Role of Electrical m/c in generation, transmission & distributing of power:

P = VI, I²R, P = I²R I

Electrical energy occupies the top most grade in energy hierarchy. It finds innumerable uses in home, commercial establishments, industry, transport, agriculture. This is because it is practically possible to transport it instantaneously, is pollution free & can be controlled very easily. Hence it is attractive over other forms of energy. The efficient use of electricity has been made possible only due to role of electrical machines in generation, transmission & distribution of power.

Electrical Energy is generated in large size thermal, hydroelectric & nuclear power stations. The key equipment in these power station is the alternator, which converts mechanical energy of the Turbine into electrical energy.

Generation voltage in india is 11kV. In some developed countries it is 33kV. If electrical energy is to be transmitted to distant areas (load centers) at this voltage, the I^2R losses would be enormous. To decrease the losses the transmission voltages are much higher 132kV, 220kV, 400kV, 765kV. This conversion of generating voltage to transmission voltage is done by step-up transformers. At load centers the voltage is stepped down to lower values suitable for consumer use. This is done by step-down transformers.

Types of Machines:

- AC machines:
  - (i) Transformers
  - (ii) Syncronous m/c
  - (iii) Induction m/c [used in industries] tube wells (agriculture) domestic home
  - (iv) Special m/c.

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• DC Machines:

(i) DC Motor

(ii) DC Generator.

} electric locomotives.

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Module - Electromechanical Energy Conversion

- An electromechanical energy conversion device is one which converts electrical energy into mechanical energy or alternatively mechanical energy into electrical energy.
- The energy storing capacity of the magnetic feild is much greater & hence electromechanical energy conversion takes place via the medium of magnetic feild.
- DC, induction, syncronous m/c are widely used for electromechanical energy conversion. When the conversion takes place from electrical to mechanical form, the device is called motor. When the mechanical energy is converted to electrical energy the device is known as generator.
- In these m/c, the following two electromagnetic phenomena takes place;
  (1) When a conductor moves in a magnetic feild, voltage is induced in the conductor.
  (2) When a current-carrying conductor is placed in magnetic feild, it experiences a mechanical force.

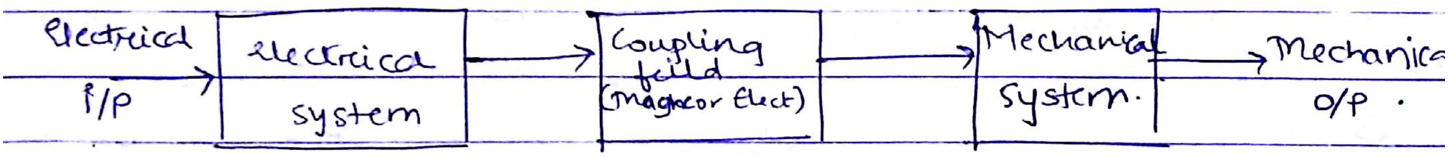


fig: electromechanical energy conversion principle.

The motoring action:

- Current is made to flow through (Armature) conductors placed in magnetic feild. A force is produced on each

* Model of an electromechanical Energy conversion device

conductor. The conductor is placed on the rotor which free to move

- An electromagnetic torque is produced on the rotor which then starts to rotate at a speed.
- The torque produced on the rotor is transfered on the shaft of the rotor & can be utilized to drive a mechanical load.
- Since the conductors are placed in magnetic field, some voltage is induced in the conductors

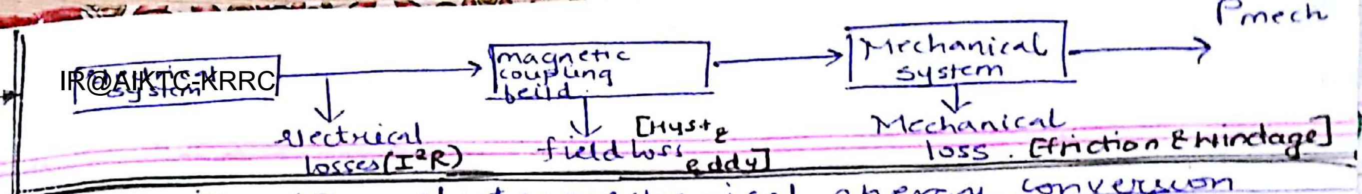
The Generation Action:

- The rotor is driven by prime mover. A voltage is induced in the rotor conductors.
- If an electrical load is connected to the winding formed by these conductors, a current will flow delivering electrical power to the load.
- Moreover the current flowing through the conductors will interact with magnetic field to produce a reaction torque which will tend to oppose the torque developed by the prime mover.

- Energy balance Equation:

- According to the principle of conversation of energy, energy can neither be created nor destroyed It is converted from one form to another.
- In the energy conversion device, the total i/p energy is equal to the sum of the following three components.

Energy dissipated, Energy stored, Useful o/p Energy.



- Thus with the electromechanical energy conversion device, the energy balance equation can be written as;

[Electrical energy i/p] = [energy to electrical losses] + [energy to feild storage in electrical system] + [Mechanical energy o/p] → ①

→ (for motoring Action)

[Mechanical energy i/p] = [Electrical energy o/p] + [total energy stored] + [total Energy dissipated] → ②.

→ (for Generating Action)

- Forms of energies :
- The various forms of energies in equation ① are.

(1) Total electrical energy i/p from the main supply.

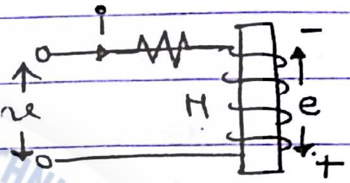
(2) Total energy dissipated
= (energy dissipated in electrical ckt as ohmic losses) +
(Energy dissipated in magnetic core as hysteresis and eddy-current loss) +
(energy dissipated in mechanical system as friction & windage losses)

3) Total energy stored in any electromechanical energy conversion device =
(Energy stored in magnetic feild Wes) +
(Energy stored in mechanical system Wms, a potential or kinetic energy)

4) All the mechanical energy does not appear as useful O/P energy. A fraction is dissipated in mechanical losses (friction & windage).

· Energy stored in magnetic feild:

· Consider a coil of 'N' turns wound around magnetic core connected to a voltage source, By KVL we have;



$$v = iR + e.$$

$e \rightarrow$ Voltage induced in the coil

$R \rightarrow$ Resistance of coil ckt.

$i \rightarrow$ Current through the coil.

$v \rightarrow$ Voltage of source.

· Instantaneous power to the system is given by,

$$p = vi = (iR + e)\, i = i^2R + ei$$

· Suppose that a dc voltage is applied to the circuit at time $t=0$ and that at end of T seconds current attained a value of 'I' amperes. The energy I/P to this system during the interval is

$$W_i = \int_0^T p\, dt = \underbrace{\int_0^T i^2R\, dt}_{\text{energy dissipated as Resistance loss}} + \underbrace{\int_0^T ei\, dt}_{\text{energy stored in magnetic feild.}}$$

so;

$$W_f = \int_0^T e\, i\, dt.$$

. By faraday's Law;

$$e = \frac{d\psi}{dt} = \frac{d}{dt}(N\phi)$$

$\psi \rightarrow$ magnetic flux linkage.

$$\therefore W_f = \int_0^T \frac{d\psi}{\cancel{dt}} . i\, \cancel{dt}$$

$$= \int_0^{\psi} i\, d\psi. \longrightarrow (3)$$

Alternatively;

$$W_f = \int_0^T \frac{d\psi}{dt} . i\, dt = \int_0^T \frac{d}{\cancel{dt}}(N\phi) - i\, \cancel{dt}$$

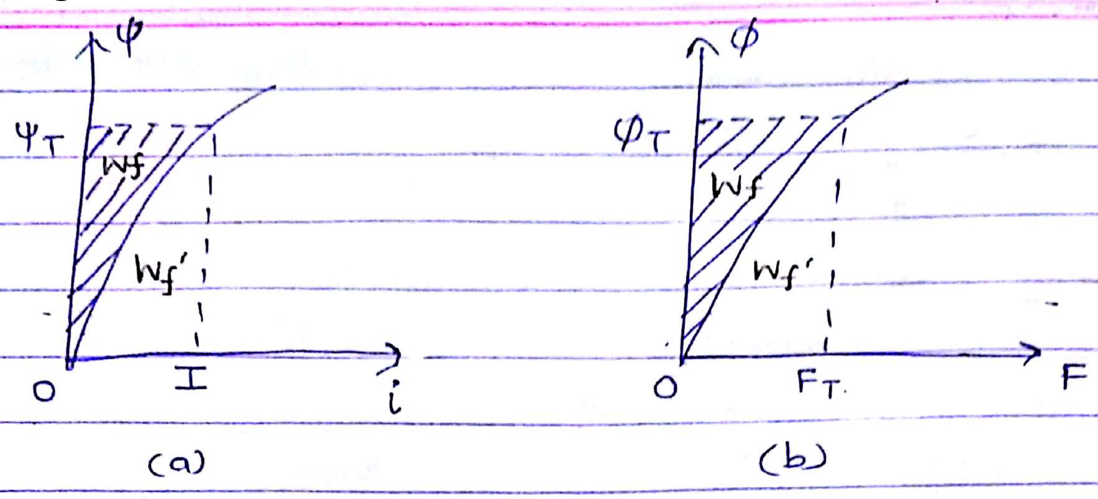
$$= \int_0^T N\, i\, d\phi$$

$$= \int_0^{\phi_T} F\, d\phi \longrightarrow (4)$$

where; $F = Ni \rightarrow$ mmf.

Equation (3) shows that the energy stored in the magnetic field is equal to the area between the ψ-i curve for the system and the flux linkage (ψ) axis.

The energy stored in the magnetic field is also equal to the area between ϕ-F curve and the flux axis as shown by the graphs below.



• Graphical representation of Energy & Co-Energy.

• Co-energy:

- The area between the magnetisation curve & the current (i) or mmf (F) axis is called Co-energy
- It is denoted by W_f'
- It has no physical significance
- However, it is used to derive expressions for force or torque developed in an electromagnetic system

For a linear system,

$$\underset{\text{(feild energy)}}{W_f} = \int_0^{\psi} i \, d\psi = \int_0^{\psi} \frac{\psi}{L} \, d\psi = \left(\frac{1}{L}\right)\frac{\psi^2}{2}$$

Since $\psi = Li$

$$\therefore W_f = \left(\frac{1}{L}\right)\frac{L^2 i^2}{2} = \frac{1}{2} L i^2$$ $\Rightarrow$ feild energy for linear magnetic system

• Co-energy is given by

$$W_f' = \int_0^i \psi \, di = \int_0^i L i \, di = \frac{1}{2} L i^2 .$$

$\Rightarrow$ Co-energy for linear magnetic system

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- Thus, for linear magnetic system, the feild energy & co-energy are equal.

$$\boxed{W_f = W_f' = \frac{1}{2} L i^2 = \frac{1}{2} \psi i = \frac{1}{2L} \psi^2}$$

- Singly - Excited System
- Consider the schematic of singly excited system where the coil is wound around a magnetic core connected to voltage source.
- The ferromagnetic rotor experiences a torque urging it towards a region where the magnetic feild is stronger.
- In other words a torque is excited on the rotor so that it tries to position itself to a given minimum reluctance for the magnetic flux.
- This reluctance is dependent of the rotor angle.
- This torque is called the reluctance torque or saliency torque due to saliency of rotor.

- Assumptions to be made:

(1) The flux linkage/current relationship is linear for any rotor position. ie $\psi = L(\theta) i$

(2) Hysteresis & eddy current losses are neglected.

(3) The coil has negligible leakage flux & all the flux follows the main magnetic path.

(4) The magnetic feild predominates and electric field effects are neglected.

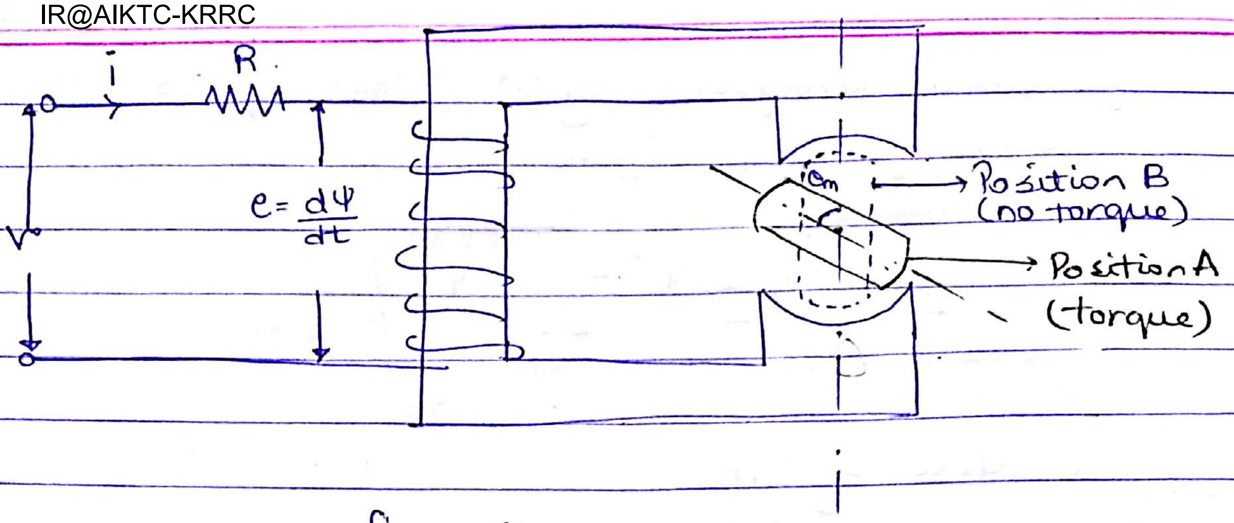

fig: Singly excited system.

- let R be the resistance of the coil
- By KVL, the instantaneous voltage equation for the coil is;

$$v = Ri + \frac{d\psi}{dt}$$

- Multipling througout by i,

$$vi = Ri^2 + i\frac{d\psi}{dt}$$

Integrating with respect to time t, from t=0 to t=t and assuming the current and flux linkages to be initially zero.

$$\int_0^t vi\,dt = \int_0^t Ri^2\,dt + \int_0^t i\,\frac{d\psi}{\cancel{dt}}\,\cancel{dt}$$

$$\therefore \int_0^t vi\,dt = \int_0^t Ri^2\,dt + \int_0^{\psi} i\,d\psi \rightarrow W_f$$

That is;

[total electrical i/p energy] = [energy to electrical losses] + [useful electrical energy]

or;

$$W_e = W_{le} + \underbrace{[W_{fe} + W_{fem}]}$$

energy for field   energy for M

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where;
$$\int_0^{\psi} i\, d\psi = W_{fe} + W_{em}.$$

(i) Static Energization:

- For any position of the rotor, a characteristic of flux linkage ψ against current i can be drawn. The fig shows the characteristic for position 'A' & 'B'
- For any stationary position of the rotor, the mechanical o/p is zero & all the useful electrical energy i/p is ~~connected~~ converted into feild energy.

$$W_f = \int_0^{\psi} i\, d\psi = \int_0^{\psi} \frac{\psi}{L}\, d\psi = \frac{\psi^2}{2L}.$$

since $\psi = Li$ for linear system;
$$W_f = \frac{L i^2}{2L} = \frac{1}{2} L i^2.$$

- The areas OCD & OEF represent the stored energy for positions A & B respectively.

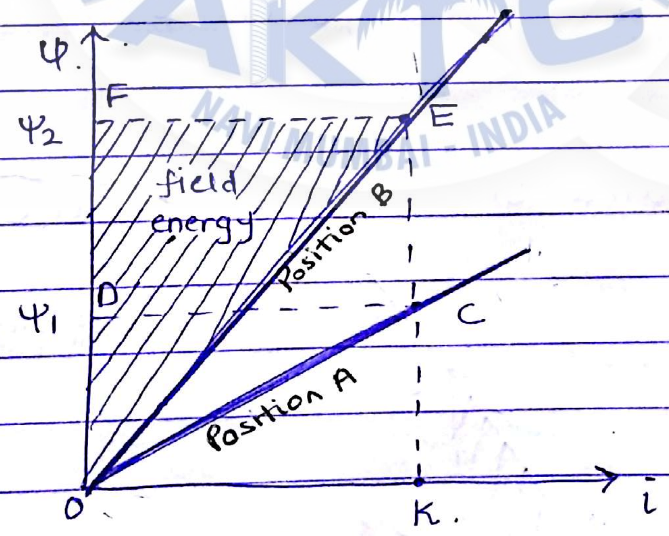

fig:

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(ii) Dyanamic Energisation :

(a) Slow Movement (c) Transient Movement.

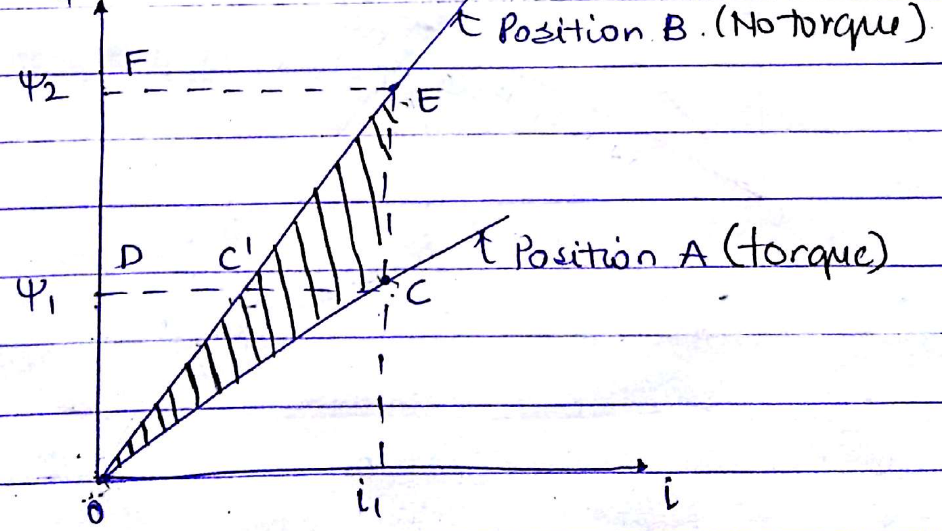
(b) Fast Movement
Instantaneous.

(a) Slow Movement :

- Here we are considering the slow movement of the rotor
- In position A, the current is i_1. & the operating point is 'C'
- During slow movement of the rotor, counter emf $\frac{d\psi}{dt}$ is very small.
- Consequently the exciting current i_1 remains fairly constant.
- The flux linkage may be assumed to increase from ψ_1 to ψ_2 at constant current i_1
- Therefore at position B the operating pt shifts to E

Change in stored energy of magnetic field (W_f) = (magnetic energy stored in position B) − (magnetic energy stored in position A)

$\therefore$ W_f = area OC'EFDO − area OCC'DO.



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- Electrical energy i/p during this change (We)

$$= \int_{\psi_1}^{\psi_2} i_1 \, d\psi$$

$$= i_1 (\psi_2 - \psi_1)$$

We $\longrightarrow$ area CEFDC'C

But;

$$W_e = W_f + W_m .$$

$$\text{area } CEFDC'C = \left[\begin{array}{l} \text{area } OC'EFDO \\ - \text{area } OCC'DO \end{array} \right] + W_m .$$

$$\therefore W_m = \text{Area } OCEC'O$$

From the above equation it is clear that the mechanical work done is equal to the area enclosed between the two ψ-i characteristics in position A & B and the vertical ψ-i locus during slow movement of the rotor.

- It is also shown that for a linear system considered half of the useful electrical energy i/p is stored in the magnetic field & the other half appears as mechanical energy o/p during slow movement of the rotor.

- Electromagnetic torque:

$$T_e = \lim_{\Delta \theta_m \to 0} \left\{ \frac{\Delta W_{fe}'}{\Delta \theta_m} \right\}_{\text{constant } i}$$

$$= \left\{ \frac{\partial W_{fe}'}{\partial \theta_m} \right\}_{\text{constant } i}$$

$$T_e = \left\{ \frac{\partial}{\partial \theta_m} \left[\frac{1}{2} L i^2 \right] \right\}_{\text{constant } i}$$

$$T_e = \frac{i^2}{2} \cdot \frac{\partial L}{\partial \theta_m}$$

or;

For a linear system L is not function of i

$$T_e = \frac{i^2}{2} \cdot \frac{dL}{d\theta_m}$$

(b) Instantaneous movement :

In this case, the rotor is assumed to move very fast from position A to B. That is, the movement is instantaneous. According to constant flux linkage theorem, flux linkages with inductive ckt cannot change suddenly. Thus during the fast movement of the rotor, the flux linkage remains constant at ψ_1 therefore the operating pt moves horizontally from C to C'.

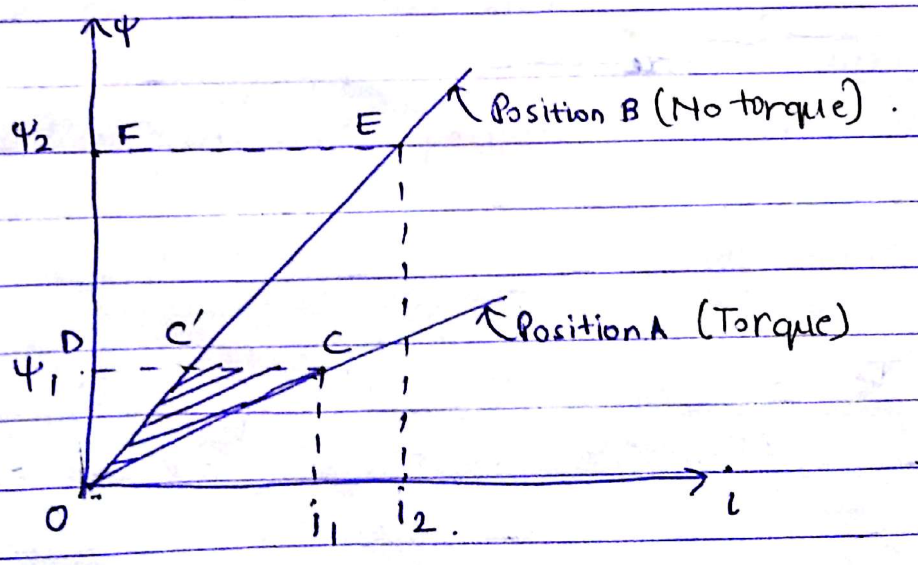

fig: Instantaneous movement of rotor.

$\frac{\partial(L^{-1})}{\partial\theta m}$ $\frac{1}{L} \times \frac{L}{L}$ $\frac{1}{L} \times L$

- Both ψ & i gradually increase to their steady state values. The final operating pt is E

Since ; $W_e = \int_{\psi_1}^{\psi_2} i \, d\psi$.

and $\psi_2 = \psi_1$ during the rapid movement,

$\therefore W_e = \int_{\psi_1}^{\psi_1} i \, d\psi$

$= 0$.

- Thus no energy is taken from the supply during fast movement of the rotor & mechanical energy o/p is obtained from the stored magnetic field energy which reduces by an equal amount.

- For incremental movement of the rotor angle θm, the energy to mechanical work corresponds to the loss of stored magnetic field energy.

- Electromagnetic torque

$$T_e = \lim_{\Delta\theta m \to 0} \left\{ -\frac{\Delta W_{fe}}{\Delta \theta m} \right\}_{\text{constant } \psi.}$$

$$= \left\{ -\frac{\partial W_{fe}}{\partial \theta m} \right\}_{\text{constant } \psi.}$$

$$= \left\{ -\frac{\partial}{\partial \theta m} \left(\frac{\psi^2}{2L} \right) \right\}_{\text{constant } \psi.}$$

?

$$T_e = -\frac{\psi^2}{2} \cdot \frac{1}{L^2} \cdot \frac{\partial L}{\partial \theta m}.$$

$$T_e = \frac{i^2}{2} \frac{\partial L}{\partial \theta m}.$$

For linear system;

$$T_e = \frac{i^2}{2}\frac{dL}{d\theta_m}.$$

- Transient Movement.

The actual rotor movement will neither be too slow nor too fast, but will lie between two extreme limits. In general, both current & flux linkage change during the movement of the rotor. The corresponding $\psi - i$ relationship is a general path from C to E as shown below:

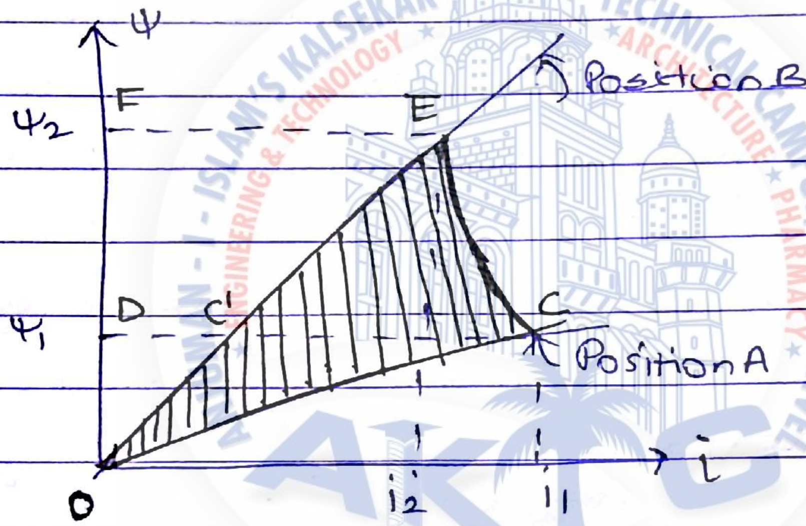

fig: Transient movement of rotor.

Here;

$$W_e = \text{area } CEFDC'C$$

$$W_f = \text{area } OEFO - \text{area } OCDO$$

But; $W_m = W_e - W_f$

$= \text{area } CEFDC - \text{area } OEFO + \text{area } OCDO$

$= (\text{area } CEFDC + \text{area } OCDO) - \text{area } OEFO.$

$= \text{area } OCEO.$

The mechanical o/p is given by the shaded part.

• Doubly excited system :

A doubly excited magnetic system has two independent sources of excitation. Eg of such systems are seperately excited dc machines, syncronous machine, loudspeakers, tachometer etc. A schematic of doubly excited system is shown in the fig. Let us assume that both stator & rotor has saliency.

• Assumptions are as for singly excited system.

• The flux linkage equations for the two winding are;

$$\psi_1 = L_1 i_1 + M i_2 \longrightarrow (1)$$

$$\psi_2 = L_2 i_2 + M i_1 \longrightarrow (2)$$

• Instantaneous voltage equations for the ~~two~~ coils are;

$$v_1 = i_1 R_1 + \frac{d\psi_1}{dt} \longrightarrow (3)$$

$$v_2 = i_2 R_2 + \frac{d\psi_2}{dt} \longrightarrow (4)$$

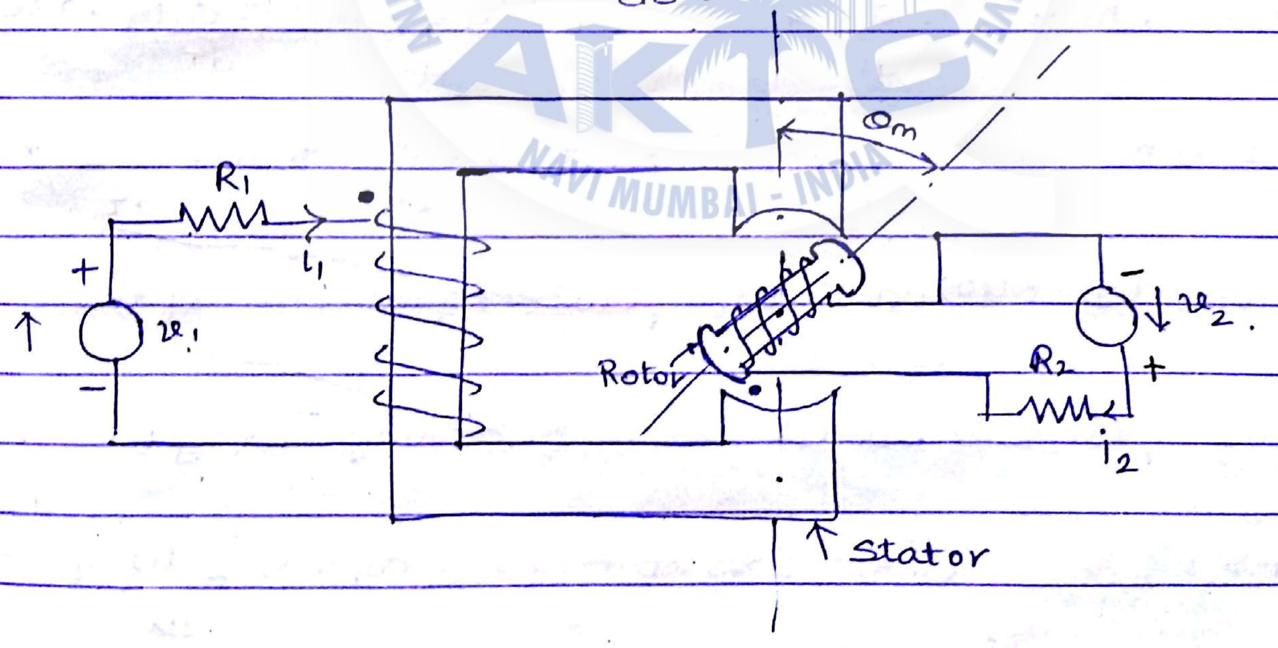

fig: doubly excited rotational magnetic system.

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- Substituting the values of ψ_1 and ψ_2 in equations (3) & (4) we get

$$v_1 = i_1R_1 + \frac{d(L_1i_1)}{dt} + \frac{d}{dt}(Mi_2) \quad \text{——— (5)}$$

$$v_2 = i_2R_2 + \frac{d(L_2i_2)}{dt} + \frac{d}{dt}(Mi_1) \quad \text{——— (6)}$$

· Now the inductances are independent of currents & dependent of the rotor angle θ_m (which is a function of time). Similarly, currents are time dependent and are not function of inductances.

$$\therefore v_1 = i_1R_1 + L_1\frac{di_1}{dt} + i_1\frac{dL_1}{dt} + M\frac{di_2}{dt} + i_2\frac{dM}{dt} \quad \text{—— (7)}$$

$$v_2 = i_2R_2 + L_2\frac{di_2}{dt} + i_2\frac{dL_2}{dt} + M\frac{di_1}{dt} + i_1\frac{dM}{dt} \quad \text{—— (8)}$$

Multiplying eq (7) by i_1 & (8) by i_2.

$$\therefore v_1i_1 = i_1^2R_1 + L_1i_1\frac{di_1}{dt} + i_1^2\frac{dL_1}{dt} + Mi_1\frac{di_2}{dt} + i_1i_2\frac{dM}{dt} \rightarrow (9)$$

$$v_2i_2 = i_2^2R_2 + L_2i_2\frac{di_2}{dt} + i_2^2\frac{dL_2}{dt} + Mi_2\frac{di_1}{dt} + i_1i_2\frac{dM}{dt} \rightarrow (10)$$

Equations (9) & (10) are the power equations of the coil.

Integrating eq (9) & (10) w.r.t time & adding we get.

$$\int (v_1i_1 + v_2i_2)\,dt = \int (i_1^2R_1 + i_2^2R_2)\,dt + \int \big(L_1i_1\,di_1 + L_2i_2\,di_2 + i_1^2\,dL_1 + i_2^2\,dL_2 + Mi_1\,di_2 + Mi_2\,di_1 + 2i_1i_2\,dM\big) \rightarrow (11)$$

So;

$$\begin{bmatrix}\text{useful electrical} \\ \text{energy} \quad i/p\end{bmatrix} = \int (e_1 i_1 + e_2 i_2)\, dt - \int (i_1^2 R_1 + i_2^2 R_2)\, dt. \quad \text{——(12)}$$

and;

$$\begin{bmatrix}\text{Energy to field} \\ \text{storage in the} \\ \text{electrical system}\end{bmatrix} + \begin{bmatrix}\text{electrical to} \\ \text{mechanical} \\ \text{energy}\end{bmatrix} = \int (L_1 i_1\, di_1 + L_2 i_2\, di_2 + i_1^2 dL_1 + i_2^2 dL_2 + M i_1\, di_2 + M i_2\, di_1 + 2 i_1 i_2\, dM) \quad \text{——(13)}$$

· Energy stored in the Magnetic field

The instantaneous value of energy stored in magnetic field depends on the inductance & current values at the instant considered. This energy may be found by considering the transducer to be stationary and the coils to be energized from zero current to the required instantaneous values of currents. There is no mechanical o/p & the W_{em} is zero. The inductance values are constant. Therefore terms dL_1, dL_2 and dM become zero.

from eq.(13) we have:

$$\int dW_{fe} = \int_0^{i_1} L_1 i_1\, di_1 + \int_0^{i_2} L_2 i_2\, di_2 + \int_0^{i_1, i_2} (M i_1\, di_2 + M i_2\, di_1)$$

$$= L_1 \frac{i_1^2}{2} + L_2 \frac{i_2^2}{2} + \cancel{M i_1 i_2 + M i_2 i_1}\ \underbrace{M \int_0^{i_1, i_2} d(i_1, i_2)}$$

$$[\text{Total } W_{fe}] = L_1 \frac{i_1^2}{2} + L_2 \frac{i_2^2}{2} + M i_1 i_2. \quad \text{——(14)}$$

$d(i_1, i_2) = i_1\, di_2 + i_2\, di_1$

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$i_1 \frac{di_1}{dt} + i_1 \frac{di_1}{dt}$

$\cancel{2 i_1 \frac{di_1}{dt}}$

$Li_1 = \psi$

• Electromagnetic torque:

Equation (14) holds for any transducer position. If the transducer rotates, the rate of change of field energy with respect to time is given by differentiating equation (14).

$$\frac{dW_{fe}}{dt} = \frac{1}{2} L_1 \frac{d i_1^2}{dt} + \frac{1}{2} i_1^2 \frac{dL_1}{dt} + \frac{1}{2} L_2 \frac{d i_2^2}{dt} + \frac{1}{2} i_2^2 \frac{dL_2}{dt} + i_1 i_2 \frac{dM}{dt} + i_1 M \frac{di_2}{dt} + i_2 M \frac{di_1}{dt}$$

??

$$\frac{dW_{fe}}{dt} = L_1 i_1 \frac{di_1}{dt} + \frac{1}{2} i_1^2 \frac{dL_1}{dt} + L_2 i_2 \frac{di_2}{dt} + \frac{1}{2} i_2^2 \frac{dL_2}{dt} + i_1 i_2 \frac{dM}{dt} + i_1 M \frac{di_2}{dt} + i_2 M \frac{di_1}{dt} \quad \text{—— (15)}$$

Integrating eq (15) w.r.t time:

$$\int dW_{fe} = W_{fe} = \int \left(L_1 i_1 \, di_1 + \frac{1}{2} i_1^2 \, dL_1 + L_2 i_2 \, di_2 + \frac{1}{2} i_2^2 \, dL_2 \right) + i_1 i_2 \, dM + i_1 M \, di_2 + i_2 M \, di \quad \longrightarrow (16)$$

This is a general expression for a moving transducer in which L_1, L_2, M, i_1 & i_2 are all varying w.r.t position & time

Comparing eq (16) with (13).

$$W_{em} = \boxed{\text{electrical to mechanical energy}} = \int \left(\frac{1}{2} i_1^2 \, dL_1 + \frac{1}{2} i_2^2 \, dL_2 + i_1 i_2 \, dM \right) \quad \text{—— (17)}$$

Differentiating eq (17) with θ_m

$$\frac{d(uv)}{dt} = u\frac{dv}{dt} + v\frac{dv}{dt}$$

Reluctance torque or Saliency torque — Co-alignment torque

$$\frac{dW_{em}}{d\theta_m} = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta_m} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta_m} + i_1 i_2 \frac{dM}{d\theta_m} \rightarrow (18)$$

as only L_1, L_2 and M are dependent on θ_m.

Eq. (18) includes the case of singly excited system when one of the ~~two~~ currents is equal to zero so that the expression for the torque becomes;

$$T_e = \frac{i^2}{2}\frac{dL}{d\theta_m}$$

co-alignment torque: two superimposed fields that try to align.

For m/c having uniform air gap, reluctance torque is not produced.

$$\frac{di_1^2}{dt} = \frac{d}{dt}(i_1 \cdot i_1) = \frac{d}{dt}\left(2 i_1 \frac{di_1}{dt}\right)$$

• Reluctance Motor:

• A elementary form of 1ø Reluctance motor is shown in the figure. It has salient poles both on stator & rotor. The reluctance or permeance of magnetic ckt depends on relative angular position of the rotor & stator.

• 1ø voltage is applied to N turns which produces pulsating flux which crosses the stator air gap. The axis of the statorie (stator direct axis - d axis) is shown by horizontal dotted lines. An axis 90° away from the d-axis is called the quadrature axis.

• Angle θ_r is the space angle between stator d-axis and the long rotor axis.

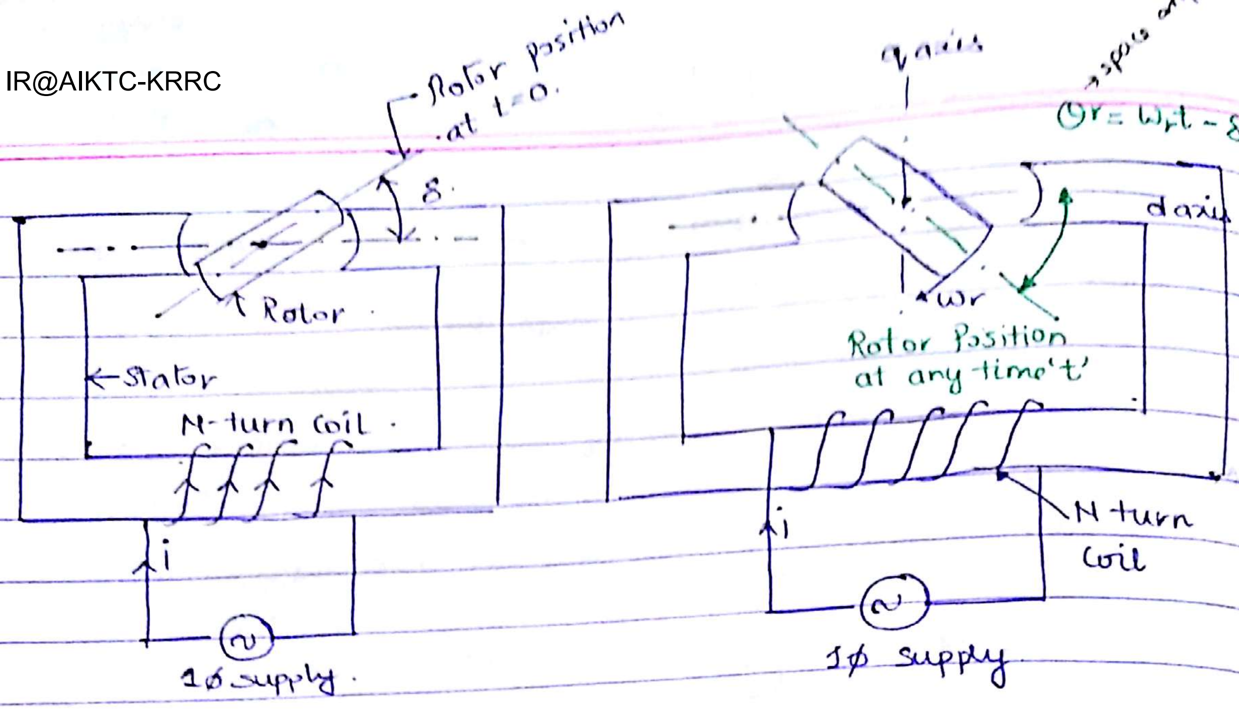


fig: Single phase reluctance motor.

- When $\theta_r = 0$; the reluctance (Rld) offered to stator flux is minimum
- When $\theta_r = 90$; the reluctance (Rlq) offered to stator flux is maximum.
- From below fig, it is clear that;

  when $\theta_r = 0, \pi, 2\pi$ etc reluctance offered is Rld.

  when $\theta_r = \pi/2, 3\pi/2$ etc reluctance offered is Rlq.

  Where;

  Rld, Rlq → direct & quadrature axis reluctances.

- The variation of reluctane with θ_r is assumed to be a sine function
- An examination of the fig will show that fig (a) is composed of fig (b) & (c)

- fig (b) reluctance is constant;

$$\therefore R_{l1} = \frac{1}{2}(R_{lq} + R_{ld}) \longrightarrow (1)$$

- fig (c) reluctance varies sinusoidally

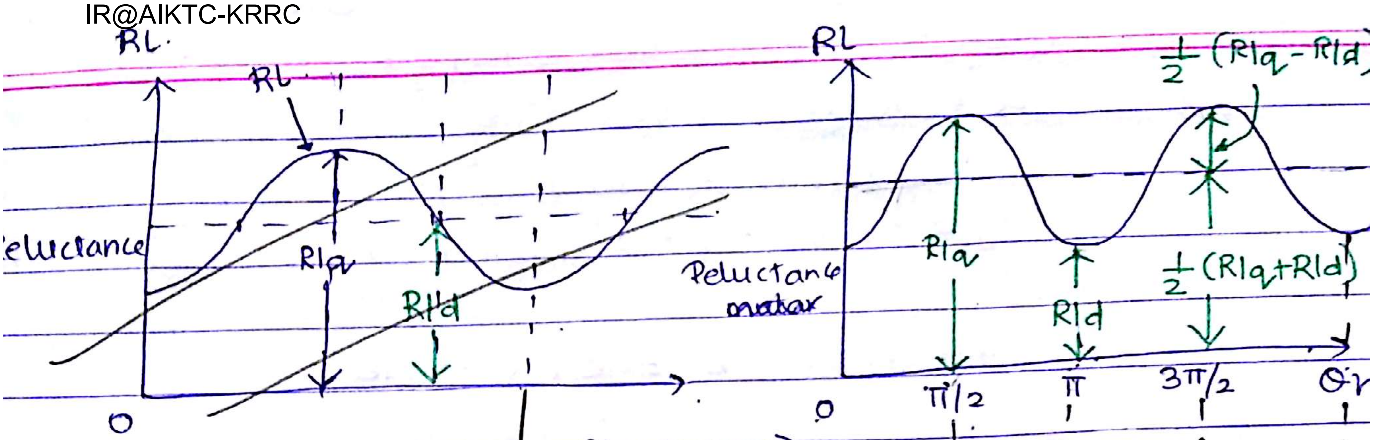


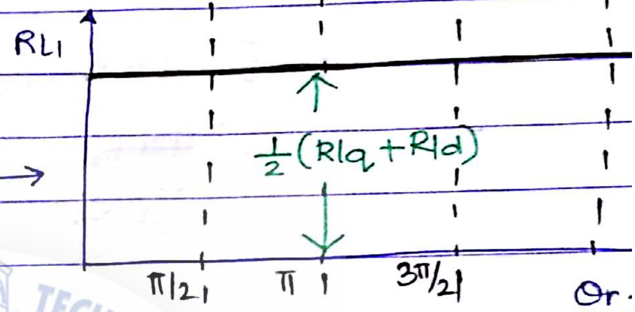
fig a →

∴ when θr = 0°,

$$R_{l2} = -\frac{1}{2}(R_{lq} - R_{ld}) = 1$$

or $R_{l2} = -\frac{1}{2}(R_{lq} - R_{ld})\cos 0°$

fig b →

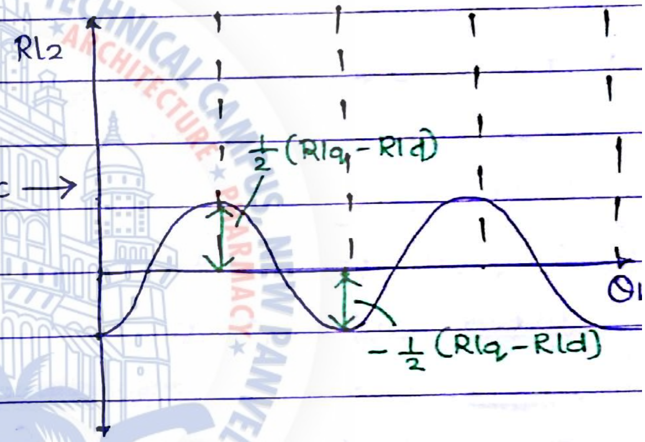


At θr = 45°;

$R_{l2} = 0$

or $R_{l2} = -\frac{1}{2}(R_{lq} - R_{ld})\cos 2\left(\frac{\pi}{4}\right)$

fig c →



At θr = 90°

$R_{l2} = \frac{1}{2}(R_{lq} - R_{ld})$

or

$R_{l2} = \frac{1}{2}(R_{lq} - R_{ld})\cos 2\left(\frac{\pi}{2}\right)$.

Therefore, the reluctance variation can be expressed as;

$$R_{l2} = -\frac{1}{2}(R_{lq} - R_{ld})\cos 2\theta r. \longrightarrow (2)$$

Sum of (1) & (2) gives

$$\boxed{R_L = R_{l1} + R_{l2} = \frac{1}{2}(R_{lq} + R_{ld}) - \frac{1}{2}(R_{lq} - R_{ld})\cos 2\theta r}$$

Torque in terms of reluctance is given by;

$$\boxed{T_e = -\frac{1}{2}\phi^2 \frac{dR_L}{d\theta r}}$$