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Principles of Analog Instruments

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AIKTC
SCHOOL OF ENGINEERING

I N D E X

Sr. No.	Title	Remarks
	Module 1.	
	<u>Principles of Analog Instruments</u>	
	- Errors in Measurement	
	- Diff between Indicating & Integrating Instruments.	
	- Moving coil and Moving Iron Ammeters & Voltmeters	
	- Extension of ranges by using shunt, Multipliers	
	- Instrument Transformer → Current Transformer, → Potential Transformer	
	- Dynamometer type Wattmeter & Power Factor Meters. → 1ϕ Electrodynamometer type Wattmeter → 3ϕ Electrodynamometer type wattmeter.	
	- Reed Moving Coil type frequency meter	
	- Principle of Double Voltmeter	
	- Double Frequency Meter	
	- Weston type Synchroscope	
	- DC Permanent Magnet Moving Coil type Galvanometers	
	- Ballistic Galvanometer	
	- AC Vibration Galvanometer.	

MODULE: PRINCIPLES OF ANALOG INSTRUMENTS

Electrical & Electronic Instruments.

- The advancement of Science & Tech. is dependend upon a parallel progress in measurement techniques.
- The reasons for this obvious.
- As Science & Tech. move ahead, new phenomena and relationships are discovered and these advances make new type of measurements imperative (crucial part or vital importance)
- New discoveries are not of any practical utility unless the results are backed by actual measurements.
- The measurements, no doubt, confirm the validity of a hypothesis but also add to its understanding
- This results in an unending chain which leads to new discoveries that require more, new and sophisticated measurement techniques. Hence modern Science & Tech. are associated with sophisticated methods of measurement.
- There are two major functions of all branches of engineering
  (i) Design of equipment and process.
  (ii) Proper Operation and maintenance of equipment & process.

Both these require measurements. This is because proper and economical design, operation and maintanance require a feedback of information

Measurement play a vital or significant role in achieving goals & objectives of Engineering because of the feedback information applied by them.

ERRORS IN MEASUREMENTS.

- Measurements done in a laboratory or at some other place always involve errors. No measurement is free from errors
- If the precision of the equipment is adequate, no matter what is accuracy is, a discrepancy (Variations) will always be observed between two measured results.
- Since errors are must in any measurements it is imperative to interpret the results of a quantitative measurement in an intelligent manner.
- An understanding and thorough evaluation of errors is essential
- No measurements that can be made with perfect accuracy but it is important to find out what accuracy actually is and how different errors have entered into the measurement.
- A study of error is a first step in finding ways to reduce them.

- The error of an instrument is the algebraic difference between the observed value and the true value of the quantity being measured

True value : The true value of quantity to be measured may be defined as the average of an infinite number of measured values when the average deviation due to various contributing factor tends to zero.

- Such an ideal situation is impossible to realise in practice and hence it is not possible to determine the "True Value"

- Errors may be expressed either as absolute or as percentage of error.
- Absolute Error : It may be defined as the difference between the expected value or true value of the variable or the measured or observed value of the variable.

$$e = Y_n - X_n$$

where e = Absolute error

Y_n = Expected value

X_n = Measured value

$$\% \text{ Error} = \frac{\text{Absolute Value}}{\text{Expected Value}} \times 100$$

$$= \frac{e}{Y_n} \times 100$$

Therefore

$$\% \text{ Error} = \left[\frac{Y_n - X_n}{Y_n}\right] \times 100$$

It is more frequency expressed as accuracy rather than error.

$$A = 1 - \left|\frac{Y_n - X_n}{Y_n}\right|$$

where A is the realitive accuracy

Accuracy is expressed as % accuracy

$$a = 100\,\% - \%\text{ error}$$

$$a = A \times 100\,\%$$

where a is the % accuracy.

SOURCES OF ERROR:

- Insufficient knowledge of process parameters and design condition
- Poor Design
- Change in process parameters, irregularities, upsets etc.
- Poor maintanance.
- Errors caused by person operating the instrument or equipment.
- Certain design limitation.

Errors may arise from different sources and are usually classified as under:

- GROSS ERRORS
- SYSTEMATIC ERRORS
- RANDOM ERRORS.

GROSS ERRORS.

- This class of errors mainly covers human mistakes in reading instruments, recording and calculating measurement results.
- The responsibility of the mistake normally lies with the experimenter. The experimenter may grossly misread the scale.

for eg: He may due to oversight read the temperature as 31.5°C while the actual reading may be 21.5°C

- He may transpose the reading while recording as

for eg: He may read 25.8°C and record 28.5°C

- So ~~ta~~ as long as human beings are involved some gross errors may definitely arise. But although complete elimination of gross errors is probably impossible, one should try to anticipate and correct them.
- Basically the point arise here is some gross errors are easily detected while others may be difficult to detect.
- Gross errors may be of any amount therefore their mathematical analysis is impossible. However they can be avoided by adopting two means They care:
- Great care should be taken in reading and recording the data 2, 3 or even more number of readings should be ~~taken~~ for the quantity under measurements.
- These readings should be taken preferably by different experimenters and the readings should be taken at a different reading point so as to avoid re-reading with the same error.
- Most commonly it is always advisable to take large number of readings assures that no gross errors has been committed.

SYSTEMATIC ERRORS

These type of errors are divided into

- Instrumental Errors
  - Inherent short-comings of instruments.
  - Misuse of the instruments.
  - Loading effects.
- Environmental Errors
- Observational Errors.

SYSTEMATIC ERRORS : A systematic error (an estimate of which is known as a measurement bias) is associated with the fact that a measured value contains an offset.

Instrumental Errors

These errors arise due to 3 main reasons

(i) Due to inherent shortcomings in the instrument

(ii) Due to misuse of the instruments

(iii) Due to loading effects of instruments.

(i) Due to inherent shortcomings in the instruments

- These errors are in instruments are inherent in instrument because of their mechanical structure.
- They may be due to construction, calibration, operation of the instruments or measuring devices and with this errors may cause the instruments to read too low or too high.

for eg: If the spring (used for producing controlling troque) of a permanent magnet instrument has become weak the instrument will always read high.

- Errors may be caused because of friction, hysteresis or even gear backlash [It is a clearance or lost motion in a mechanism caused by gaps between the parts].
- While making precision measurements, we must recognize the possibility of such errors as it is often possible to eliminate them or atleast reduce them to great extent.

*Calibration : It is a process of finding a relationship between 2 quantities that are unknown.

(a) The procedure of measurement must be carefully planned. Substitution methods or calibration against standards may be used for this purpose.

(b) Correction factor should be applied after determining the instrumental errors

[Correction ~~error~~ factors: A factor that is multiplied with the result of an equation to correct for a known amount of systematic errors].

(c) The instrument may be re-*calibrated carefully.

ii] Misuse of Instruments

- There is an old saying that instruments are better than the people who use them.
- So the errors caused in measurements are due to the fault of the operator than that of the instrument
- A good instrument used in unintelligent manner may give rise to errors
- Use of improper practice may not cause a permanent damage to the instrument but are most likely to cause errors.
- There are some certain ill practices like using the instruments contrary to [opposite or mutually opposed] manufacturer's instruction and Specifications which in addition leads to produce errors that causes permanent damage to the instruments as a result of overloading and overheating that may ultimately results in failure of the instrument and sometimes system itself

iii] **Loading Effects**

- One of the most common errors committed by beginners is the improper use of an instrument for measurement work.

for eg: a well calibrated voltmeter may give a misleading voltage reading when connected across a high resistance circuit

Example A voltmeter having a sensitivity of 1000 Ω/V reads 100V on its 150V scale when connected across an unknown resistor in series with a multi-ammeter when the multiammeter reads 5mA Calculate:

(a) apparent resistance of the unknown resistor.

(b) actual resistance of the unknown resistor &

(c) error due to the loading effect of voltmeter.

Given : V_T (total voltage) = 100V

I_T (total current) = 5mA

Total circuit resistance

$$R_T = \frac{100}{5 \times 10^{-3}}$$

$$= 20 K\Omega$$

Resistance of voltmeter

$$R_V = 1000 \times 150$$

$$= 150 K\Omega$$

Unknown resistance

$$R_x = \frac{R_T R_V}{R_V - R_T}$$

$$= \frac{20 \times 150}{150 - 20} = 23.077 K\Omega$$

$$\text{Percentage error} = \frac{\text{measured value} - \text{true value}}{\text{true value}} \times 100$$

$$= \frac{20 - 23.077}{23.077} \times 100$$

$$= -13.33\ \%$$

Referring to the same example with the milliammeter reads 800 mA and voltmeter reads 40V on its 150V scale then

Total circuit resistance

$$R_T = \frac{40}{800 \times 10^{-3}}$$

$$= 50\Omega$$

Resistance of voltmeter

$$R_V = 1000 \times 150$$

$$= 150 K\Omega$$

$$\text{Unknown resistance } R_x = \frac{R_T R_V}{R_V - R_T}$$

$$= \frac{50 \times 150 \times 10^3}{150 \times 10^3 - 50}$$

$$= 50.017\ \Omega$$

$$\text{Percentage error} = \frac{50.0 - 50.017}{50.017} \times 100$$

$$= -0.034\ \%$$

- Therefore errors caused by loading effects of the meters can be avoided by ~~use~~ using them intelligently.

So for eg: when measuring a low resistance by ammeter-voltmeter method a voltmeter having a very high value of

resistance should be used.
- It is mainly preferred to use those methods which results in negligible or no loading effects.

Environment Error

- These errors are due to conditions extenal to the measuring device including conditions in the area surrounding the instrument.
- Effects of temp, pressure, humidity, dust, vibrations or of external magnetic or electrostatic fields.

Measures to consider and employed so as to eliminate these undesirable effects are :

1) Arrangements should be kept as near as possible and as constant possible

For eg : Temperature can be kept constant by keeping the equipment in a temp. controlled enclosure.

2) Using equipment which is immune (not liable) to these effects.

For eg : Variations in resistance with temp. can be minimized by using renistance materials which have a very low renistance temp coefficient

3) Employing techniques which eliminate the effect of these disturbances.

For eg : The effect of humidity dust etc.. can be entirely eliminated by hermetically sealing the equipment.

4) In case it is suspected that extenal magnetic or electrostatic fields that can effect the readings of the instruments, magnetic or electrostatic shields.

5) Applying computed corrections.

Observational Errors

- There are many sources of observational errors for an example the pointer of a voltmeter rests slighty above the surface of the scale. Thus an error on account of ~~par~~ PARALLAX will be incurred unless the line of vision of the obsever is exactly above the pointer.
- To minimize parallax errors, highly accurate meters are provided with mirrored scale.
- We can eliminate this error by having the pointer and the scale in the same plane.
- Human factors involved in measurement the sensing capabilities of individual observers affects the accuracy of measurement. No two persons observe the same situation in exactly the same way where small details are concerned.

For eg: There are observational errors in measurements involving timing of an event [One observer may tend to anticipate the signal and read too soon]

[Different experimenters may produce different results especially when sound and light measurements are involved since no two observers posses the same physical response]

In order to avoid Observational Errors

- Modern electrical instruments have digital display of output which completely eliminates the errors on account of human observational or sensing powers as the output is in the form of digits.

- RANDOM ERRORS

- There has been consistently found that experimental results show variation from one reading to another even after all systematic errors have been accounted for.
- Mainly these errors are due to ~~mo~~ multitude of small factors which change or fluctuate from one measurement to another.
- The quantity being measured is affected by many disturbances throughout the universe which are always unaware. Hence the disturbance about which are unaware are lumped together and called "Random" or "Residual" errors.
- Since these errors remain even after the systematic errors have been taken care of or pooper measured steps are considered though we call these errors are called random errors

Ex: Random error

A spring balance might show variations in measurements due to fluctuations in temp, condition of loading & unloading.

Limiting Errors OR Guarantee Errors

- The accuracy and precision of an instrument depends upon its design, the material used and the work that goes into making the instruments.
- Perhaps an instrument used for an application requiring a high degree of accuracy that has to use expensive materials and a high skilled workmanship.

- The economical production of any instrument requires the proper choice of material, design and skill
- In most of the instrument the accuracy is guranteed to be within a certain percentage of full scale reading thus the manufacturer has to specify the deviations from the nominal value of a particular quantity.
- Hence the percentage which shows the deviation of a quantity from its specified value is called as limiting error.

OR

The limits of deviations from the specified value is termed as limiting error or guranteed error.

For eg: If the value of a capacitor specified by the manufacture is 4.7 MF with a tolerance of ±5% then the actual value of capacitance is guranteed to be within the limits.

$$C = 4.7\,MF \pm (5\%\ of\ 4.7\,MF)$$

$$C = 4.7\,MF \pm 0.235\,MF$$

$$C = 4.935\,MF\ \&\ 4.465\,MF$$

- Considering other example where the manufacture may specify that an instrument is accurate with ±2% of full scale deflection reading is guranteed to be within ±2% of perfectly accurate reading. But for readings that are less than the full scale reading the limiting errors increases.

Thus the value of limiting error is mathematically expressed as

$$A_t = A_s \pm \delta A$$

where A_t = actual or true value

A_s = specified or rated value

δA = Limiting error or tolerance

LIMITING ERRORS

- Relative limiting Error
- Combination of Quantities with Limiting Error
  1) Sum of two quantities
  2) Difference of two quantities
  3) Product of two quantities
  4) Division of two quantities
  5) Power of a factor.

Combination of Quantities with Limiting Error.

- Sum of the two quantities.

let a_1 and b_1 be the two quantities that are added to obtain the result as A_T

let the relative Increament

ANALOG INSTRUMENTS

- An analog instrument device is one in which the output or display is a continous function of time and bears a constant relation to its input.

CLASSIFICATION OF ANALOG INSTRUMENTS

- The analog instruments may be classified according to the quanrity they measure.

for eg: An instrument meant for measurement of current is classified as an Ammeter while an instrument that measures voltage is classified as Voltmeter. Similarly we have wattmeters, power factor meters, frequency meters etc.

Analog instruments depends for their operation on one of the many effects produced by current and voltage and thus ~~many effects produced by current~~ classified according to which of the effect is used for working.

Effect	Instruments
Magnetic Effect	Ammeters, Voltmeters, wattmeters integrating meters.
Heating Effect	Ammeters, Voltmeters & Wattmeters.
Electrostatic Effect	Voltmeters.
Induction Effect	AC ammeters, voltmeters, wattmeters energy meters.
Hall Effect	flux meters, ammeters & Poynting vector wattmeter.

Analog Instruments are classified as

- Indicating
  - Electromechanical Instruments
  - Electronic Instruments
- Integrating
- Recording.

Indicating Instruments

- Instruments which indicate the magnitude of quantity being measured.
- They generally make use of a dial and a pointer for this purpose.
- The analog indicating instruments may be divided in 2 groups.
  - Electromechanical Instruments
  - Electronic Instruments.

Integrating Instruments

- These instruments totalize events over a specified period of time.
- The summation, which they gave is the product of time and an electrical quantity for eg: Ampere hour meter, Energy meter (watt hour)
- The integration (summation value) is generally given bay a register consisting of a set of a pointers and dials.

Recording Instruments

- It gives a continous record of the quantity being measured over a specified period.
- The variation of the quantity being measured are recorded by a pen attached to the moving system of the instrument the moving

system is operated by the quantity being measured) on a sheet of paper fixed or moving.

For eg: We may have a recording voltmeter in a sub-station which keeps record of the variations of supply voltage during the day.

Difference Between Indicating & Integrating Instruments

Sr.No	Indicating Instruments	Integrating Instruments
1.	Indicating instruments are those which indicate magnitude of a quantity being measured.	1. Integrating instruments totalize events over a specified period of time.
2.	They ~~are~~ generally make use of a dial and a pointer for this purpose eg: voltmeter, ammeter frequency meter etc.	2. The summation which they give is the product of time and electrical quantity for eg: Amphere hour meter, Energy meter

Essentials of Indicating Instruments

Three types of torques are needed for the satisfactory operation of an indicating instruments

- Deflecting Torque
- Controlling Torque
- Damping Torque.

1) Deflecting Torque.

- The deflecting torque is required for moving the pointer from its zero position.
- The system producing the deflectric force is called deflecting system or Moving system.
- The deflecting torque can be produce by utilizing any of the effects such as.

Magnetic Effect: When a current carrying conductor is placed in uniform magnetic field, it experinces a force which causes to move it. This effect is mostly used in many instruments like moving iron acttraction and repulsion type, permanent magnet moving coil instruments etc.

Thermal Effect: The current to be measured is passed through a small element which heats it to cause rise in temperature which is converted to an emf by a thermocouple attached to it.

[When two dissimilar metals are connected end to end to form a closed loop and the two junctions formed are maintained at different temperatures, then emf is induced which causes the flow of current through the closed circuit which is called as thermocouple.

Electrostatic Effect: When two plates are charged there is a force exerted between them, which moves one of the plate. This effect is used in electrostatic instruments which are normally voltmeters.

Induction Effect: When a non-magnetic conducting disc is placed in a magnetic field produced by electromagnets which are excited by alternating currents, an emf is induced in it.

- The deflecting system of an instrument converts the electric current or potential into a mechanical torque called deflecting torque.
- The deflecting torque acts as the prime mover responsible for the deflection of the pointer.

2) Controlling Torque.

- This torque is required in an indicating instrument in order that the current produces deflection of the pointer proportional to its magnitude.
- The system producing a controlling force ~~or~~ torque is called controlling system.

The functions of the controlling system are :

(i) To produce a force equal and opposite to the deflecting force at the final steady position of pointer in order to make the deflection of the pointer definite for a particular magnitude of current.

In absence of controlling torque (system), the pointer will ~~shoot~~ swing beyond the final steady position for any magnitude of current and thus the deflection will be indefinite.

(ii) In the absence of a controlling system the pointer will not come back to zero when current is removed.

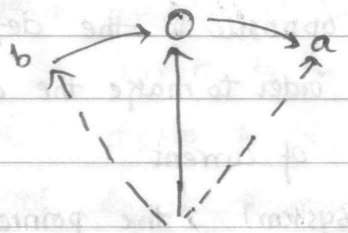
- Controlling torque is usually provided by springs.

3) Damping Torque.

- When deflecting torque is applied the pointer deflects and come to rest a position where the deflecting torque is balanced by controlling torque.
- Since deflecting and controlling systems have inertia, pointer cannot immediately settle to its final position but swings aheat ~~it~~ of it

*Damp: It is an influence upon the oscillatory s/m to prevent, restrict or to reduce its oscillations.

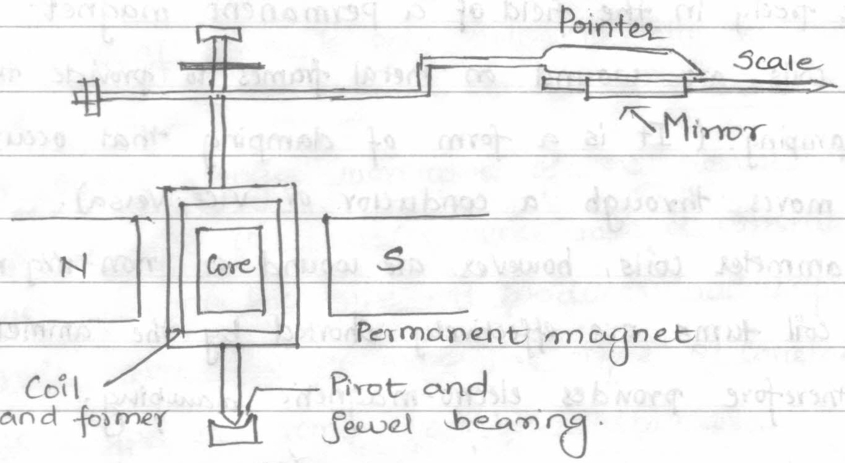
- The pointer thus oscillates about its final steady position with decreasing amplitude till its kinetic energy is dissipated in friction. and therefore it will settle down at its final steady position.
- If extra forces are not ~~requ~~ provided to "damp" these oscillations of the moving system will take considerable time to ~~stalle~~ settle to the final position and hence time consumed in taking readings will be very large
- Therefore damping torque is necessary so that the moving system or deflecting s/m comes to its equillibrium position rapidly and smoothly without any ~~ossci~~ oscillations.

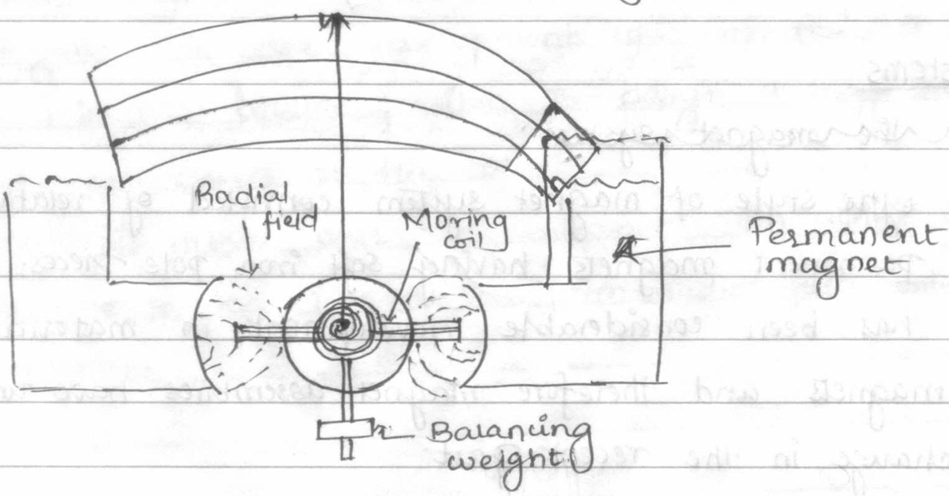


Moving Coil Instrument

The permanent magnet moving coil instrument is most acurate type for dc measurements.

Construction of PMMC Instruments.





CONSTRUCTION OF PMMC [Permanent Magnet Moving Coil].

Moving Coil

- The moving coil is wound with many turns of enamelled or silk covered copper wire
- The coil is mounted on a rectangular aluminium former which is pivoted on jewelled bearings.
- The coils move freely in the field of a permanent magnet.
- Most voltmeter coils are wound on metal frames to provide the required electro-magnetic damping. (It is a form of damping that occurs when a magnetic field moves through a conductor or vice. versa).
- Most of the ammeter coils, however are wound on non-~~mg~~ magnetic formers, because coil turns are effectively shorted by the ammeter shunt. the coil itself, therefore provides electro-magnetic damping.

Magnet Systems

- ~~In Earlier the magnet system.~~
- Earlier the ~~syte~~ style of magnet system consisted of relatively long U shaped permanent magnets having soft iron pole pieces.
- Now there has been considerable development in materials for permanent magnets and therefore magnet assemblies have undergone a lot of change in the recent past.
- Owing to development of materials like Alcomax and Alnico which have a high coercive force (is a measure of the ability of a ferromagnetic material to withstand an external magnetic field without becoming demagnetized.), it is possible to use smaller lengths and high field intensities.
- The flux densities used in PMMC instruments vary from $0.1\,Wb/m^2$ to

$1\ wb/m^2$ Thus in small instruments it is possible to use a small coil having small number of turns and hence a reduction in volume is achieved.

- Alternatively in instruments having a large scale length ~~to accomodate large~~ it is possible to increase the air gap length to accomodate large number of turns.
- The angular span of scale is only upto 90°. Thus the movement of the coil is restricted.
- In order to obtain longer movement of the pointer and a longer angular swing of the coil, a concentric magnet construction is shown. Since magnet is concentric type, it produces radial flux pattern which extends over 250° or more. This type of construction is used for maney panel type and some portable instruments.
- In recent years with the development of improved magnetic materials like Alnico, it has become feasible to design a magnetic s/m in which the magnet itself serves as the core.
- The moving coil moves over the magnet. The active sides of the moving coil are located in the uniform radial field between pole pieces and steel yoke.

- This arrangement has an advantage of being relatively unaffected by the external magnetic fields.

Control: when the coil is supported between two jewel bearings the control torque is provided by two phosphor bronze hair springs.

- These springs also serve to lead current in and out of the coil.
- The control torque is provided by the ribbon suspension
- This method is comparatively new and is claimed and its claimed to be advantageous as it eliminates bearing friction.

Damping: Damping torque is produced by movement of aluminium formes moving in the magnetic field of the permanent magnet.

Pointer & Scale:

- The pointer is carried by the spindle and moves over a graduated scale.
- It has a light weight.
- It has a fine blade which helps to reduce parallax errors in the reading of scale. Such errors can be further reduced by careful alignment of the pointer blade and its reflection in the mirror adjacent to scale.

Torque Equation.

- The deflecting torque for a moving coil instrument is given by.

$$T_d = NBldI$$

where N = Number of turns in the coil

B = Flux density in air gap wb/m²

I = Current through moving coil (Amp)

l = Length of horizontal side (width) of coil M

$$G = NBld = \text{Displacement constant}$$

$$\therefore T_d = GI$$

- The controlling torque provided by spring is given by

$$T_c = K\theta$$

Where K = Spring constant θ = final steady state deflection of moving coil in rad.

- Final Steady deflection [Balanced deflection]

$$T_c = T_d \qquad [\because T_c = K\theta]$$

$$\therefore K\theta = GI$$

$$\theta = \frac{G}{K} I$$

or

$$I = \frac{K}{G}\theta$$

As the deflection is proportional to the current passing through the meter (K & G are being constants) we get a uniform (linear) scale for the instrument.

ERRORS IN PMMC:

1. Weakening of permanent magnets due to ageing and temp effects.
2. Weakening of springs due to ageing and temp effects.
3. Change of resistance of moving coil with temp.

Advantages of PMMC Instruments:

- The scale is uniformly or linearly divided
- The power consumption is very low as 25 MW to 200 μW
- The torque weight ratio is high which gives a high accuracy of about 2% of full scale deflection.

*Permeability : It is the measure of the ability of a material to support the formation of a magnetic field

**reluctance : the property of a magnetic ckt of opposing the passage of magnetic flux lines.

Moving Iron Instruments.

- The most common ammeters and voltmeters for lab used at power frequencies are the moving iron instruments.
- It can measure dc as well as ac.

General Principle of Moving Iron Instruments.

- A plate or vane of soft iron of high *permeability (measure of the ability of a material to support the formation of magnetic field) steel forms the moving element of the s/m This iron vane is so situated that it can move in a magnetic field produced by a stationary coil.
- The coil is excited by the current or voltage under measurement. When the coil is excited, ~~by the~~ it becomes electromagnet and the iron vane moves in such a way so as to increase the flux of electromagnet.
- This is because the vane tries to ocupy a position of minimum reluctance. Thus the force produced is always in such direction so as to increase the inductance (the property of an elec conductor that causes emf to be generated by change in current) of coil (this is because the inductance is inversely proportional to **reluctance of magnetic ckt of coil).

Classification of Moving Iron Instruments.

Moving Iron instruments are of 2 types.

→ Attraction Type.

→ Repulsion Type
- → Radial vane type
- → Co-axial Vane type.

Moving Iron Attraction type Instrument

- In an moving iron attraction type instrument the coil is flat disc or a sector eccentrically mounted.
- When the current flows through the coil, a magnetic field is produced and the moving iron moves from the weaker outside the coil to the stronger field inside it or in other words, the moving iron is attracted in
- The controlling torque is provided by springs but gravity control can be used for panel type of instruments which are vertically mounted.
- Damping is provided by air friction with the help of a light aluminium piston (attached to the moving system) which moves in a fixed chamber closed at one end or with the help of a vane (attached to the moving system).

Moving Iron Repulsion type Instrument

- In the repulsion type, there are two vanes inside the coil one fixed and other movable.
- These are similarly magnetised when the current flows through the coil and there is a force of repulsion between the two vanes resulting in the movement of the moving vane.

Two different designs are in common use:

(i) Radial Vane Type:

- In this type, the vanes are radial strips of iron
- The strips are placed within the coil
- The fixed vane is attached to the coil and movable one to the spindle of the instrument.

(ii) Co-axial Vane Type:

- In this type of Instrument the fixed and moving vanes are sections of co-axial cylinders
- The Controlling torque is provided by springs Gravity control can also be used in vertically mounted instruments.
- The damping torque is produced by air friction
- The operating magnetic field in moving iron instruments is very weak and therefore eddy current damping is not used in them as introduction of permanent magnet req. for eddy current ( ) damping would distort the magnetic field.

Q. Explain MI Iron Instrument is unpolarized Instrument.

- A plate or vane of soft iron or of high permeability steel forms the moving element of s/m This iron vane is so situated that it can move in a magnetic field produced by stationary coil.
- The coil is current excited by the current or voltage under measurement
- When the coil is excited it becomes an electromagnet and the iron vane moves in such a way so as to increase the flux of electromagnet.
- This is because the vane tries to occupy a position of minimum reluctance thus the force produced is always in such direction so as to increase the inductance of coil.
- So it is clear that whatever may be the direction of the current in the coil of the instrument, the iron vanes are so magnetized that there is always a force of attraction type and repulsion in the repulsion type of ~~instrstur~~ instruments.
- Thus moving iron instruments are unpolarized instruments. i.e. they are independent of the direction in which the current passes Therefore these instruments can be used ~~as~~ on both ac and dc.

Torque Equation of Moving Iron Instrument

- Consider a small increament in current supplied to the coil of the instrument. Due to this current let $d\theta$ be the deflection under the deflecting torque Td. Due to such deflection some mechanical work will be done.

$$\therefore \text{ Mechanical work} = T_d \, d\theta$$

- There will be a change in the enegy stored in the magnetic field due to change in inductance.
- This is because the vane tries to occupy the position of minimum reluctance hence the force is always in such a direction so as to increase the inductance of coil.
- The inductance is inversely proportional to the reluctance of the magnetic ckt of coil.

Let I = initial current

L = increament Instrument inductance.

θ = Deflection

dI = Increase in current

$d\theta$ = Change in deflection

dL = change in inductance.

In order to effect an increament dI in the current, there must be an increase in the applied voltage given by

$$e = \frac{d(LI)}{dt}$$

$$e = I\frac{dL}{dt} + L\frac{dI}{dt}$$ [As both I & L are changing]

The electrical energy supplied is given by

$$eI\,dt = \left[I\frac{dL}{dt} + L\frac{dI}{dt}\right] I\,dt$$

$$eI\,dt = I^2 dL + IL\,dt$$

The stored energy increases from

$$\frac{1}{2}LI^2 \quad \text{to} \quad \frac{1}{2}(L+dL)(I+dI)^2$$

Hence the Change in the Stored energy is given by

$$= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$$

$$= \frac{1}{2}(L+dL)(I^2 + 2I\,dI + dI^2) - \frac{1}{2}LI^2$$

Neglecting higher order terms

$$IL\,dI + \frac{1}{2}I^2 dL$$

Now from principle of the conservation of energy

Electrical Energy supplied = Increase in stored energy + Mechanical done work.

$$I^2 dL + IL\,dI = IL\,dI + \frac{1}{2}I^2 dL + T_d\,d\theta$$

$$T_d \cdot d\theta = \frac{1}{2} I^2 dL$$

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \qquad T_d = T_c$$

While the controlling torque is given by

$$T_c = K\theta$$

K = Spring constant

$$K\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\theta = \frac{1}{2} \frac{I^2}{K} \frac{dL}{d\theta}$$

Thus the deflection is proportional to the square of the current through the coil and instrument gives square law response.

Errors in Moving Iron Instruments. [Errors occurs with both DC & AC]

I] Hysteresis error.

- This errors occurs as the value of flux density is different for the same current for ascending and descending values.
- The value of flux density is higher for descending values of current and therefore the instrument tends to read higher for descending values of current (and voltage) than for ascending values.
- This error can be minimized by making the iron parts small so that they demagnetize themselves quickly.
- Another method is to work the iron parts at low values of flux density so that the hysteresis effects are small.

ii) Temperature Error.

- The temperature error arises due to the effect of temperature on the temperature coefficient of the spring.
- This error is of order of 0.02 % per °C change in temperature
- Errors can cause due to self heating of the coil and due to which change in resistance of the coil.
- So coil and series resistance must have low temperature coefficient

~~iii) Stray magnetic field error.~~
~~-The operating~~

[Errors with AC only]

iii) Frequency error :

- These are related to ac operation of the instrument.
- The change in the frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This cause errors in the instrument.

Advantages of Moving iron Instruments

- These instruments can be used for both a.c and dc supply.
- Errors due to friction are quite small as torque weight ratio is quite high.
- These instruments are rebuilt owing to simple construction and also there are no current carrying moving parts.
- They have acuracy within the limits of both precision and industrial grades.

Disadvantages of Moving Iron Instruments

- These instruments are subjected to serious errors due to hysteresis, frequency changes and stray magnetic field.
- There is a difference between dc and ac calibration on account of effect of inductance of the meter and eddy currents.

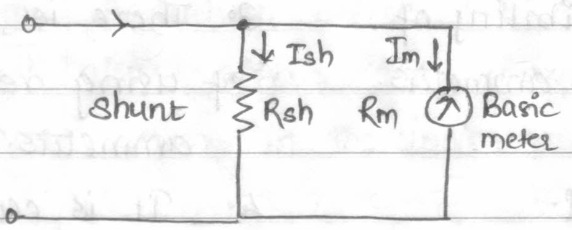
Difference Between Attraction & Repulsion Type Moving Iron Instruments:

Attraction Type	Repulsion Type.
1. It has lower inductance than repulsion type.	1. It has a higher inductance than attraction type.
2. Attraction type voltmeter accurate over a wide range of frequency.	2. Repulsion type voltmeter is accurate over a small range of frequency
3. There is greater possibility of using attraction type ammeters.	3. There is a lesser possibility of using repulsion type ammeters.
4. It is not economical.	4. It is economical.
5. Nearly uniform scale is not easily obtained.	5. Nearly uniform scale is easily obtained.

Shunt and Multipliers For Moving Coil Instrument

Ammeter Shunt:

- The basic movement of a dc ammeter is a PMMC galvanometer d'Arsonval galvanometer.
- The coil winding of a basic movement is small and light, and can carry very small currents since the construction of an accurate instrument with a moving coil to carry currents greater than 100mA is impracticable owing to the bulk and weight of the coil that would be required.
- When heavy currents are to be measured, the major part of the current is bypassed through a low resistance called "shunt"
- The fig shows the basic movement (meter) and its shunt to produce an ammeter.



Basic ammeter ckt

The resistance of the shunt can be calculated using conventional circuit analysis.

where R_m = Internal resistance of Basic meter

R_{sh} = Resistance of shunt Ω

$I_m = I_{fs}$ = full scale deflection current of movement A

I_{sh} = shunt current A

I = current to be measured.

Since the shunt resistance is in parallel with the meter movement, the voltage drop across shunt and movement must be the same

$$I_{sh} R_{sh} = I_m R_m$$

$$R_{sh} = \frac{I_m R_m}{I_{sh}} \quad \text{--- (1)}$$

But $I_{sh} = I - I_m$ --- (2)

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

From eq (1)

$$\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}}$$

Substituting $I_{sh} = I - I_m$

$$\frac{I - I_m}{I_m} = \frac{R_m}{R_{sh}}$$

$$\frac{I}{I_m} - 1 = \frac{R_m}{R_{sh}}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

The ratio of the total current to the current in the movement (basic meter) is called multiplying power of shunt.

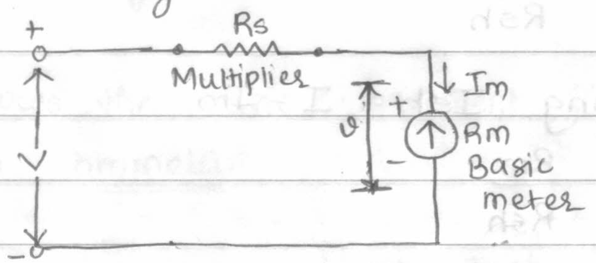
∴ Multiplying power

$$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

∴ Resistance of shunt $R_{sh} = \frac{R_m}{m-1}$

Voltage Multipliers.

- The basic voltmeter is nothing but D'Arsonval galvanometer the resistance is required to be connected in series with the basic meter to use it as a voltmeter this series resistance is called multiplier
- The main function of the multiplier is to limit the current through the basic meter so that the meter current does not exceed the full scale deflection value.
- The voltmeter measures the voltage across the two points of a circuit or a voltage across a circuit component



The voltmeter must be connected across two points or a component to measure the potential difference, with the proper polarity.

The multiplier resistance can be calculated as

Let R_m = Internal resistance of coil ie meter

R_s = Series multiplier resistance

I_m = Full scale deflection current

V = Full range voltage to be measured

from fig.

$$V = I_m (R_m + R_s)$$

$$V = I_m R_m + I_m R_s$$

$$I_m R_s = V - I_m R_m$$

$$R_s = \frac{V}{I_m} - R_m$$

The multiplying factor for multiplier is the ratio of full range voltage to be measured and the drop across the basic meter

v = drop across the basic meter

$= I_m R_m$

m = multiplying factor

$$m = \frac{V}{v}$$

$$= \frac{I_m (R_m + R_s)}{I_m R_m}$$

$$m = 1 + \frac{R_s}{R_m}$$

Hence multiplier resistance can also be expressed as

$$R_s = (m-1) R_m$$

Thus to increase the range of voltmeter 'm' times, the series resistance required is $(m-1)$ times the basic meter resistance. This is nothing but the extension of ranges of a voltmeter.

INSTRUMENT TRANSFORMER

- Transformers are static devices which works on principle of Faraday's Law of mutual induction and are used in ac systems for the measurement of current, voltage, power and energy.
- They are also used in connection with measurement of power factor, frequency and for indication of *synchronism
- Instrument transformers have a wide range of application in protection circuits of power systems for the operation of over current, under voltage, earth fault and various other types of relays.
- Transformers used in conjuction with measuring instruments for measurement purposes are called Instrument Transformer
- The transformer used for measurement of current is called a "Current Transformer" or simply "CT"
- Transformer used for measurement of voltage is called a "Potential Transformer" or simply "PT"

Use of Instrument Transformer.

- In power systems, currents and voltages handled are very large and therefore direct measurement is not possible as these currents and voltages are ~~far~~ far too large for any meter of reasonable size and cost.
- The solution lies in stepping down these currents and voltages with the help of instrument transformer so that

they could be metered with instruments of moderate size.
For eg: In power transmission the line voltage can be in excess of 200 KV the current can be in access of 7KA. so instruments which can measure directly that kind of voltage and current are not available even though they are made available working with them at such high current and high voltages is not desierable in safety point of view. Hence it is necessary to reduce this voltage and current magnitude. In order to bring them to the level where they can be safely measured using again transformers this are called Instrument Transformers.

INSTRUMENT TRANSFORMER

→ POTENTIAL TRANSFORMER

→ CURRENT TRANSFORMER

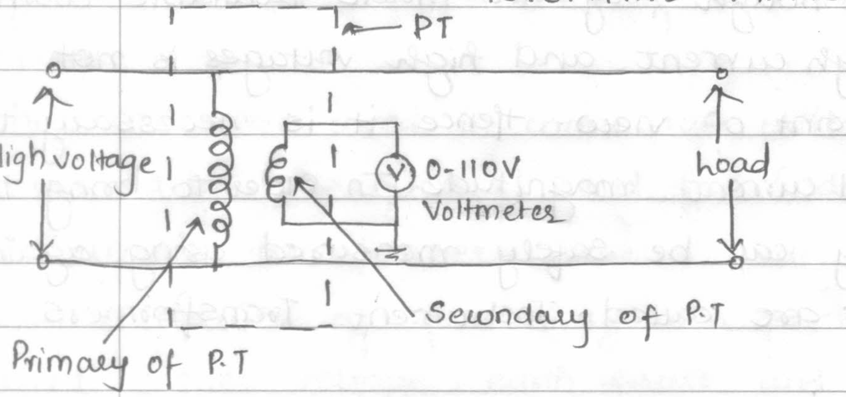
POTENTIAL TRANSFORMER

- The basic principle of these transformers is same as current transformers.
- The high alternating voltage is reduced in a fixed propotion for the measurement purpose with the help of potential transformers.
- The construction of these transformers is similar to the normal transformer.
- The windings are of low power rating windings.
- The primary winding of the transformer is connected across the line carrying the voltage to be measured while the secondary.

is connected to low range voltmeter coil. One end of secondary is always grounded for safety.

- The secondary winding is designed so that a voltage of 100V to 120V is delivered to the instrument load.
- The normal secondary voltage reading is 110V

POTENTIAL TRANSFORMER

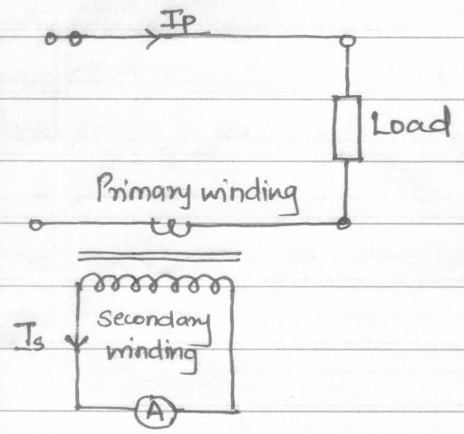


its ratio can be specified as

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

CURRENT TRANSFORMER

- The current transformer is used with its primary winding connected in series with line carrying the current current to be measured and therefore the primary current is dependant upon the load connected to the system and is not determined by the load connected on the secondary winding of the current transformer.
- The primary winding consists of very few turns and therefore there is no appreciable voltage drop across it.
- The secondary winding of the current transformer has large number of turns, the exact number being determined by the turns ratio. The ammeter or wattmeter current coil are connected directly across the secondary winding terminals.
- Thus the current transformer operates its secondary winding nearly under short circuit conditions.
- One end of the secondary is always grounded for safety.



Current Transformer.

Electrodynamometer Type Instruments

- The electrodynamometer type instrument is a transfer instrument. A
- A transfer instrument is one which is calibrated with a dc source and used without any modifications for ac measurements.
- Such transfer instrument has same accuracy for ac and dc measurements
- The electrodynamometer type instruments are often used in accurate ac voltmeters and ammeters.
- With some little modifications, it can be used as a wattmeter for the power measurements.

CONSTRUCTION

Fixed coils: The necessary field required for the operation of the instrument is produced by the fixed coils.

- A uniform field is obtained near the centre of the coil due to division of coil into two sections these coils are air cored.
- Fixed coils are wound with the fine wire for using as voltmeter. while for ammeters and wattmeters it is wound with heavy wire
- The coils are usually varnished. they are clamped in place against the coil supports. This makes the construction is rigid.
- Ceramic is usually used for mounting supports. If metal parts would have been used then it would weaken the field of the fixed coil.

Moving coil : The moving coil is wound either as a self sustaining ~~(table to continue in a health state~~ or else on a metallic former.

- If metallic former is used, then it would induce eddy currents in it.
- The construction of moving coil is made light as well as rigid. It is air cored.

Controlling : The controlling torque is provided by springs. These springs acts as leads to the moving coil.

Moving system : The moving coil is mounted on an alluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case of high accuracy is desired.

Damping : The damping torque is provided by air friction, by a pair of alluminium vanes which are attached to the spindle at the bottom. They move in sector shaped chambers. As operating field would be distorted by eddy currents damping, it is not employed.

Shielding : The field produced by these instruments is very weak. Even earth's magnetic field considerably affect the reading. So shielding is done to protect it from stray magnetic fields.

- It is done by enclosing in a casing of high permeability alloy.

Cases and Scales: - Laboratory standard instruments are usually contained in polished wooden or metal cases which are rigid
The case is supported by adjustable levelling screws.
A spirit level may be provided to ensure proper levelling.

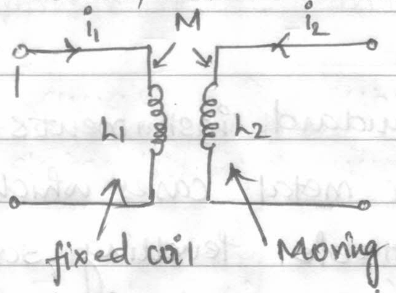
- For using electrodynamometer instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them upto desired limit.

- The electrodynamometer instrument can be used as a voltmeter by connecting the fixed and moving coils in series with a high non-inductive resistance. It is most accurate type of voltmeter.

- For using electrodynamometer instrument as wattmeter to measure the power, the fixed coils acts as current coil and must be connected in series with the load. The moving coil acts as voltage coil or pressure coil and must be connected to the supply terminals.
- When the current passes through the fixed and moving coils, both the coils produce the magnetic fields.
- The field produced by the fixed coil is proportional to the load current while the field produced by fixed coil is proportional to the voltage. As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to load.

Torque Equation

The electrodynamomet instrument can be represented by an equivalent circuit

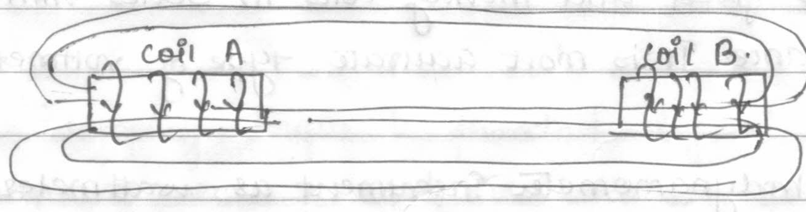


Mutual Inductance principle is based on the working of electrodynamometer instrument and that depends on

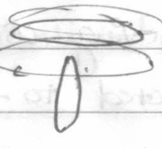
- Relative position
- Medium
- Orientation.

Theory : Mutual Inductance

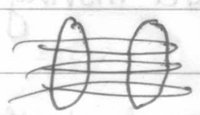
Any change in current through coil A produces emf in coil B this phenomena is called mutual inductance.



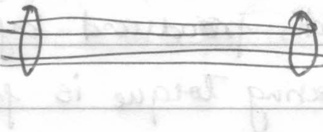
If the coils are placed.



Mutual inductance will be minimum since the flux produced by the coils has no proper orientation.



flux will be maximum



The distance b/w the two coils are large so in such case the mutual inductance will be less.

Now from the ckt we get

The flux linkages in coil 1 (fixed coil) are.

$$\phi_1 = L_1 i_1 + M i_2 \quad -(1)$$

$L_1 i_1$ ↓ Self inductance due to current i_1 in the fixed coil

$M i_2$ → Mutual inductance. because of the moving coil is mutually induced with the fixed coil.

The flux linkages in coil 2 (moving coil) are

$$\phi_2 = L_2 i_2 + M i_1 \quad -(2).$$

Now $e_1 = \frac{d\phi_1}{dt}$ —(3)

$$e_2 = \frac{d\phi_2}{dt} \quad -(4)$$

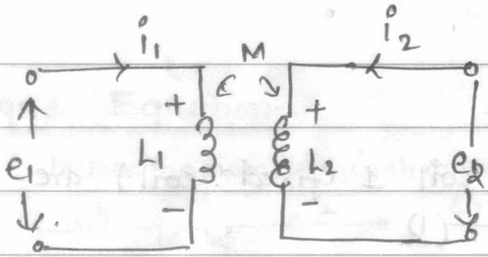
Now the varrying quantities.

M	i_1	i_2
$M+dM$	i_1+di_1	i_2+di_2

If the source is connected to both the ends.

If energy is supplied. $e_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} + i_2 \frac{dM}{dt}$ —(5) ~~Since mutual~~ ~~coil is getting varied~~. Because of mutual inductance. moving

$$e_2 = L_2 \frac{di_2}{dt} + M\frac{di}{dt} + i_1 \frac{dM}{dt} \quad -(6)$$



Now if energy is supplied.

∴ electrical input energy will be $eidt$.

$E_1 = L_1 \not{i_1} di_1 + M i_1 \not{dt}\, di_2 + i_1 i_2\, dM$

$E_2 = L_2 i_2 di_2 + M i_2 di_1 + i_1 i_2\, dM$.

Total Energy will $E = E1 + E2$.

→ Now their will be a change in this energy.

→ $\frac{1}{2} L_1 i_1^2$

→ $\frac{1}{2} L_2 i_2^2$

→ $M i_1 i_2$

And new stored energy will be here. i_1, i_2, M which is getting varied.

$\frac{1}{2} L_1 [i_1 + di_1]^2$

$\frac{1}{2} L_2 [i_2 + di_2]^2$

$[M + dM]\,[i_1 + di_1]\,[i_2 + di_2]$

Now from the principle of conservation of energy.

Energy i/p = Energy stored + Mech. energy

Mech energy = Energy i/p − Energy stored.

$$= \frac{1}{2}L_1 i_1^2 + \frac{2}{2} L_1 i_1 di_1 + \frac{1}{2} L_1 di_2^2 - \frac{1}{2} L_1 i_2^2$$

$$+ \frac{1}{2} L_2 i_2^2 + \frac{2}{2} L_2 i_2 di_2 + \frac{1}{2} L_2 di_2^2$$

$$- \frac{1}{2} L_2 i_2^2 + M i_1 i_2 - M i_1 i_2$$

$$+ M i_1 di_2 + M i_2 di_1 + i_2 i_1 dM$$

$$\boxed{L_1 i_1 di + L_2 i_2 di_2 + i_1 i_2 dM + M i_1 di_2 + M i_2 di_1}$$

Mech energy = Mech. work done + $T_d \, d\theta$.

$$i_1 i_2 \, dM = T_{di} \, d\theta$$

$$T_{di} = i_1 i_2 \frac{dM}{d\theta}.$$

The controlling torque is provided by springs hence.

$$T_c = K\theta.$$

$$T_d = T_c$$ [since deflecting torque must be equal to controlling torque].

$$I_1 I_2 \frac{dM}{d\theta} = K\theta.$$

$$\theta = \frac{I_1 I_2}{K} \frac{dM}{d\theta}.$$

For DC operation: For dc currents I_1 & I_2

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

The controlling torque is provided by springs hence

$$T_c = K\theta$$

$$T_d = T_c$$

$$I_1 I_2 \frac{dM}{d\theta} = K\theta$$

$$\theta = \frac{I_1 I_2}{K} \cdot \frac{dM}{d\theta}$$

Thus deflection is proportional to the product of two currents and the rate of change of mutual inductance.

For AC operation: In ac operation, the total deflecting torque over a cycle must be obtained by integrating Ti over one period

Average deflecting torque over one cycle is

$$T_d = \frac{1}{T}\int_0^T T_i \, dt$$

T= Time period of one cycle

If two currents are sinusoidal and displaced by a phase angle ϕ then

$$i_1 = I_{m1} \sin \omega t$$

$$i_2 = I_{m2} \sin (\omega t - \phi)$$

$$T_d = \frac{dM}{d\theta} \cdot \frac{1}{T}\int_0^T I_{m1} \sin\omega t \cdot I_{m2} \sin(\omega t - \phi) \, d(\omega t)$$

$$= \left[\frac{I_{m1} I_{m2}}{2}\right] \cos\phi \frac{dM}{d\theta}$$

$$= I_1 I_2 \cos\phi \frac{dM}{d\theta}$$

where I_1, I_2 are the rms values of two currents are as

$$I_1 = \frac{I_{m1}}{\sqrt{2}} \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

$$T_c = K\theta$$

Hence in steady state $T_c = T_d$

$$I_1 I_2 \cos\phi \frac{dM}{d\theta} = K\theta$$

$$\theta = \frac{I_1 I_2}{K} \cos\phi \frac{dM}{d\theta}$$

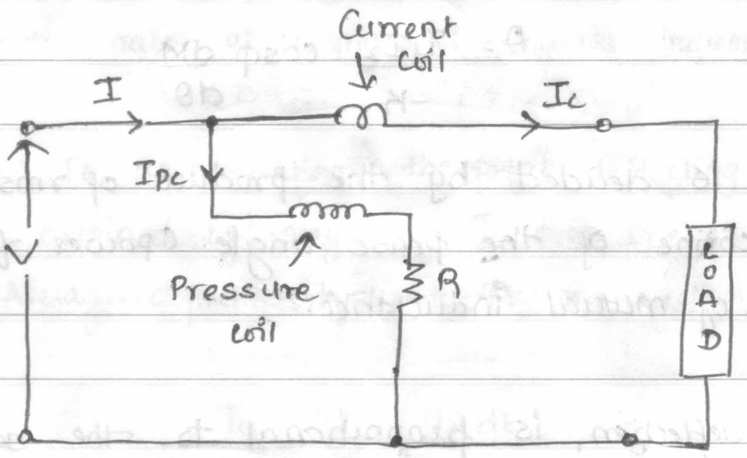
Thus the deflection is decided by the product of rms values of two currents, cosine of the phase angle (power factor) and rate of change of mutual inductance.

- For dc use the deflection is proportional to the square of current and the scale is non-uniform.
- For ac use the instantaneous torque is proportional to the square of the instantaneous current.

The i^2 is +ve and as current varies, the deflecting torque also varies.

Single Phase Dynamometer Wattmeter

- An electrodynamometer type wattmeter is used to measure power.
- It has two coils, fixed and coil which is current coil and a moving coil which is pressure coil or voltage coil.
- The current coil carries the current of ckt while pressure coil carries current proportional to the voltage in the ckt
- This is achieved by connecting a series resistance in voltage circuit
- The connections of an electrodynamometer wattmeter in the ckt are shown



I_c = Current through current coil

I_{pc} = Current through pressure coil

R = Series resistance

V = RMS value of supply voltage

I = RMS value of current

Torque Equation.

According to theory of electrodynamic instruments,

$$T_i = i_1 i_2 \frac{dM}{d\theta} \quad -(1)$$

let v = Instantaneous voltage

$= V_m \sin \omega t$

$= \sqrt{2} V \sin \omega t \quad -(2)$

Due to high series resistance, pressure coil is treated to be purely resistive

The current I_{pc} is in phase with V as pressure coil is purely resistive.

i_{pc} = Instantaneous value $= \frac{v}{R_p}$ where $R_p = r_{pc} + R$

$$i_{pc} = \frac{\sqrt{2}V}{R_p} \sin \omega t$$

$$= \sqrt{2} I_{pc} \sin \omega t$$

If current coil current lags the voltage by angle ϕ then its instantaneous value is

$$i_c = \sqrt{2} I_c \sin(\omega t - \phi)$$

$i_1 = i_c$ and $i_2 = i_{pc}$ hence

$$T_i = [\sqrt{2} I_{pc} \sin \omega t][\sqrt{2} I_c \sin(\omega t - \phi)] \frac{dM}{d\theta}$$

$$= 2 I_c I_{pc} \sin(\omega t) \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$T_i = I_c I_{pc} [\cos\phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta}$$

Thus the instantaneous torque has a component of power which varies as twice the frequency of current and voltage.

$$T_d = \text{Average deflecting torque} = \frac{1}{T}\int_0^T T_i \, d(\omega t)$$

$$= \frac{1}{T}\int_0^T I_c I_{pc}\left[\cos\phi - \cos(2\omega t - \phi)\right]\frac{dM}{d\theta}\, d(\omega t)$$

$$T_d = I_c I_{pc} \cos\phi \frac{dM}{d\theta}$$

where $I_{pc} = \frac{V}{R_p}$.

For spring controlled wattmeter

$$T_c = K\theta$$

$$T_d = T_c$$

$$I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K\theta$$

$$\theta = \frac{1}{K} I_c I_{pc} \cos\phi \frac{dM}{d\theta}$$

$$= K_1 I_c I_{pc} \cos\phi$$

$$K_1 = \frac{1}{K}\frac{dM}{d\theta}$$

$$\theta = K_1 I_c \frac{V}{R_p} \cos\phi$$

$$= K_2 P$$

$K_2 = \frac{K_1}{R_p}$ and $P = V I_c \cos\phi$ = Power

$$\theta \propto P$$

Thus the wattmeter deflection when calibrated gives the power consumption of the circuit.

Power factor :

POWER FACTOR METERS:

The power in single phase ac circuit is given by

$$P = VI\cos\phi$$

Where $\cos\phi$ = Power factor of the circuit

Thus by using precise voltmeter, ammeter and wattmeter in the circuit, the readings of V, I and P can be obtained. Then the power factor can be calculated as

$$\cos\phi = \frac{P}{VI}$$

- This method is not accurate since the errors in all the meters together cause error in power factor calculation.
- Similarly the method is not suitable for the circuits whose power factor is varying according to circuit and load conditions.
- Hence it is necessary to have a meter which can directly indicate the power factor of the circuit.
- Such a meter which indicates the instantaneous power factor of the circuit is called power factor meter.

Basic Construction of Power Factor Meter

- The basic construction of power factor meter is similar to wattmeter.
- It has two circuits current circuit and a voltage circuit.
- The current circuit carries current or fraction of current in the circuit whose power factor has to be measured.
- The voltage coil is split into two parallel paths, one inductive and one non-inductive.

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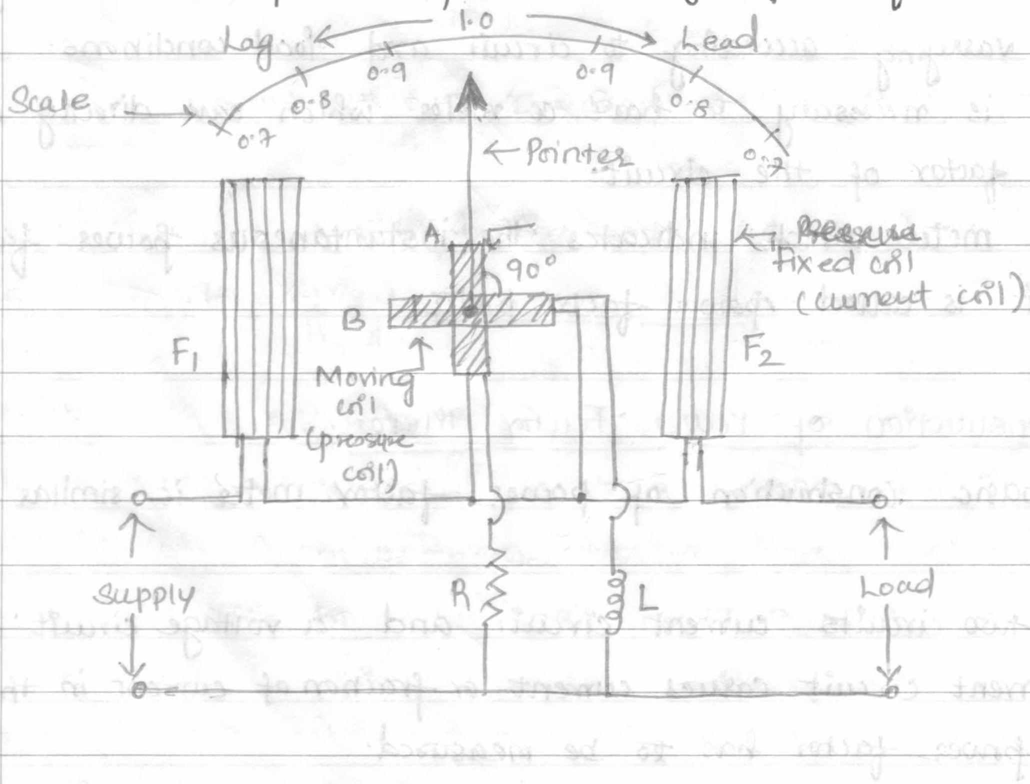
- The voltage coil is split into two parallel paths, one inductive and one non-inductive.

- The current in the two paths are propotional to the voltage of the circuit. Thus the deflection depends upon the phase difference between the main current through current circuit and the currents in the two branches of the voltage ie. power factor of the circuit.
- Single Phase Electrodynamometer Power factor Meter
- Three Phase Electrodynamometer Power factor Meter

- Single Phase Electrodynamometer Power factor Meter

The construction of electrodynamometer type power factor meter is similar to the construction of electrodynamometer type wattmeter.

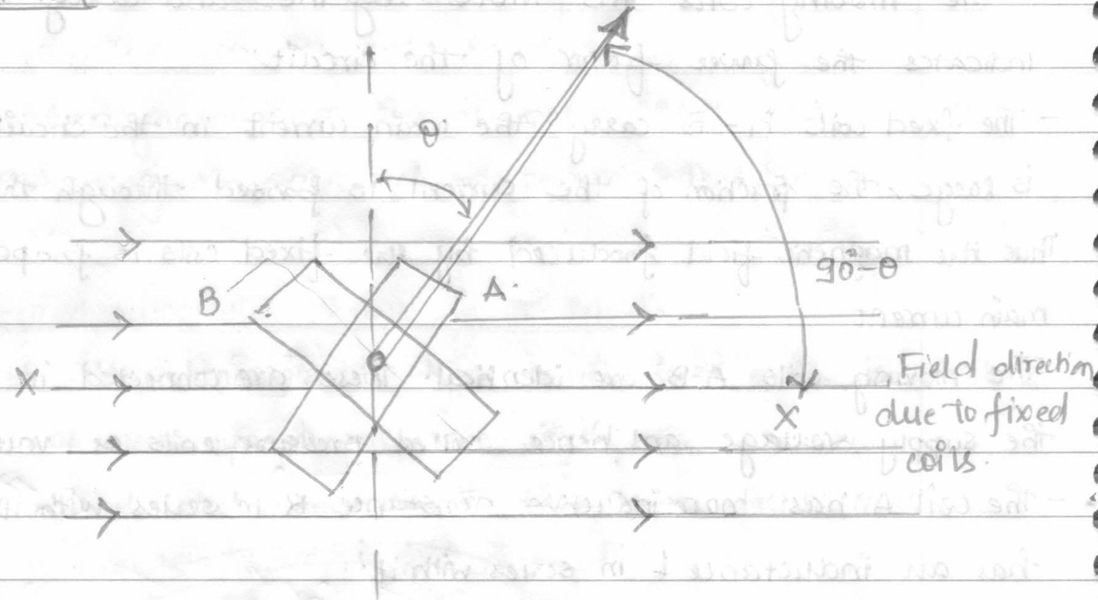
The basic construction of electrodynamometer type power factor meter



- $F_1 - F_2$ are the two fixed coils which are connected in series.
A-B are the two moving coils which are rigidly connected to each other so that their axes are at 90° to each other.
- The moving coils AB move together and carry the pointer which indicates the power factor of the circuit.
- The fixed coils $F_1 - F_2$ carry the main current in the circuit. If the current is large, the fraction of the current is passed through the fixed coils. Thus the magnetic field produced by the fixed coils is proportional to the main current.
The moving coils A-B are identical. These are connected in parallel across the supply voltage and hence called pressure coils or voltage coils.
- The coil A has non-inductive resistance R in series with it while the coil B has an inductance L in series with it.
- The value of R and L are so adjusted that the coils A and B carry equal currents at normal frequency.
So at normal frequency. $R = \omega L$
- The current through coil A is in phase with the supply voltage while the current through coil B lags the supply voltage by nearly 90° due to highly inductive nature of the circuit.
- Due to L, current through coil B is frequency dependent while current through coil A is frequency independent.
- The currents in the coil A and B are equal and produce the magnetic fields of equal strength, which have phase difference of 90° between them.
- The coils are also mutually perpendicular to each other.
- The controlling torque is absent. The contacts to the moving coils

with the help of extremely fine ligaments which give no controlling effect on the moving system.

WORKING OF METER



- Assume that the current through coil B lags the voltage exactly by 90°
- Also assume that the field produced by the fixed coils is uniform and in the direction X-X as shown in fig.
- Due to interactions of the fields produced by the currents through various coils, both coils A and B experiences a torque
- The windings are arranged in such a manner that the torques experienced by coils A and B are opposite to each other. Hence the pointer attains an equilibrium position when these two torques are equal.
- The torque on each coil, for a given coil current will be maximum when the coil is parallel to the field produced by $F_1 - F_2$ ie direction X-X.

Let ϕ = Power factor angle

θ = Angle of deflection.

The θ is measured from the vertical axis, in the equilibrium position

Torque on coil A

$$T_A = KVI \cos\phi \cos(90° - \theta)$$

K = Constant

- The current through coil A is in phase with system voltage V and it moves in a magnetic field
- And the magnetic field is proportional to system current I and $\frac{dM}{d\theta}$ which is generally constant for radial field is not constant for parallel field and is proportional to $\cos(90° - \theta)$
- Similarly current in coil B lags the supply voltage by 90° and moves in the same field

Hence torque on B is proportional to $\cos(90° - \phi)$ ie. $\sin\phi$ and $\cos\theta$

$$T_B = KVI \sin\phi \cos\theta$$

In equilibrium position $T_A = T_B$

$$\cos\phi \cos(90° - \theta) = \sin\phi \cos\theta$$

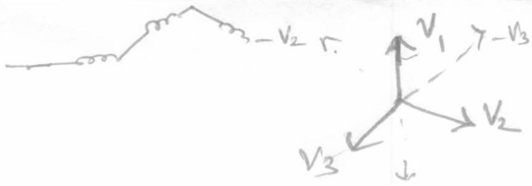
$$\sin\theta = \tan\phi \cos\theta$$

$$\tan\theta = \tan\phi$$

$$\theta = \phi$$

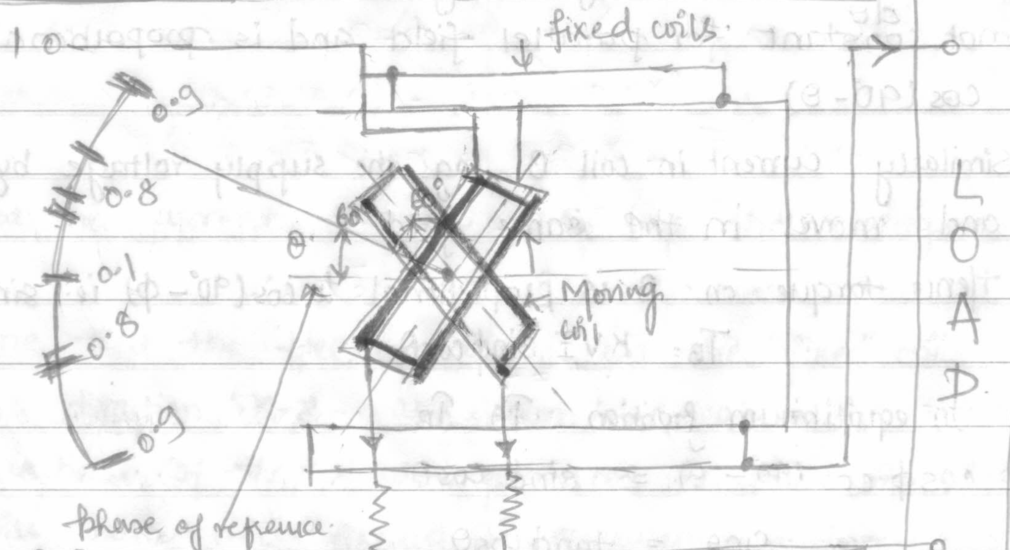
Thus the angular position taken up by the moving coils is equal to the system power factor angle.

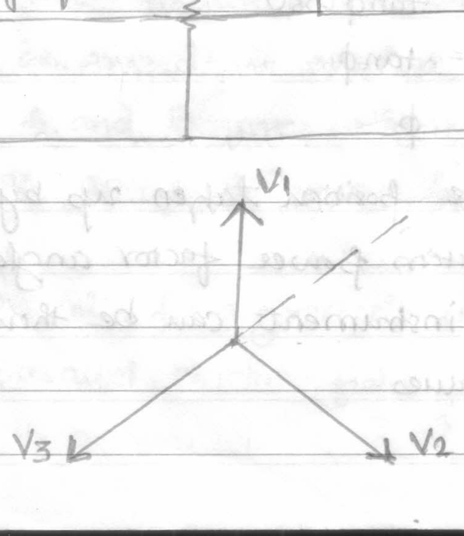
The scale of the instrument can be then calibrated in terms of power factor values.

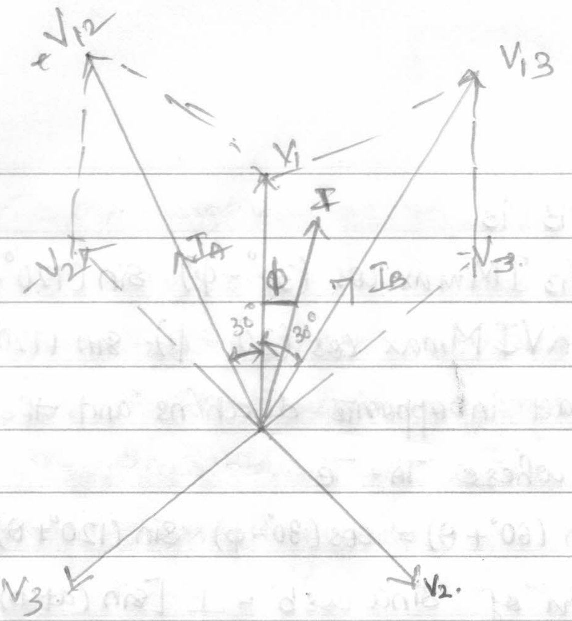


- The operation of the instrument is dependent on the specific supply frequency. ~~If the frequency~~
- If the frequency is different or it contains harmonics then inductance of choke coil changes, due to which there will be serious errors in the instrument reading.
- Thus the operation of the meter is not dependent on the values of current and voltage but dependent on the frequency and the waveform.

<u>Three phase Electrodynamometer Power Factor Meter</u>







- The meter is useful for balanced loads
- The two moving coils are so placed that the angle between their planes is 120°. They are connected across two different phases of the supply circuit. Each coil has series resistance.
- ~~There is no need~~
- The required phase displacement between current I_A and I_B in the two moving coils can be obtained from the supply itself.
- Voltage applied across coil A is V_{12} and as its circuit is resistive current I_A is in phase with voltage V_{12}
- Voltage applied across coil B is V_{13} and the current I_B is in phase with V_{13} as the circuit is resistive.

let ϕ = Phase

θ = Angular deflection from plane of refrence

$V_1 = V_2 = V_3 = V$.

• Torque acting on coil A is

$$T_A = KV_{12}\, I\, M_{max} \cos(30° + \phi) \sin(60° + \theta)$$

$$T_A = \sqrt{3} KVI M_{max} \cos(30° + \phi) \sin(60° + \theta)$$

- Torque action on coil B is

$$T_B = KV_{13}\, I\, M_{max} \cos(30^\circ - \phi)\ \sin(120^\circ + \theta)$$

$$T_B = \sqrt{3}\, KVI\, M_{max} \cos(30^\circ - \phi)\ \sin(120^\circ + \theta).$$

- Torque T_A and T_B act in opposite directions and the moving system take up a position where $T_A = T_B$

$$\therefore \cos(30^\circ + \phi)\ \sin(60^\circ + \theta) = \cos(30^\circ - \phi)\ \sin(120^\circ + \theta)$$

Using the relation of $\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

$$\frac{1}{2}[\sin(90+\theta+\phi) + \sin(30+\theta-\phi)] = \frac{1}{2}[\sin(150+\theta-\phi) + \sin(90+\theta+\phi)]$$

$$\therefore \sin(30+\theta-\phi) = \sin(150+\theta-\phi)$$

[Substitute $\theta - \phi = x$]

$$\sin(30+x) = \sin(150+X)$$

$$\frac{\cos x}{2} + \frac{\sqrt{3}}{2}\sin x = \frac{\cos x}{2} - \frac{\sqrt{3}}{3}\sin x.$$

$$\therefore \sqrt{3}\sin x = 0.$$

$$\therefore x = 0.$$

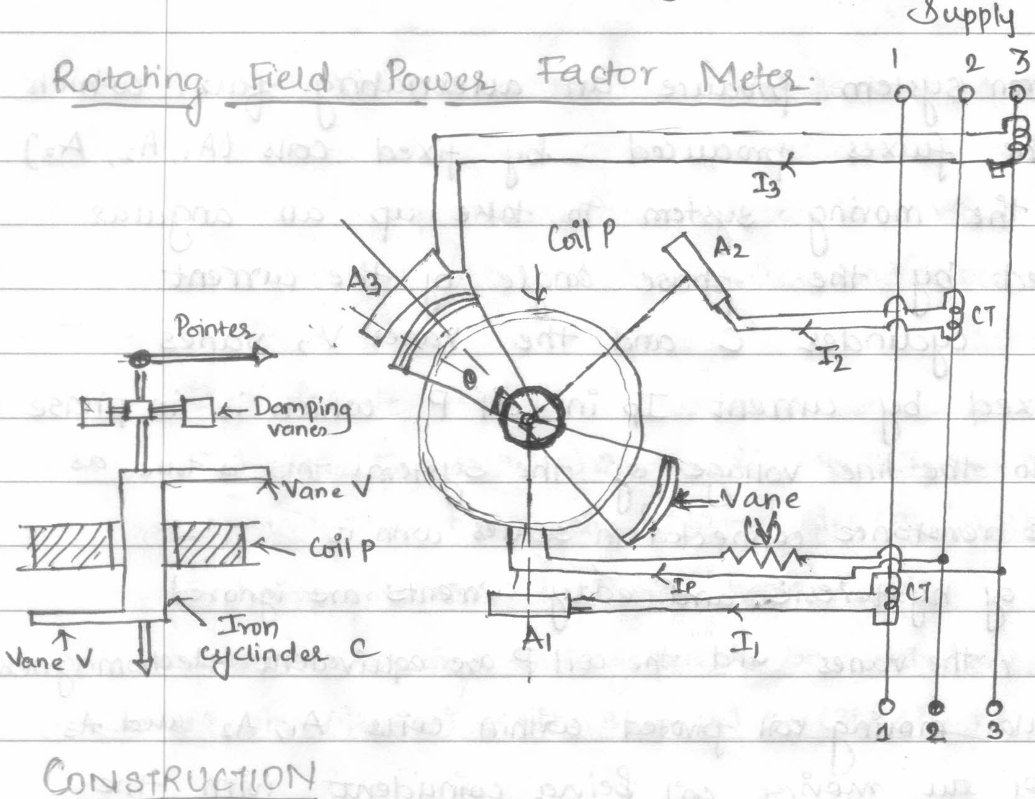
$$\theta = \phi.$$

- Thus the angular deflection of the pointer from the plane of reference is equal to the phase angle of circuit to which the meter is connected.
- Since the currents in the two moving coils are equally affected by any change of frequency, three phase power factor meter is independent of waveform and frequency supply.

MOVING IRON POWER FACTOR METER.

- These moving iron power factor meter can be divided into 2 categories based on their operations of the instrument depends upon
  - → Rotating magnetic field
  - → Number of alternating fields.

Rotating Field Power Factor Meter.



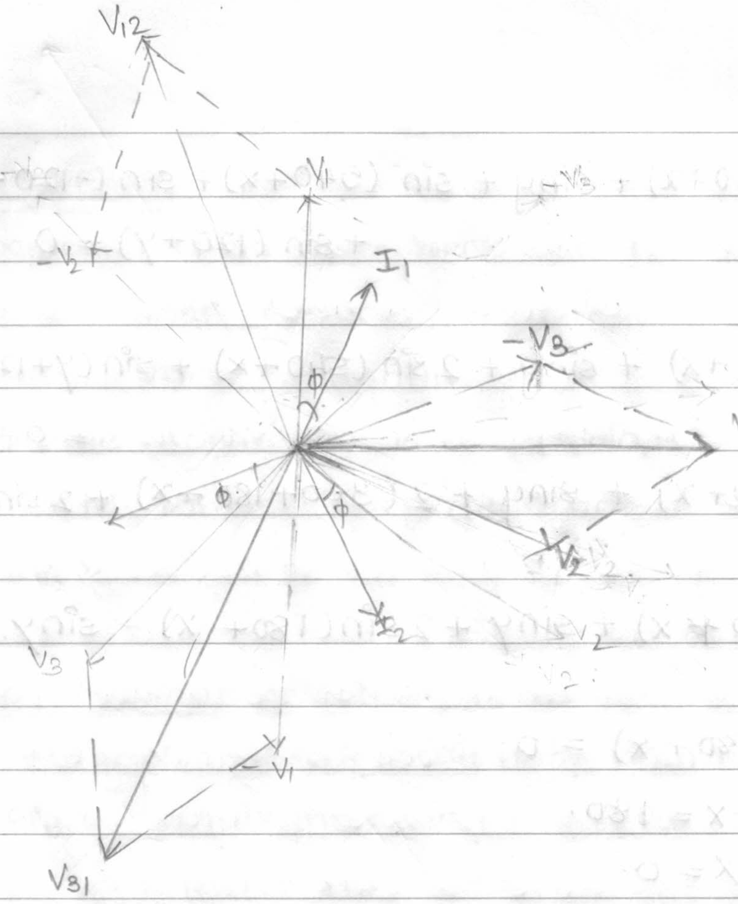
CONSTRUCTION

- A_1, A_2, A_3 are the three fixed coils, with their axes displaced by 120° from each other and intersecting on the centre line of the instrument.
- Three coils (A_1, A_2 A_3) are connected respectively to lines 1, 2 and 3 of 3ϕ supply via CT. (current transformer).
- P is a fixed coil connected in series with high resistance across one pair of lines say for eg 2 and 3

- Iron cylinder C is placed inside the coil P.
- Two sector shaped iron vanes V are fixed to this cylinder and these two vanes are 180° apart in space.
- Spindle carries damping vanes and a pointer. Here ~~no control springs are absent~~ their is an absence of control springs.

WORKING:

- Coil P and iron system produce an alternating flux, which interacts with the fluxes produced by fixed coils (A_1, A_2, A_3) which causes the moving system to take up an angular position determined by the phase angle of the current.
- Considering the cylinder C and the vanes V, vanes V to be magnetized by current I_p in coil P which is in phase and proportional to the line voltage of the system this is true as coil P has a large resistance connected in series with it.
- Then if the effects of hysteresis and eddy currents are ignored the iron cylinder, the vanes and the coil P are equivalent electromagnetically to a rectangular moving coil pivoted within coils A_1, A_2 and A_3 the centre line of the moving coil being coincident with the axes of the iron vanes.



Total deflecting torque

$$T_d \propto [I_1 I_p \cos(90-\phi)\sin(90^\circ+\theta)] + I_2 I_p \cos(330^\circ-\phi)\sin(210^\circ+\theta)$$
$$+ I_3 I_p \cos(210-\phi)\sin(330^\circ+\theta)]$$

- For steady state deflection total torque is zero

Also considering the system to be balanced ie $I_1 = I_2 = I_3$ we have.

$$\cos(90-\phi)\sin(90^\circ+\theta) + \cos(330^\circ-\phi)\sin(210^\circ+\theta) + \cos(210^\circ-\phi)\sin(330^\circ+\theta) = 0.$$

Using relation

$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\frac{1}{2}[\sin(180+\theta-\phi) + \sin(\theta+\phi)] + \frac{1}{2}[\sin(540+\theta-\phi)] + \sin(-120+\theta+\phi)]$$

$$+\frac{1}{2}[\sin(540+\theta-\phi) + \sin(120+\theta+\phi)] = 0.$$

Let $\theta - \phi = x$ and $\theta + \phi = y$.

$\therefore \sin(180+x) + \sin y + \sin(540+x) + \sin(-120+y) + \sin(540+x)$
$+ \sin(120+y) = 0$

$\therefore \sin(180+x) + \sin y + 2\sin(540+x) + \sin(y+120) + \sin(y-120)$
$= 0$

$\therefore \sin(180+x) + \sin y + 2(360+180+x) + 2\sin y \cos 120° = 0$

$\sin(180+x) + \sin y + 2\sin(180+x) - \sin y = 0$

$3\sin(180+x) = 0$

$180+x = 180$

$x = 0$

$\theta = \phi$

Therefore the deflection of iron vane from the refrence axis is directly measure of phase angle between each line current and the corresponding phase voltage.

GALVANOMETERS.

- A galvanometer is an instrument used for detecting the presence of small currents or voltages in a circuit or for measuring their magnitudes.
- Their principle application is in bridges and potentiometers measurements. where there function is to indicate zero current.

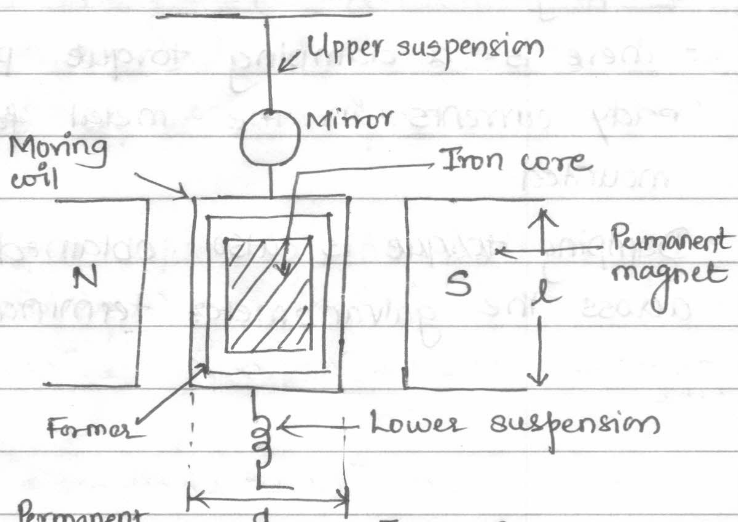
3 Types of Galvanometers

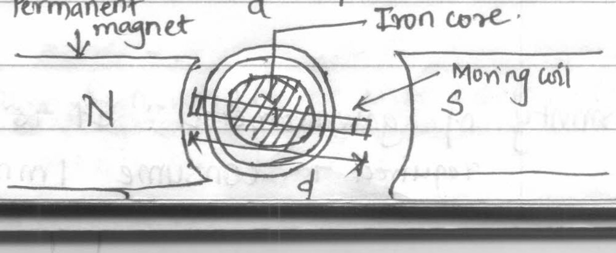
1] Permanent Magnet moving coil type galvanometer [D' Arsonval Galvanometer]
2] Ballistic Galvanometer
3] Vibration Galvanometer.

1] D'Arsonval Galvanometer.

- Construction

1] Moving Coil
2] Damping
3] Suspension
4] Indication
5] Zero setting





- Moving Coil
- It is the current carrying element either in shape of rectangular or circular consists of a number of turns of fine wire
- The coil is suspended so that it is free to turn about its vertical axis of symmetry
- It is arranged in a uniform, radial, horizontal permanent magnet and iron core
- The iron core is spherical in shape if the coil is circular but it is cylindrical if the coil is rectangular.
- The iron core is used to provide a flux path of low reluctance and therefore to produce a ~~flux~~ strong magnetic field for the coil to move in which increases the deflecting torque and hence the *sensitivity of galvanometer.

- Damping
- There is a damping torque present owing to production of eddy currents in the metal former on which the coil is mounted
- Damping ~~torque~~ is also obtained by connecting a low resistance across the galvanometer terminals.

*Sensitivity of galvanometer: It is defined as the current in 1μA required to consume 1mm of deflection on scale.

- **Suspension:**
- The coil is supported by a flat ribbon suspension which also carries current to the coil
- Their is an another current connection in a sensitive galvanometer is a coiled wire which is called as the lower suspension that has a negligible torque effect.
- Such type of galvanometers must be levelled carefully so that the coil hangs straight and centrally.
- Some of the portable galvanometers which do not require exact levelling have "taut suspensions"* consisting of straight flat strips kept under tension for both top and bottom.

- **Indication**
- The suspension carries a small mirror upon which a beam of light is cast.
- The beam of light is reflected on a scale upon which the deflection is measured.

Zero Setting

A torsion* head is provided for adjusting the position of the coil and also for zero setting.

* Torsion head: It is a balance from which the wire suspended and which is usually graduated with an angular scale

* Taut suspension: suspended ~~ass~~ very tightly.

Torque Equation :

Let l = length of coil

d = width of coil

N = Number of turns in the coil

B = Flux density in air gap wb/m^2

i = Current through moving coil, in Amp.

K = Spring constant of suspension, Nm/rad

θ_F = Final steady state deflection of moving coil, rad

Force on each side of coil = $NBil\sin\alpha$.

α = angle between direction of magnetic field and conductor

As field is radial $\alpha = 90°$

$\therefore$ Force on each side of coil = $NBil$

~~Deflecting torque $T_d = Gi$~~

~~where G = displacement constant.~~

~~$G = NBld$~~

~~$\therefore T_d = NBldi$~~

Deflecting torque T_d = force × distance

$= NBil\,d$

$A = ld$

= area of coil m^2

where N, B, A are constants for galvanometers.

Deflecting torque

$T_d = Gi$

where $G = NBA$

$= NBld$

Now the controlling torque exerted by the suspension at deflection θ_F is $T_c = K\theta_F$

For final steady state deflection

$$T_c = T_d$$

$$K\theta_F = Gi$$

$$\theta_F = \frac{Gi}{K}$$

BALLISTIC GALVANOMETER.

- A ballistic galvanometer is used for measurement of quantity of electricity (charge) passed through it.
- In magnetic measurements, this quantity of electricity is due to an instantaneous emf induced in a *search coil connected across the ballistic galvanometer.
- The instantaneous emf is induced in the search coil when the flux linking with the search coil is changed.
- The quantity of electricity passing through the galvanometer is proportional to the emf induced and hence to the change in flux linking with the search coil.
- The galvanometer can therefore be calibrated to read the charge directly.

Search coil (also known as inductive sensor) is a sensor which measures the variation of the magnetic flux.

CONSTRUCTION OF BALLISTIC GALVANOMETERS

- Ballistic galvanometer is normally of the D'Arsonval type, since it is least affected by external magnetic field
- The ballistic galvanometer does not show steady deflection (as in case of current galvanometer) when in use owing to the transitory nature of the current passing through it.
- It oscillates with decreasing amplitude. The amplitude of the first deflection being proportional to the charge passing.

The relationship between charge and the deflection is given as

$$Q = K\theta \quad -(1).$$

where Q = charge in μ Coulombs

K = Galvanometer sensitivity in μC per mm deflection and is determined from the calibration of galvanometer.

θ = deflection in millimeter (mm)

Equation (1) holds good only if the discharge of the electricity through the galvanometer has been completed before any appreciable deflection of the system takes place. Hence, the moving system of such a galvanometer must have a large moment of inertia compared to the restoring moment due to the suspension.

- This condition can be satisfied if the time taken by the charge to pass is small the period of the undamped oscillations of the galvanometer is large.
- The time period of undamped oscillations of a galvanometer is given as

$$T = 2\pi\sqrt{\frac{J}{K}}$$

where T = time period of undamped oscillations.

J = moment of inertia.

K = Galvanometer sensitivity. (control constant of the suspension)

- The moment of inertia (J) of the moving system should be large
- The control constant (K) of the suspension should be small.

The above conditions can be satisfied by attaching small weights to the moving system in order to increase its moment of inertia and by using suspension of smaller stiffness so as to decrease the control constant K.

- The damping of the galvanometer should be small so that the first deflection (swing) is large. After the first deflection (coil) has been observed, electromagnetic damping may be used to bring the movement rapidly to rest.
- A switch to short circuit in the galvanometer is provided to save time in bringing the movement to rest.
- Other considerations in the construction of ballistic galvanometers are that the moving coil should be free from magnetic material, the suspension strip should be carefully chosen and mounted
- The terminals, coil and connections with the galvanometer should be of copper throughout to avoid thermo electric effects
- The suspension is non-conducting and the current is fed into the deflection coil by delicate spirals of very thin copper strips.

Thermo electric effect: It is the direct conversion of temp. differences to electric voltage and vice-versa

Torque Equation:

- The moving system deflects only after the charge has completely passed through the meter
- The moving system has to deflect in order that the energy imparted to it by the charge is dissipated gradually in friction and electromagnetic damping.
- But during the actual motion, there is no deflecting torque as there is not current through the coil as the charge would have already passed.

Suppose Q is the charge to be measured

$$\therefore Q = \int i\,dt$$

$$\text{or } i = \frac{dQ}{dt}$$

let the instant of time during which the charge Q passes be defined as the time between $t = t_0 = 0$ and $t = t_1$

Between the time interval t_0 to t_1, there is no motion of the coil of the galvanometer and the equation of motion is

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = Gi$$

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = G\frac{dQ}{dt}$$

$$\frac{d^2\theta}{dt^2} + \frac{D}{J}\frac{d\theta}{dt} + \frac{K}{J}\theta = \frac{G}{J}\frac{dQ}{dt}$$

Integrating

$$\frac{d\theta}{dt}\Big|_{t_0=0}^{t_1} + \frac{D}{J}\theta\Big|_{t_0=0}^{t_1} + \frac{K}{J}\int_0^{t_1}\theta\, dt = \frac{G}{J}Q$$

As θ is zero between t_0 & t_1

$$\frac{d\theta}{dt} = \frac{G}{J}Q$$

After passage of charge no current passes through the coil and so deflecting torque is zero

Thus the equation of motion after time t_1 is

$$J\frac{d^2\theta}{dt^2} + D\frac{d\theta}{dt} + K\theta = 0$$

The solution of this equation is given by

$$\theta = \exp\left(\frac{-Dt}{2J}\right)[\sin(\omega_d t + \phi)] + \theta_F$$

As galvanometer does not show any steady state deflection

$\theta_F = 0$

Also since damping is small $\omega_d = \omega_n$

$$\therefore \theta = F\exp\left(\frac{-Dt}{2J}\right)[\sin(\omega_n t + \phi)]$$

Differentiating the above equation

$$\frac{d\theta}{dt} = F\omega_n \exp(-D/2J)[\cos(\omega_n t + \phi)] - F\left(\frac{D}{2J}\right)\exp\left(\frac{-D}{2J}\right)[\sin(\omega_n t + \phi)]$$

The initial conditions at $t=0$ are $\theta = 0$ and $\frac{d\theta}{dt} = \left(\frac{G}{J}\right)Q$

$F\sin\phi = 0$ or $\phi = 0$

$$F = \frac{G Q}{J \omega_n}$$

$$\theta = (G / J\omega_n)\, Q \exp(-Dt/J) \sin \omega_n t$$

$$\therefore \theta = (G/J) \sqrt{J/K}\, Q \exp(-Dt/2J) \sin \sqrt{K/J}\, t$$

$$= A\, Q \exp(-Dt/2J) \sin (2\pi / T_0) t$$

$$\left[\text{As } \omega_n = \sqrt{\frac{K}{J}} = \frac{2\pi}{T_0}\right]$$

where $A = \left(\frac{G}{J}\right)\left(\sqrt{\frac{J}{K}}\right)$.

Thus deflection θ is proportional to charge Q at any instant the motion of the galvanometer is oscillatory with a decreasing amplitude.

VIBRATION GALVANOMETER

- CONSTRUCTION, WORKING, TORQUE EQUATION

- Vibration galvanometers are suitable for use at power and low audio frequencies. In general vibration galvanometers are available for various frequencies between 5Hz to 1000 Hz. But in most of the applications vibration galvanometer are used at frequencies below 250 Hz.
- Vibration galvanometers operates with a moving system which may be either moving magnet or moving coil and the basic moving system is tuned at operating frequency to *mechanical resonance
- The damping is very small in these galvanometers this is required in order to obtain a sharp resonance curve so that the galvanometer is selective.

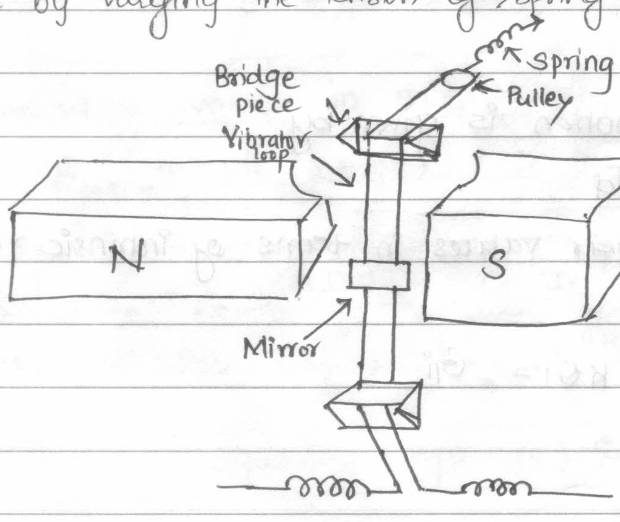
Mechanical Resonance: It is the tendency of a mechanical system to respond at greater amplitude when the frequency of its oscillations matches the system's natural frequency of vibration.

CONSTRUCTION:

- The fig shows the construction of Duddell's moving coil vibration galvanometer
- The moving coil consists of a fine bronze or platinium silver wire. This wire passes over a small pulley at the top and is pulled tight by a spring attached to the pulley, the tension of the spring can be adjusted by tuning a milled head attached to the spring.
- Two ivory bridge pieces are used to stretch the loop of wire & the distance between these pieces is adjustable.
- When a.c current flows through, the moving coils vibrate and thus the reflected beam from the mirror spreads a band of light upon a scale.

<u>Tuning</u> Tuning means adjustment of the natural frequency of the moving system so that it is equal to the frequency of the current passing through the coil.

- In this case of galvanometer coarse tuning is done by varying the distance between the bridge pieces.
- This varies the length of the loop which is free to vibrate and thus varies the natural frequency of the system. Fine adjustment of tuning is done by varying the tension of spring



- When the galvanometer is tuned (ie. when it is under mechanical resonance conditions) the amplitude of vibrations is very large and consequently a very wide band of light is observed on the side.

WORKING:

- When an alternating current is passed through the moving coil, an alternating deflecting torque is produced which makes the coil vibrate with a frequency equal to the frequency of the current passing.
- On account of inertia of the moving parts, the amplitude of vibration is small. However if the natural frequency of the current, mechanical resonance is obtained and the moving system vibrates with the large amplitude.

TORQUE EQUATION.

- The 4 basic galvanometer constants are namely.

(i) Displacement constant (G)
(ii) Constant of inertia (J)
(iii) Damping constant (D)
(iv) Control constant (K)

Now the equation of motion is given by.

$$T_i + T_D + T_C = T_d$$

Thus replacing terms by their values in terms of intrinsic constant of galvanometer we get

$$\frac{Jd^2\theta}{dt^2} + \frac{Dd\theta}{dt} + K\theta = Gi$$

Let the a.c current flowing through the coil is given by.

$$i = I_m \sin \omega t \quad \text{— (1)}$$

Putting value of i in eq^n (1) we get

$$\frac{J d^2\theta}{dt} + \frac{D d\theta}{dt} + K\theta = G (I_m \sin \omega t) \quad \text{— (2)}.$$

~~eq~~ eq^n (2) is II^nd order differential eq^n

The expression for θ i.e. solution of eq^n (2) consists of two parts added together namely Particular integral (P.I) and complementary function (CF)

A galvanometer is underdamped system, hence the complementary function

$$\theta = Fe^{\frac{-D}{2J}t} [\sin(\omega d t + \phi)] \quad \text{— (3)}$$

To find ~~Par~~ (P.I) consider eq^n (2) again. The P.I for eq^n (2) is in the form

$$\theta = A \sin(\omega t - \alpha)$$

$$\frac{d\theta}{dt} = A\omega \cos(\omega t - \alpha) \quad \text{and}$$

$$\frac{d^2\theta}{dt^2} = -A\omega^2 \sin(\omega t - \alpha)$$

Substituting values in eq^n (2) we get

$$J[-A\omega^2 \sin(\omega t - \alpha)] + D[A\omega \cos(\omega t - \alpha)] + K[A \sin(\omega t - \alpha)]$$
$$= G (I_m \sin \omega t) \quad \text{— (4)}$$

When $\omega t = \alpha$ $AD\omega = G[I_m \sin \alpha]$ — (5)

When $(\omega t - \alpha) = \frac{\pi}{2}$, $-AJ\omega^2 + AK = G[I_m \cos \alpha]$ — (6)

Squaring eqn (5) & (6)

$$A^2 D^2 \omega^2 + A^2 (K - J\omega^2)^2 = G^2 I_m^2 (\sin^2\alpha + \cos^2\alpha)$$

$$\therefore A^2 D^2 \omega^2 + A^2 (K - J\omega^2)^2 = G^2 I_m^2 \quad \text{— (7).}$$

Solving for A

$$A^2 [(D\omega)^2 + (K - J\omega^2)^2] = G^2 I_m^2$$

$$A^2 = \frac{G^2 I_m^2}{(D\omega)^2 + (K - J\omega^2)^2}$$

$$A = \frac{G I_m}{\sqrt{(D\omega)^2 + (K - J\omega^2)^2}} \quad \text{— (8)}$$

Now dividing eqn (5) by eqn (6) we get

$$\frac{AD\omega}{-AJ\omega^2 + AK} = \frac{G[I_m \sin\alpha]}{G[I_m \cos\alpha]}$$

Rearranging terms

$$\frac{AD\omega}{-A[K - J\omega^2]} = \frac{\sin\alpha}{\cos\alpha}$$

$$\tan\alpha = \frac{D\omega}{[K - J\omega^2]}$$

$$\alpha = \tan^{-1}\left[\frac{D\omega}{(K - J\omega^2)}\right] \quad \text{— (9)}$$

Hence particular integral can be written as

$$\theta = A \sin(\omega t - \alpha)$$

ie. $$\theta = \frac{G I_m}{\sqrt{(D\omega)^2 + (K - J\omega^2)^2}} \sin\left[\omega t - \tan^{-1}\left[\frac{D\omega}{K - J\omega^2}\right]\right] \quad \text{— (10)}$$

Hence the complete solution of eqn (2) can be written as

$$\theta = CF + PI$$

$$\theta = Fe^{-\frac{D}{2J}t} \sin(\omega dt + \phi) + \frac{GI_m}{\sqrt{(D\omega)^2 + (K - J\omega^2)^2}} \sin\left[\omega t - \tan^{-1}\left(\frac{D\omega}{K - J\omega^2}\right)\right] \quad -(11)$$

The first term in above solution represents the transient part of the solution. This term is effective only for deflection ~~for~~ few/ fir for very few first vibrations after instrument is switched on.

The deflection of the galvanometer is given by.

$$\theta = \frac{GI_m}{\sqrt{(D\omega)^2 + (K - J\omega)^2}} \sin(\omega t - \alpha)$$

As the amplitude of vibration is given by

$$A = \frac{GI_m}{\sqrt{(D\omega)^2 + (K - J\omega^2)^2}}$$

The amplitude can be increased for particular value of I_m of current i by increasing G or by decreasing $[(D\omega)^2 + (K - J\omega^2)]^2$

General Observation:

1) The amplitude can be increased by increasing G

But as we know $G = NBA$

To increase amplitude of vibration, G can be increased by using coil of large area A and greater number of ~~of~~ turns. value of G can be increased by increasing flux density B. Thus in vibration galvanometer powerful magnets are preferred to give greater amplitude of vibration increasing sensitivity of galvanometer.

2) The amplitude of Vibration can be increased by decreasing value of $[(D\omega)^2 + (K - J\omega^2)^2]$ term. Hence three constants are involved. For an existing galvanometer consider that J and D are fixed. Then K can be varied by adjusting length and tension of suspension of moving system. To have, value of the above term, minimum condition is

$$K - J\omega^2 = 0$$

ie. $K = J\omega^2$

$$\omega = \sqrt{\frac{K}{J}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{J}}$$

But this nothing but natural undamped frequency f_n of the moving system

Hence for maximum ~~apl~~ amplitude of vibration

$$f = f_n$$

In this way, the vibration galvanometer can be tuned by varying length and tension of the moving system to have natural frequency of moving system (ie f_n) equal to the supply frequency