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..... **“Kinematic and Dynamic Modelling of a Drone/robot following a trajectory using MATLAB”** .....

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# Kinematic and Dynamic Modeling of a Quadcopter following a trajectory using MATLAB

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**Abstract** - Our project consists of plotting various outputs of kinematic and dynamic motion with respect to time. This output is obtained by making a quadcopter to trace a prescribed trajectory in the MATLAB ra13 simulation platform. The centre of the mass of the quadcopter is maintained at the centre of the quadcopter and the following governing kinematic and dynamic equations are prepared. These equations derived are used for the inception of the kinematic and dynamic modelling of the quadcopter made to trace the trajectory. The outputs of the kinematic and dynamic modelling are made the input for the correction of error input and tuning of the PID control system. This closed function PID control system is used for the stabilization of the quadcopter while following a given trajectory and optimize the output of the kinematic and dynamic modelling process for tuning purpose of the control. **Index Terms** - List key index terms here. No more than 5.

## I. INTRODUCTION

A helicopter is a flying vehicle which uses rapidly spinning rotors to push air downwards, thus creating a thrust force keeping the helicopter aloft. Conventional helicopters have two rotors. These can be arranged as two coplanar rotors both providing upwards thrust, but spinning in opposite directions (in order to balance the torques exerted upon the body of the helicopter). The two rotors can also be arranged with one main rotor providing thrust and a smaller side rotor oriented laterally and counteracting the torque produced by the main rotor. However, these configurations require complicated machinery to control the direction of motion; a swashplate is used to change the angle of attack on the main rotors. In order to produce a torque the angle of attack is modulated by the location of each rotor in each stroke, such that more thrust is produced on one side of the rotor plane than the other. The complicated design of the rotor and swashplate mechanism presents some problems, increasing construction costs and design complexity. A quadrotor helicopter (quadcopter) is a helicopter which has four equally spaced rotors, usually

arranged at the corners of a square body. With four independent rotors, the need for a swashplate mechanism is alleviated. The swashplate mechanism was needed to allow the helicopter to utilize more degrees of freedom, but the same level of control can be obtained by adding two more rotors. The development of quadcopters has stalled until very recently, because controlling four independent rotors has proven to be incredibly difficult and impossible without electronic assistance. The decreasing cost of modern microprocessors has made electronic and even completely autonomous control of quadcopters feasible for commercial, military, and even hobbyist purposes. Quadcopter control is a fundamentally difficult and interesting problem. With six degrees of freedom (three translational and three rotational) and only four independent inputs (rotor speeds), quadcopters are severely underactuated. In order to achieve six degrees of freedom, rotational and translational motion are coupled. The resulting dynamics are highly nonlinear, especially after accounting for the complicated aerodynamic effects. Finally, unlike ground vehicles, helicopters have very little friction to prevent their motion, so they must provide their own damping in order to stop moving and remain stable. Together, these factors create a very interesting control problem. We will present a very simplified model of quadcopter dynamics and design controllers for our dynamics to follow a designated trajectory. We will then test our controllers with a numerical simulation.

Unmanned air vehicles (UAVs) are self-propelled aerial robots. They can be equipped with various instruments and payloads, making them capable of performing various civilian or military tasks. Among existing small UAVs, we find quadrotors which are Vertical Take-Off and Landing (VTOL) four rotor helicopters. They are controlled simply by changing the rotation speed of the four rotors. The front and rear rotors (2, 4) rotate in a clockwise direction while the left and right rotors (1, 3) rotate in a counter-clockwise direction to balance the torque created by the spinning rotors. The up/down motion

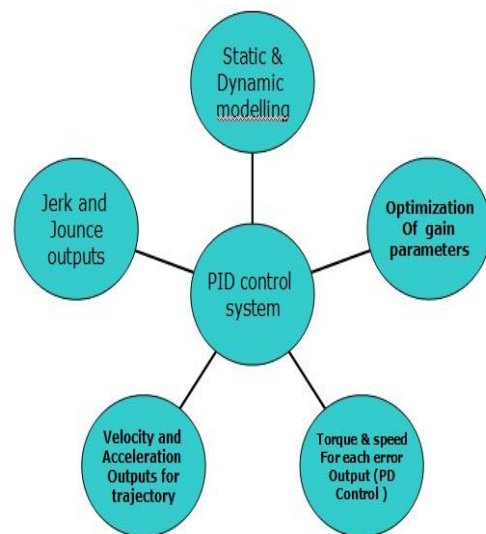
is achieved by increasing/decreasing the rotors speed while maintaining an equal individual speed. The forward/backward, left/right motions are achieved through a differential control strategy of rotors speed. Thanks to this configuration, quadrotors are able to hover, takeoff, and land in small areas and enable them to perform tasks that fixed-wing craft are unable to do.

## 2.PROBLEM DEFINITION

- to carry out mathematical analysis of a quadcopter following various forms of path
- to derive generalized governing kinematic and dynamic equations of the quadcopter assuming its centre of gravity to be at the centre of the quadcopter
- to carry out body frame analysis on various co-ordinate scales:-
  - Earth-frame(E-frame) analysis:- in this form of analysis the earth surface is taken as the reference and the x cordinte is assumed to be the north pole , the y cordinate is assumed to be the east and the z cordinate is assumed towards the earth.This also known as absolute or generalized cordinates system.
  - body -frame analysis (B-frame):-in this form of analysis the x, y & z cordinate is set as per the orientation and is more convinient way of rigid body motion analysis.
- to carry rigid body motion analysis usng MATLAB ra13 for simulation of yaw motion.
- to carry trajectory analysis of quadcopter tracing a path using MATLAB ra13 for four conditions:-
  - hovering
  - pitching
  - rolling
  - yawing
 this all orientations are conditional for the simulation of positonal error of quadcopter in a with respect to a time frame in seconds in 1 dimensional and 3 dimensional mode of results.
- to evaluate the error with respect to time over the output error function from the kinematic and dynamic modelling for a PID control sytem.
- to set adequte gain parameters for a PID control system so as to optimize stabalization parameter of a quadcopter and prevent overshoot of the system by adequate tuning of the PID gain parameters.

## 3.OBJECTIVE OF WORK

- design and develop kinematic and dynamic simulation models for a quadcopter moving over a trajectory
- study the positon and time variables of the motion of quadcopter in 1 dimensional and 3 dimensional time variant scale
- study and obtain the change in position , velocity and acceleration of the quadcopter motion with respect to the variant time scale
- study and obtain the results of third and fourth order derivative motions such as jerk and jounce and its error cost function analysis (J).
- enable PID control system with automatic tuning to avoid steady state error by setting adequte and proper gain parameters viz:  $k_p$  ,  $k_d$  ,  $k_i$  so as to avoid overshoot to large error function value.



## 4.METHODOLOGY

The following steps will be adopted to prepare kinematic and dynamic model and get ouptut results on MATLAB ra13:-

- study various thrust equations and propelling conditions for:-
  - hovering
  - rolling
  - pitching
  - yawing

- study , analyse and evaluate the governing kinematic and dynamic equations for computation of position and time variable of a moving quadcopter over a trajectory.
- computing the above results on MATLAB ra13 simulation platform and obtain respective results of position versus time analysis by writing algorithms and commands on the simulation platform
- computing the inertia and body frame equations of rigid body dynamics on MATLAB ra13 simulation platform.
- comparing the results and obtain the error function as result of deviation of quadcopter displacement with the reference trajectory.
- optimizing the error function in the form "cost-function analysis" to enable the PID control system with automatic Tuning
- decide gain parameters by testing against the steady state error for automatic tuning of PID Control system.
- take the output of the kinematic and dynamic modelling and make it as input for evaluating the error function with respect to time to nullify the steady state error.
- develop and build SIMULINK\* model of kinematic and dynamic model and control inputs to the PID control system on MATLAB ra13 simulation platform.

#### 5. EXPECTED OUTCOMES

- 1D and 3D time frame analysis of variation of position of quadcopter with the reference trajectory for:-
  - hovering
  - rolling
  - pitching
  - yawing
- rigid body analysis for yawing motion of quadcopter
- thrust calculations for respective motor torque each for hovering , rolling ,

pitching ,and yawing motions of quadcopter moving over a trajectory.

- adequate input coupling of modelling data for the optimization of gain parameters for proper automatic tuning of PID control system to nullify steady state error in the input data.
- full fledged SIMULINK\* model of the kinematic and dynamic modelling and simulation , control inputs and PID control for input data and its automatic tuning for gain parameters on the platform of MATLAB ra13 simulation.

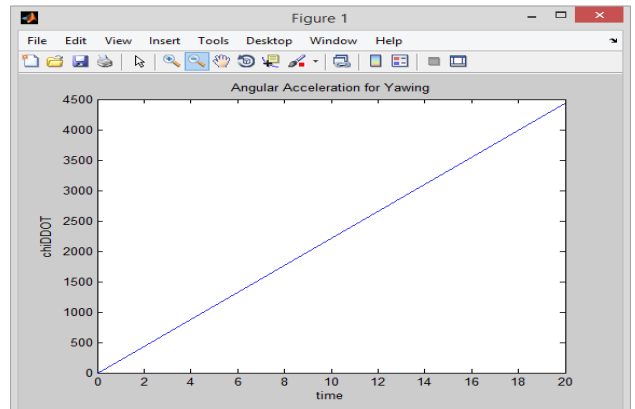
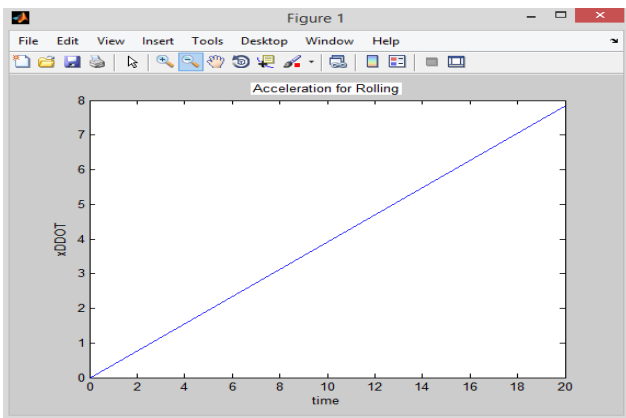
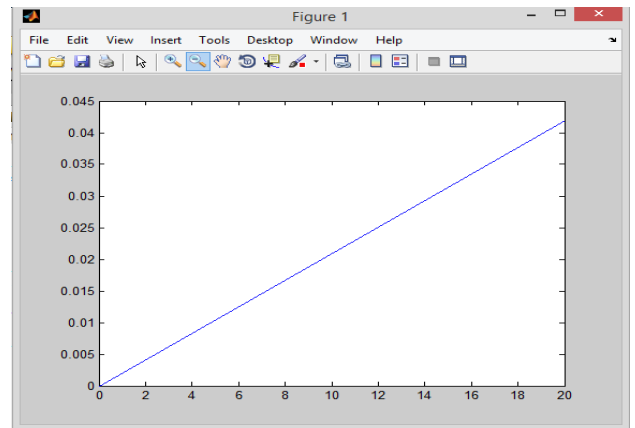
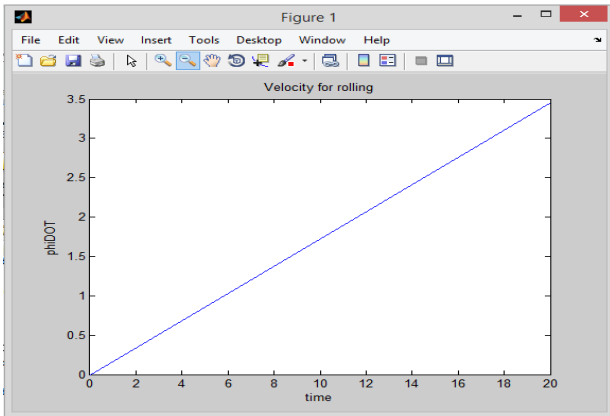
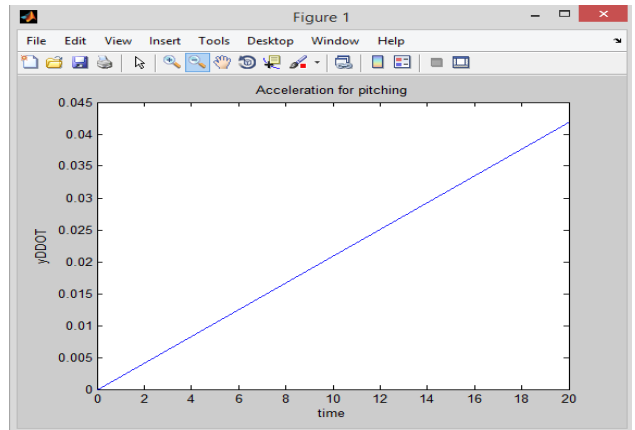
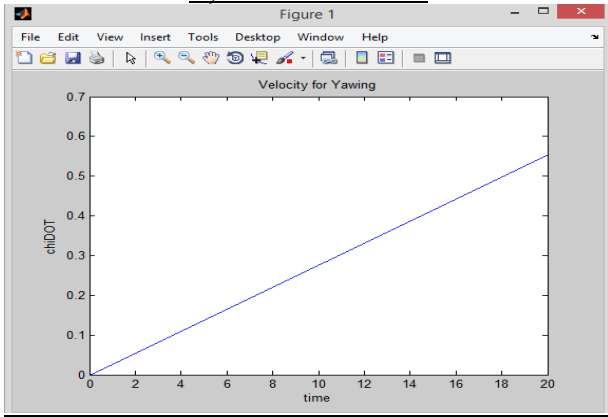
#### 6. VALIDATION OF WORK

Validation accounts for feasibility of the work criteria set for the project and expected outcomes of the project our measure of validation of :-

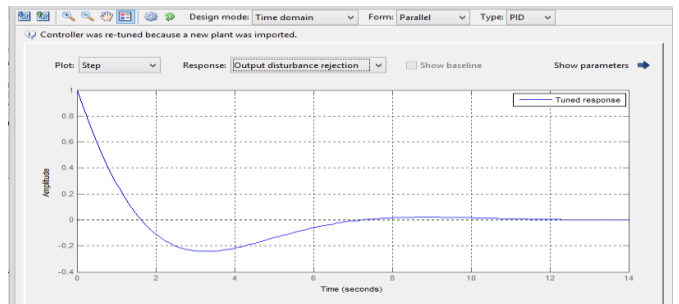
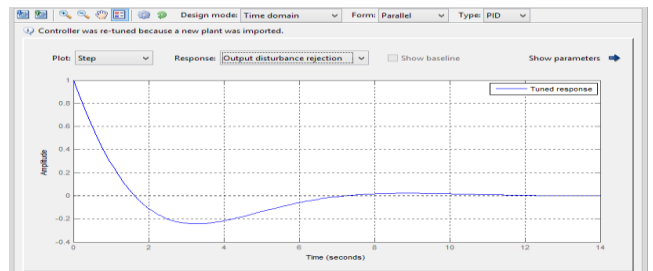
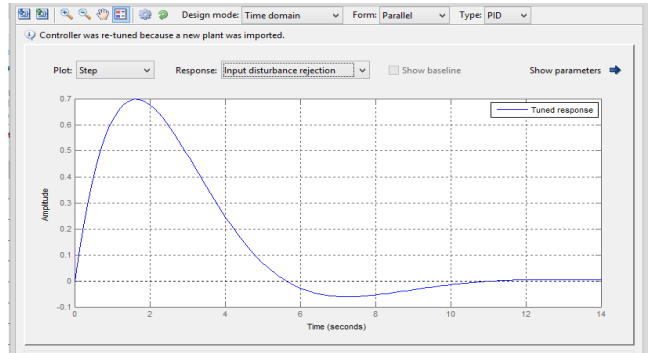
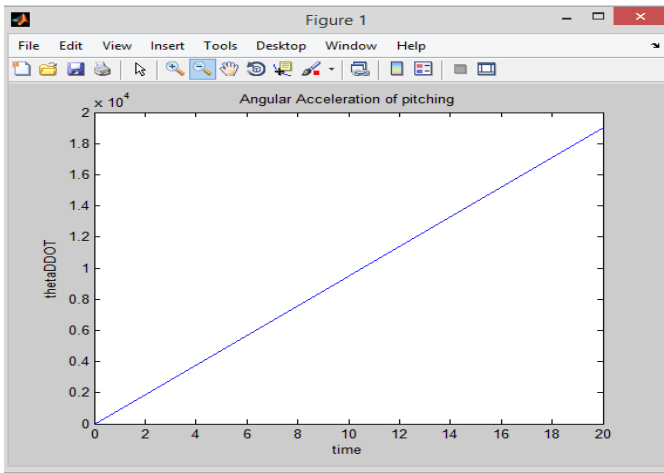
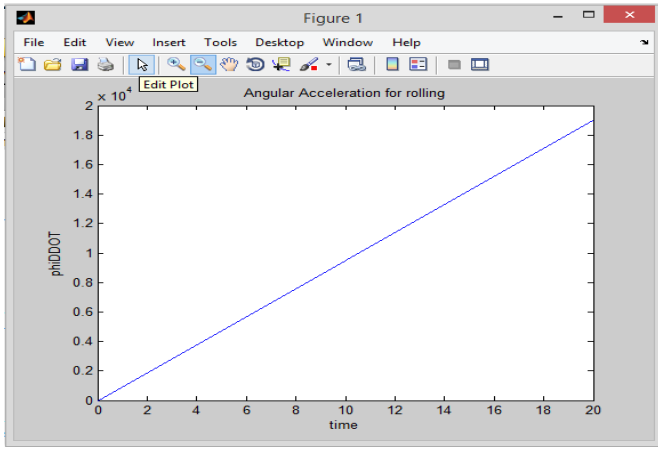
- the experimentation and computational results almost can be equilised.
- since because the computational platform is in real time analysis and multi scale modelling done on MATLAB ra13 simulation platform the results can be easily aggregated to the actual ones.
- The tuning of PID control system through automatic tuning system being done by analysng and optimizing gain parameters may show a better version of results as far as stabilizing and precise operation and motion of a quadcopter is concerned on a given trajectory.
- usage of B-frame analysis and "exact differential equations" for implementation of kinematic and dynamic modelling ensures the interpolation of solutions for the equations rather than approximation.
- developing a SIMULINK\* model ensures interpretation of results and control inputs and improves feasibility for changing output variables with varying inputs instead referring the entire algorithm.
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#### 7. OUTPUT OBTAINED

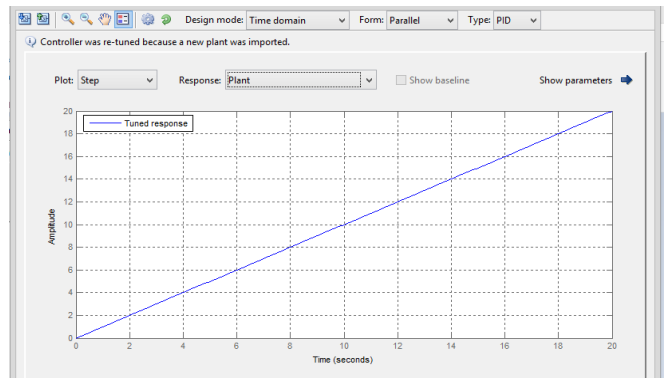
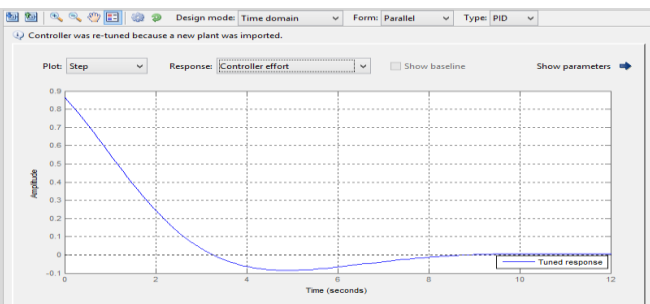
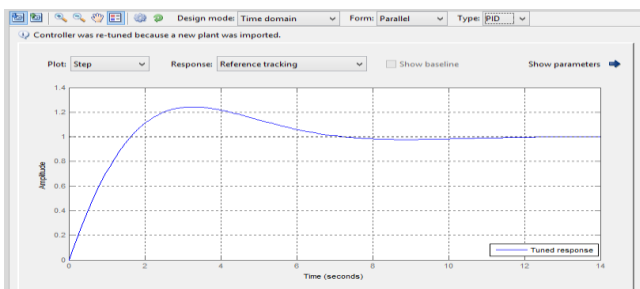
## Dynamic Simulations:-



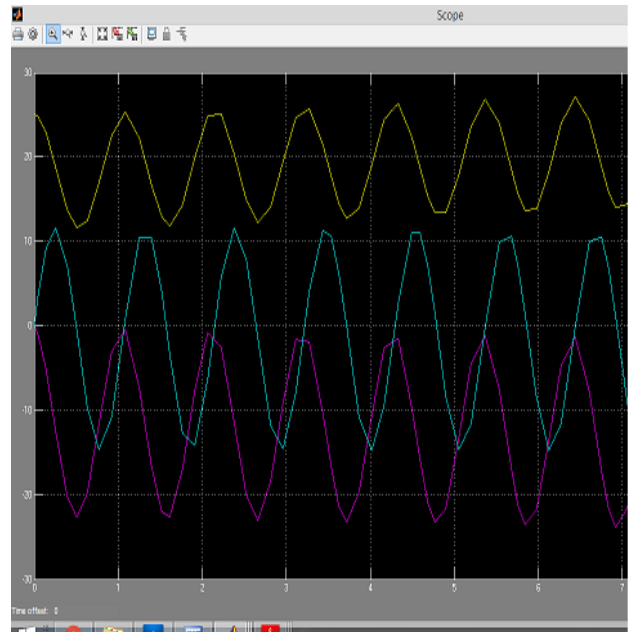
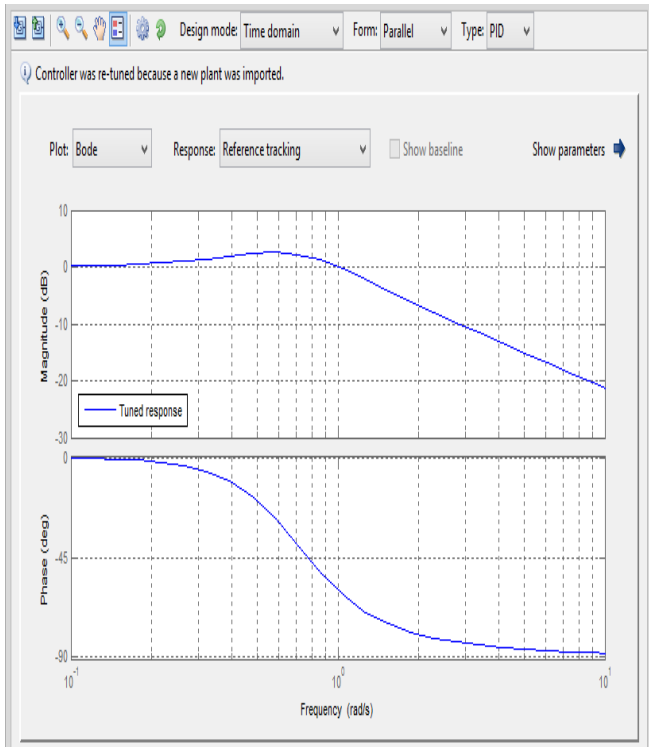




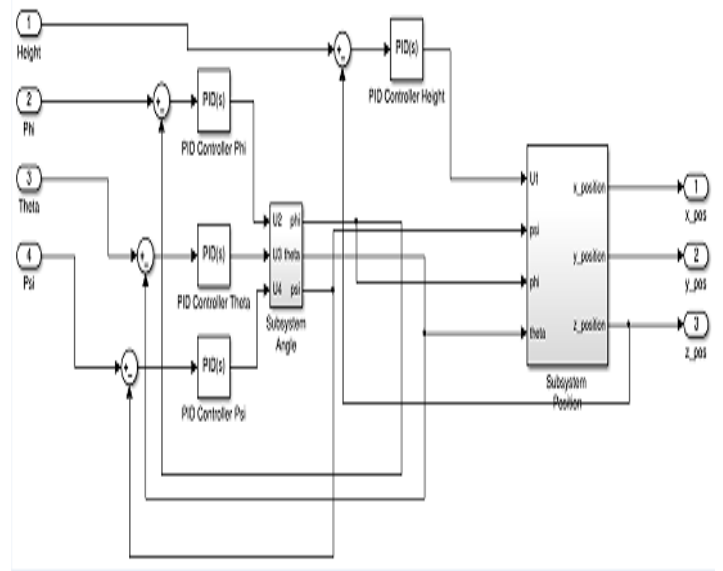
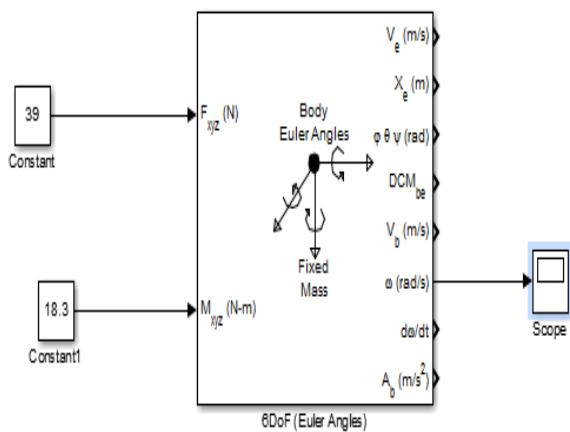
**PID Plant model analysis:-**



**Bode Plot Stability Analysis for phase margin and gain margin:-**



**SIMULINK Model result using Forward Kinematics:-**



## ACKNOWLEDGMENT

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