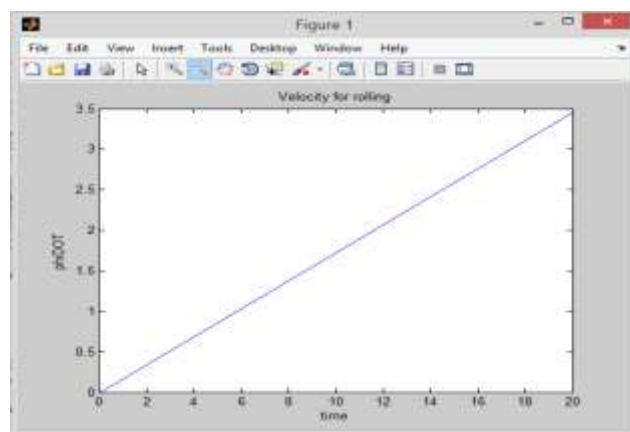
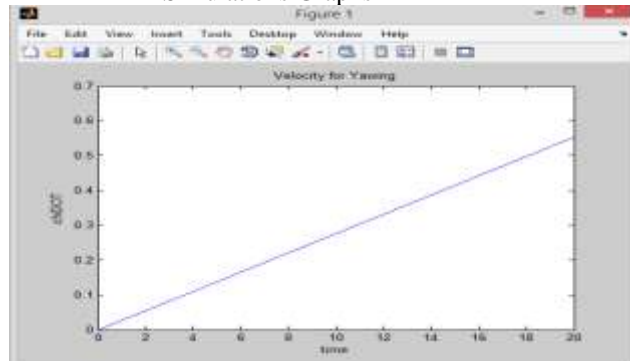
#### VI. VALIDATION

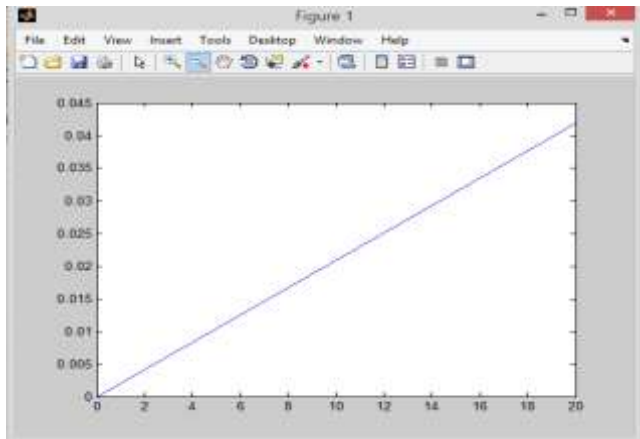
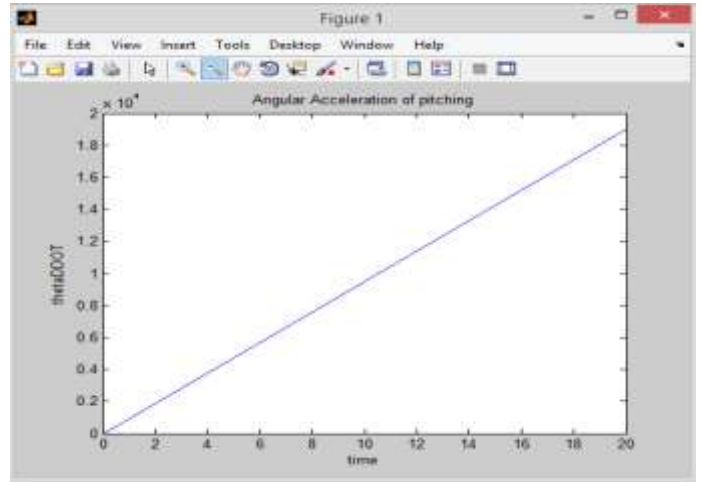
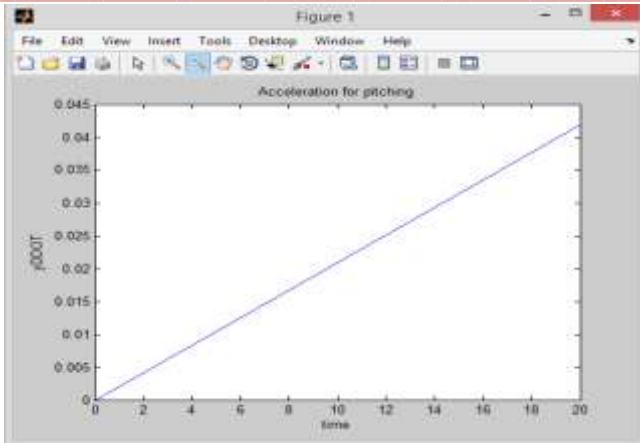
Validation accounts for feasibility of the work criteria set for the project and expected outcomes of the project our measure of validation of:

- The experimentation and computational results almost can be equalized.
- Since because the computational platform is in real time analysis and multi scale modeling done on MATLAB ra13 simulation platform the results can be easily aggregated to the actual ones.
- The tuning of PID control system through automatic tuning system being done by analyzing and optimizing gain parameters may show a better version of results as far as stabilizing and precise operation and motion of a quad copter is concerned on a given trajectory.
- Usage of B-frame analysis and "exact differential equations" for implementation of kinematic and dynamic modeling ensures the interpolation of solutions for the equations rather than approximation.

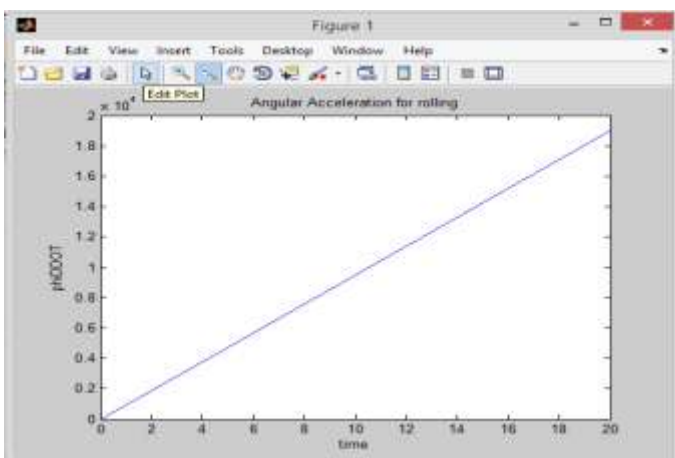
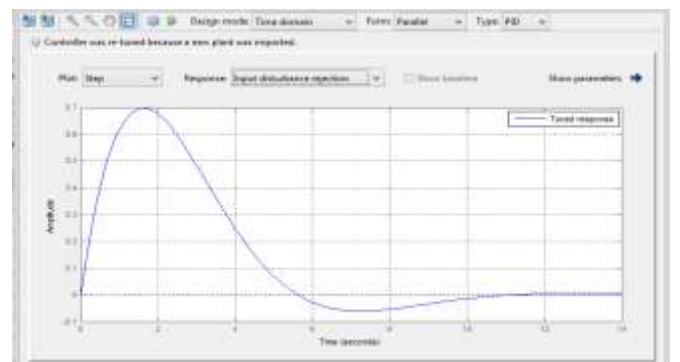
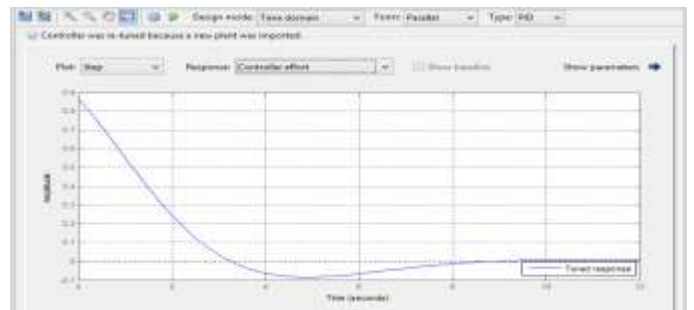
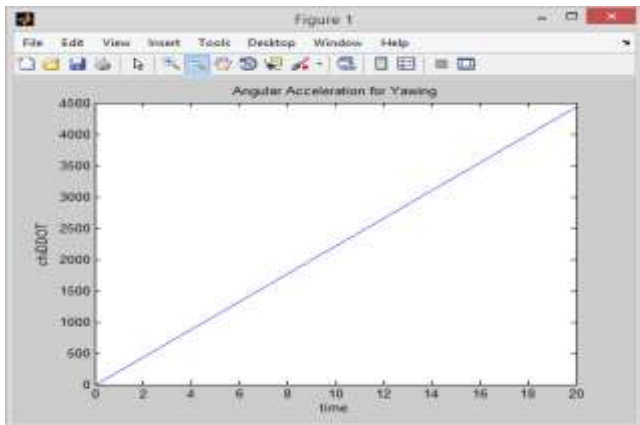
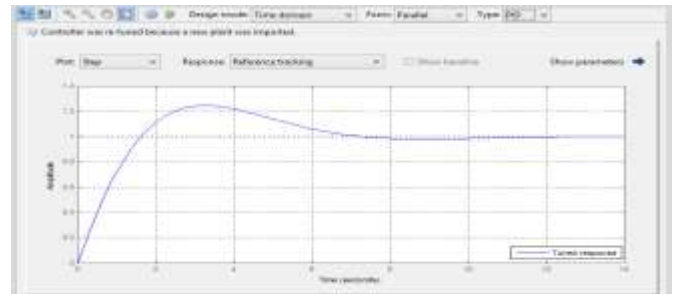
- Developing a SIMULINK\* model ensures interpretation of results and control inputs and improves feasibility for changing output variables with varying inputs instead referring the entire algorithm.

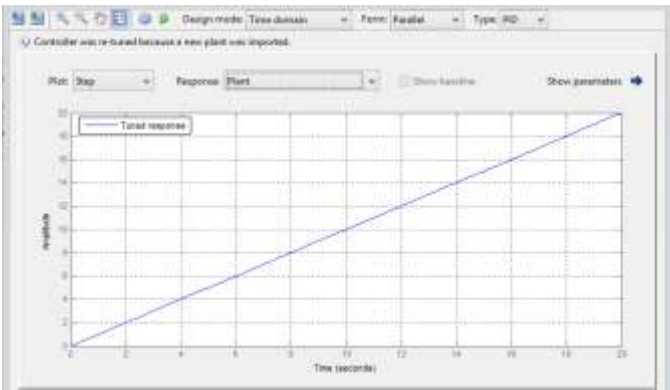
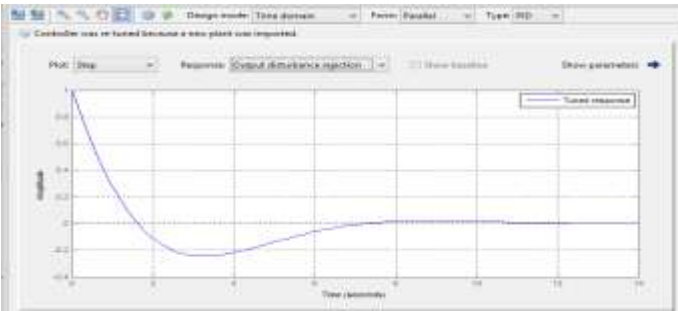
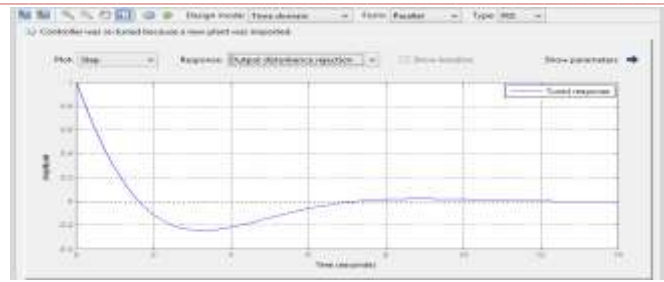
#### VII. MATLAB OUTPUTS Simulations Graphs



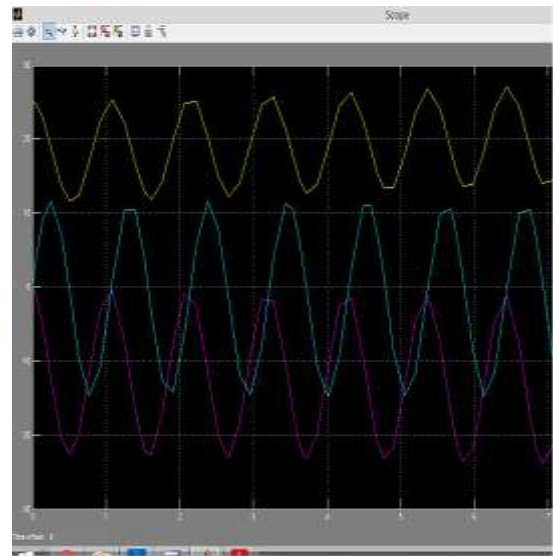
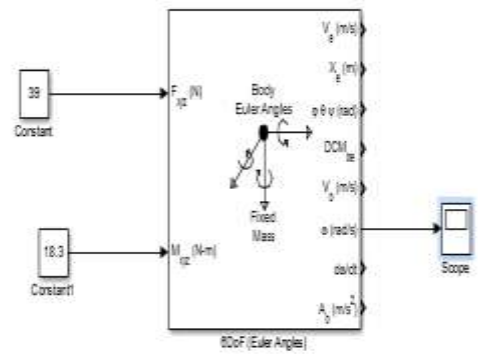


PID Plant model analysis:-

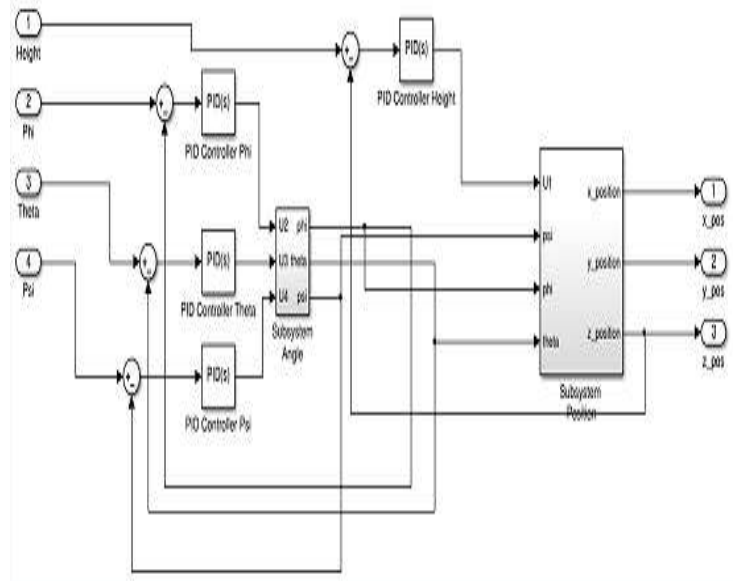
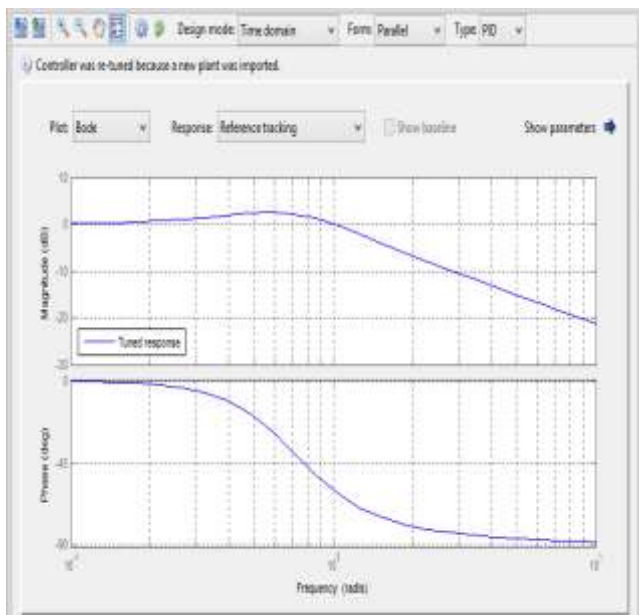




SIMULINK Model result using Forward Kinematics:-



Bode Plot Stability Analysis for phase margin and gain margin:-



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