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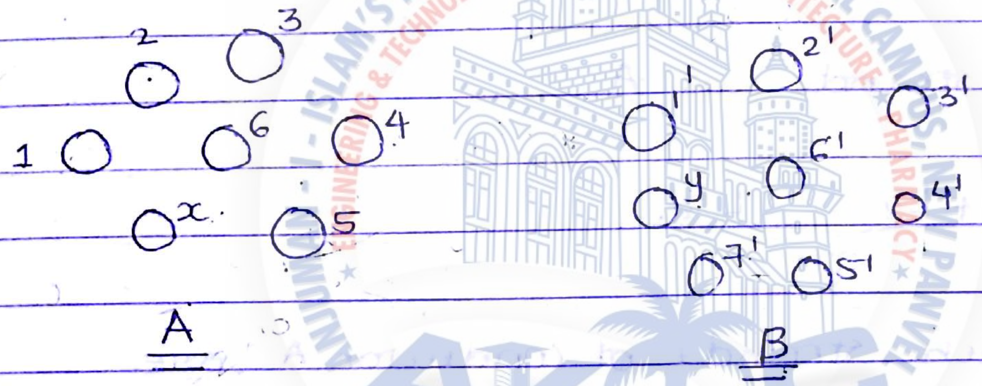
* Inductance of composite conductor lines
– Self & Mutual GMDs.

- Consider a 1ϕ line consisting of two parallel conductors A & B, conductor A consists of x no & B consists of y strands.
- Let conductor A & B carry currents I & $-I$ respectively.

- Current carried by conductor $A = \frac{I}{x}$.

- Current carried by conductor $B = \frac{-I}{y}$.



- Flux linkages for strand 1 in conductor A.

$$\psi_1 = 2\times10^{-7}\,\frac{I}{x}\left[\log_e\frac{1}{r_1'} + \log_e\frac{1}{d_{12}} + \log_e\frac{1}{d_{13}} + \dots + \log_e\frac{1}{d_{1x}}\right]$$
$$-\,2\times10^{-7}\,\frac{I}{y}\left[\log_e\frac{1}{d_{11'}} + \log_e\frac{1}{d_{12'}} + \log_e\frac{1}{d_{13'}} + \dots + \log_e\frac{1}{d_{1y}}\right]$$

wb.turn/m.

$$\psi_1 = 2\times10^{-7}\left[\frac{I}{x}\log_e\frac{1}{(r_1'.d_{12}.d_{13}\dots d_{1x})} - \frac{I}{y}\log_e\frac{1}{(d_{11'}, d_{12'}, d_{13'}\dots d_{1y})}\right]$$

$$= 2\times10^{-7}\,I.\ \frac{\sqrt[y]{(d_{11'}, d_{12'}, d_{13'}\dots d_{1y})}}{\sqrt[x]{(r_1', d_{12}, d_{13}\dots d_{1x})}}$$ wb turn/m.

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- Inductance of strand 1 of conductor A.

$$L_1 = \frac{\psi_1}{I/x} = 2\times10^{-7}x \log_e \frac{\sqrt[y]{d_{11}', d_{12}', d_{13}', d_{14}' \ldots d_{1y}'}}{\sqrt[x]{r', d_{12}, d_{13}, d_{14}, \ldots d_{1x}}} \text{ H/m}$$

- $r' = d_{11}$

- $$L_1 = 2\times10^{-7}x \log_e \frac{\sqrt[y]{d_{11}', d_{12}', d_{13}', d_{14}' \ldots d_{1y}}}{\sqrt[x]{d_{11}, d_{12}, d_{13}\, d_{14} \ldots d_{1x}}} \text{ H/m}$$

Similarly for strand2;

- $$L_2 = 2\times10^{-7}x \log_e \frac{\sqrt[y]{d_{21}'\ d_{22}'\ d_{23}'\ d_{24}' \ldots d_{2y}}}{\sqrt[x]{d_{21}, d_{22}, d_{23} \ldots d_{2x}}} \text{ H/m}$$

- Avg L of strands in A

$$L_{av} = \frac{L_1 + L_2 \ldots L_x}{x}$$

- Since x such strands of conductor A are electrically in parallel

inductance

$$L_A = \frac{L_{av}}{x} = \frac{L_1 + L_2 \ldots L_x}{x^2}$$

$$= \cancel{2\times10^{-7}\log_e}\left[\cancel{\frac{L_1}{x}} + \cancel{\frac{L_2}{x}} + \cancel{\frac{L_x}{x}}\right]$$

$$= 2\times10^{-7}\ \cancel{\log_e}\left[\frac{\log_e \frac{\sqrt[y]{d_{11}', d_{12}', d_{13}' \ldots d_{1y}'}}{\sqrt[x]{d_{11}, d_{12}, d_{13} \ldots d_{1x}}}}{x} + \frac{\log_e \frac{\sqrt[y]{d_{21}', d_{22}', d_{23}' \ldots}}{\sqrt[x]{d_{22}, d_{21}, d_{23} \ldots}}}{x}\right]$$

$$= 2\times10^{-7} \log_e \frac{\sqrt[xy]{(d_{11}', d_{12}', d_{13}' \ldots d_{1y})}}{\sqrt[x^2]{d_{11} \cdot d_{12} \cdot d_{13} \ldots d_{1x}}} + \log_e \frac{\sqrt[xy]{d_{21}' \cdot d_{22}' \cdot d_{23}'}}{\sqrt[x^2]{d_{22} \cdot d_{21} \cdot d_{23}}}$$

$$\frac{\log_e(b)}{a} = \log_e \sqrt[a]{(b)}$$

Mutual GMD

$$= 2\times10^{-7} \log_e \frac{\boxed{\sqrt[xy]{(d_{11}'\, d_{12}'\, d_{13}' \ldots d_{1y})(d_{21}'\, d_{22}'\, d_{23}' \ldots d_{y}')}}}{\boxed{\sqrt[x^2]{(d_{11}\cdot d_{12}\, d_{13} \ldots d_{1x})(d_{22} : d_{21}\cdot d_{23} \ldots d_{2y})}}}$$

Self GMD (GMR)

→ let GMD → Dm and GMR → Ds

- $L_A = 2\times10^{-7} \log_e \dfrac{D_m}{D_{SA}}$ H/m.
- $L_B = 2\times10^{-7} \log_e \dfrac{D_m}{D_{SB}}$ H/m.

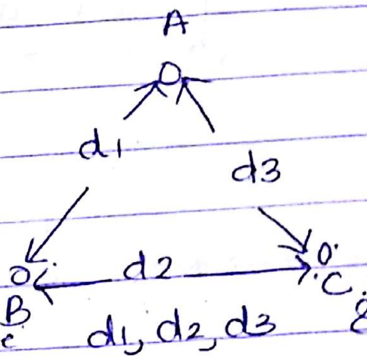
$D_{SA} = D_{SB} = D_s$.

inductance of loop

$$\therefore L_A + L_B = L = 4\times10^{-7} \log_e \frac{D_m}{D_s} \text{ H/m}.$$

* Inductance of 3ϕ overhead lines

• Unsymmetrical Spacing:



→ Consider 3ϕ line with conductor A, B, C each having radius 'r' meters.

→ Let the spacing between them be d_1, d_2, d_3 & the current flowing through them be I_A, I_B, I_C.

→ flux linkages of conductor A due to its own current I_A & currents I_B & I_C.

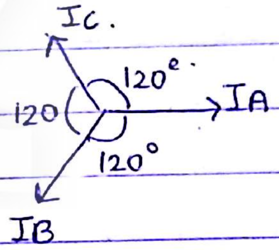
$$\psi_A = 2\times10^{-7}\left[I_A \log_e \frac{1}{r'} + I_B \log_e \frac{1}{d_1} + I_C \log_e \frac{1}{d_3}\right] \text{ wb-turn/meter}$$

$$\psi_B = 2\times10^{-7}\left[I_B \log_e \frac{1}{r'} + I_A \log_e \frac{1}{d_1} + I_C \log_e \frac{1}{d_2}\right]$$

$$\psi_C = 2\times10^{-7}\left[I_C \log_e \frac{1}{r'} + I_A \log_e \frac{1}{d_3} + I_B \log_e \frac{1}{d_2}\right].$$

→ If system is balanced;

$$I_A = I_B = I_C = I.$$



$I_A = I\angle 0° = I$

$I_B = I\angle -120° = I(-0.5 - j0.866)$

$I_C = I\angle 120° = I(-0.5 + j0.866)$

$1\angle 120°$

$\rightarrow -0.5 - 0.866i$

$$\rightarrow \psi_A = 2\times10^{-7}\left[I \log_e \frac{1}{r'} + I(-0.5 - j0.866)\log_e \frac{1}{d_1} + I(-0.5 + j0.866)\log_e \frac{1}{d_3}\right]$$

$$= 2\times10^{-7}\, I\left[\log_e \frac{1}{r'} - 0.5\log_e \frac{1}{d_1} - j0.866\log_e \frac{1}{d_1} - 0.5\log_e \frac{1}{d_3} + j0.866\log_e \frac{1}{d_3}\right]$$

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$\log_e A^B = B \log A$. $-\log_e(d_1 d_3)^{-1/2} = -\frac{1}{2}\log d_1 d_2$

$$= 2\times10^{-7} I\left[\log_e\frac{1}{r'} - 0.5\left[\log_e\frac{1}{d_1} + \log_e\frac{1}{d_3}\right] + j0.866\left[\log_e\frac{1}{d_3} - \log_e\frac{1}{d_1}\right]\right]$$

$$= 2\times10^{-7} I\left[\log_e\frac{1}{r'} - 0.5\log_e\frac{1}{d_1 d_3} + j0.866\log_e\frac{d_1}{d_3}\right]$$

$(d_1 d_3)^{-1}$

$$= 2\times10^{-7} I\left[\log_e\frac{1}{r'} - \frac{\log_e 1/d_1 d_3}{2} + j\sqrt{3}\times 0.5\log_e\frac{d_1}{d_3}\right]$$

$1/2$

$\log_e (d_1 d_3)^{-1/2} \rightarrow -\frac{1}{2}\log_e d_1 d_2$

$$= 2\times10^{-7} I\left[\log_e\frac{1}{r'} \boxed{-\log_e \sqrt[2]{1/d_1 d_3}} + j\sqrt{3}\log_e\sqrt{d_1/d_3}\right]$$

~~$\log A = \log A^{-1}$~~

$$= 2\times10^{-7} I\left[\log_e\frac{1}{r_1} + \log_e\sqrt{d_1 d_3} + j\sqrt{3}\log_e\sqrt{d_1/d_3}\right]$$

$$\rightarrow L_A = \frac{\psi_A}{I_A} = 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log_e\sqrt{d_1 d_3} + j\sqrt{3}\log_e\sqrt{\frac{d_1}{d_3}}\right]$$

Similarly $L_B = 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log\sqrt{d_1 d_2} + j\sqrt{3}\log_e\sqrt{\frac{d_2}{d_1}}\right]$ H/m

$$L_C = 2\times10^{-7}\left[\log_e\frac{1}{r'} + \log\sqrt{d_3 d_2} + j\sqrt{3}\log_e\sqrt{\frac{d_3}{d_2}}\right]$$ H/m.

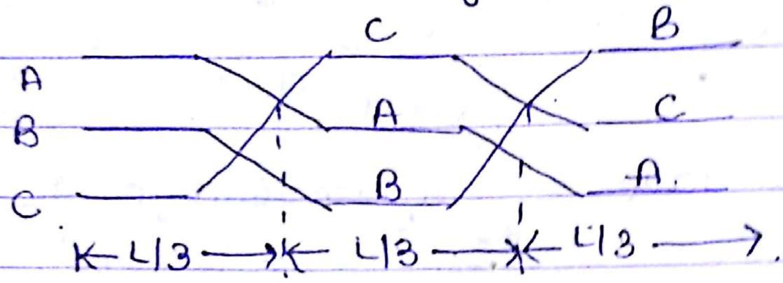
If conductors are not equidistant;

- Inductances & flux linkages are different. ∴ voltage drop is different ∴ Power Xmitted/ conductor is different.

$$\log_e(d_1 d_3)^{-1/2} = -\frac{1}{2}\log_e(d_1 d_3)$$

$$= -\log_e\sqrt{d_1 d_3}.$$

- This is overcome by transposition of conductors.



- Each of the 3 possible arrangements of conductors exist for 1/3rd of the total length of line.

(i) The effect of transposition is that each conductor has the same average inductance

$$L = \frac{L_A + L_B + L_C}{3}$$

$$= \frac{2\times10^{-7}}{3}\Big[3\log_e \frac{1}{r'} + 2\left(\log_e\sqrt{d_1 d_3} + \log_e\sqrt{d_1 d_2} + \log_e\sqrt{d_2 d_3}\right)$$

$$+ j\sqrt{3}\left(\log_e\sqrt{d_1/d_3} + \log_e\sqrt{d_2/d_1} + \log_e\sqrt{d_3/d_2}\right)\Big]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{3}\log_e\sqrt{d_1^2 d_2^2 d_3^2} + \frac{j\sqrt{3}}{3}\log_e\sqrt{\frac{d_1}{d_3}\cdot\frac{d_2}{d_1}\cdot\frac{d_3}{d_2}}\right]$$

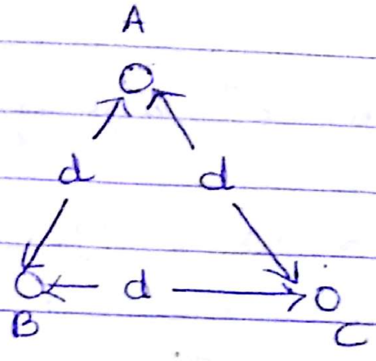
$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{3}\log_e d_1 d_2 d_3 + \frac{j\sqrt{3}}{3}\log_e(1)\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \log_e\sqrt[3]{d_1 d_2 d_3} + \frac{j\sqrt{3}}{3}(0)\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \log_e\sqrt[3]{d_1 . d_2 d_3}\right] \quad H/m$$

$$\boxed{L = 2\times10^{-7}\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r'}} \quad H/m = L_A = L_B = L_C.$$

(ii) If the conductors are equispaced:



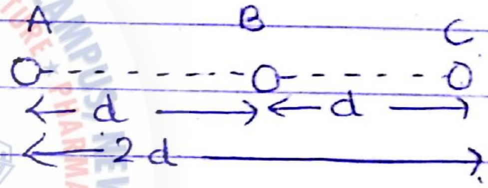
→ Substituting $d_1 = d_2 = d_3 = d$.

$$L = 2\times10^{-7} \log_e \frac{\sqrt[3]{d\times d\times d}}{r'} \quad H/m.$$

$$L = 2\times10^{-7} \log_e \frac{d}{r'} \quad H/m.$$

For stranded conductor r' will be replaced by D_s (self GMD) (GMR)

(iii) When the conductors of 3ϕ xmission line are in the same plane.



→ In this position we have
$d_1 = d$; $d_2 = d$.
$d_3 = 2d$.

Substituting;
∴ $d_1 = d_2 = d$ & $d_3 = 2d$ in above expression we get:

$$L_A = 2\times10^{-7}\left[\log_e \frac{1}{r'} + \log_e \sqrt{d\times 2d} + j\sqrt{3}\log_e \sqrt{\frac{d}{2d}}\right].$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{2}\log_e 2d^2 + j\sqrt{3}\log_e \sqrt{1/2}\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{2}\log_e 2d^2 + j\frac{\sqrt{3}}{2}\log_e (2)^{-1}\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{2}\log_e 2d^2 + j0.8660\log_e (2)^{-1}\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{2}\log_e 2d^2 - j0.866\log_e 2\right].$$

$\log_e \sqrt{2}.d$

$\log_e \sqrt{2} + \log_e d$

$\frac{1}{2}\log_e 2 + \log_e \frac{d}{r_1}$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r_1} + \log_e \sqrt{2}\,d - j0.866\log_e 2\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r_1} + \log_e d + \log_e \sqrt{2} - j0.866\log_e 2\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{d}{r_1} + \frac{1}{2}\log_e 2 - j0.866\log_e 2\right] \text{ H/m}$$

$$L_B = 2\times10^{-7}\left[\log_e \frac{1}{r_1} + \log_e \sqrt{d\times d} + j\sqrt{3}\log_e \sqrt{d/d}\right] \text{ H/m}$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r_1} + \log_e d + 0\right]$$

$$\boxed{L_B = 2\times10^{-7}\log_e \frac{d}{r_1} \text{ H/m}}$$

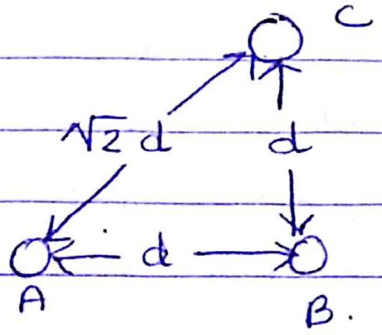
$$L_C = 2\times10^{-7}\left[\log_e \frac{1}{r_1} + \log_e \sqrt{2d\times d} + j\sqrt{3}\log_e \sqrt{\frac{2d}{d}}\right]$$

$$\boxed{L_C = 2\times10^{-7}\left[\log_e \frac{d}{r_1} + \frac{1}{2}\log_e 2 + j0.866\log_e 2\right] \text{ H/m}}$$

(iv) When the conductors are at the corners of a right angled triangle.

→ Substitute ~~LA & LB~~ $d_1 = d_2 = d$

and $d_3 = \sqrt{2}\, d$.



$$\rightarrow L_A = 2\times10^{-7}\left[\log_e \frac{1}{r'} + \log_e \sqrt{\sqrt{2}\, d \times d} + j\sqrt{3}\log_e \sqrt{\frac{d}{\sqrt{2}\, d}}\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{1}{r'} + \frac{1}{2}\log_e \sqrt{2} - j0.866\log_e \sqrt{2}\right] \text{ H/m}$$

$$\rightarrow L_B = 2\times10^{-7}\left[\log_e \frac{1}{r'} + \log_e \sqrt{d \times d} + j\sqrt{3}\log_e \sqrt{\frac{d}{d}}\right]$$

$$= 2\times10^{-7}\log_e \frac{d}{r'} \text{ H/m}.$$

$$\rightarrow L_C = 2\times10^{-7}\left[\log_e \frac{1}{r'} + \log_e \sqrt{\sqrt{2}\, d \times d} + j\sqrt{3}\log_e \sqrt{\frac{\sqrt{2}\, d}{d}}\right]$$

$$= 2\times10^{-7}\left[\log_e \frac{d}{r'} + \frac{1}{2}\log_e \sqrt{2} + j0.866\log_e \sqrt{2}\right] \text{ H/m}$$

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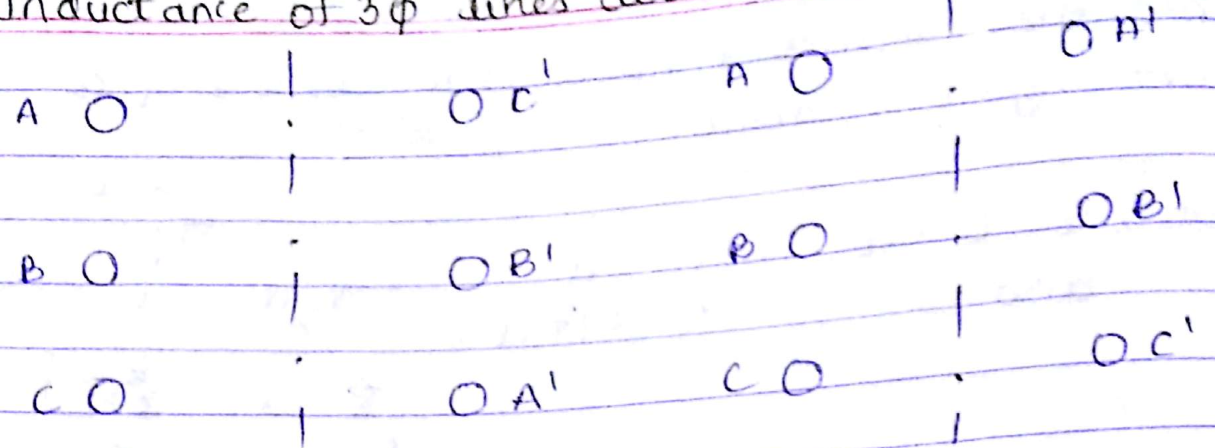
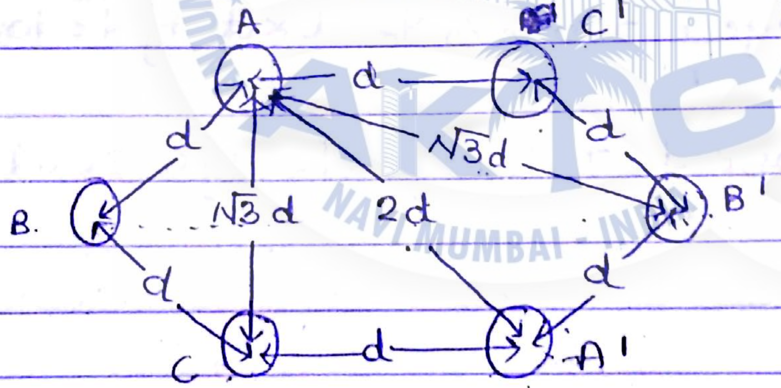
* Inductance of 3ϕ lines with more than one circuit



- This arrangement has less mutual inductance hence less GMD & more GMR.
- This arrangement has more mutual inductance hence ~~less~~ more GMD.

* Inductance of 3ϕ double circuit line with symmetrical spacing.

- Consider a 3ϕ double ckt connected in parallel conductors A, B, C forming one circuit conductors A', B', C' forming another circuit



flux linkages of conductor

$$\psi_A = 2\times10^{-7}\left[I_A\left(\log_e\frac{1}{r'}+\log_e\frac{1}{2d}\right)+I_B\left(\log_e\frac{1}{d}+\log_e\frac{1}{\sqrt{3}\,d}\right)+I_C\left(\log_e\frac{1}{\sqrt{3}\,d}+\log_e\frac{1}{d}\right)\right] \text{ wb-turns/m}$$

$$= 2\times10^{-7}\left(I_A\log_e\frac{1}{r'2d}\right)+(I_B+I_C)\left(\log_e\frac{1}{d^2\sqrt{3}}\right)$$

$$= 2\times10^{-7}\, I_A\log_e\frac{1}{2r'd} - I_A\log_e\frac{1}{\sqrt{3}\,d^2} \quad (\because I_A+I_B+I_C=0)$$

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$$= 2\times10^{-7}\, I_A \left(\log_e \frac{1}{2r'd} - \log_e \frac{1}{\sqrt{3}\ d^2} \right)$$

$$= 2\times10^{-7}\, I_A \log_e \frac{\sqrt{3}\ d^{\cancel{2}}}{2r'\cancel{d}}$$

$$\psi_A = 2\times10^{-7}\, I_A \log_e \frac{\sqrt{3}\ d}{2r'} \text{ wb turns/m.}$$

$$\rightarrow L_A = \frac{\psi_A}{I_A} = 2\times10^{-7} \log_e \frac{\sqrt{3}\ d}{2r'} \text{ H/m.}$$

→ Inductance of L_B & L_C will have same values becauses the phases are symmetrically placed

→ Since conductors are electrically in parallel, inductance of each phase;

$$= \frac{1}{2} L_A = 10^{-7} \log_e \frac{\sqrt{3}\ d}{2r'} \text{ H/m.}$$

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* Capacitance of 3ϕ overhead lines.

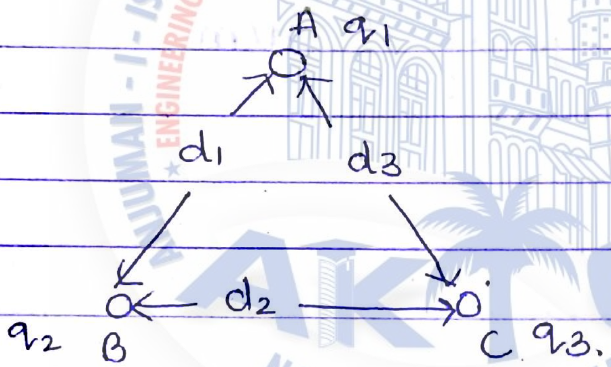
- Unsymmetrically spaced line:

→ For untransposed unsymmetrical 3ϕ line the capacitances between conductor to neutral of three conductors are different.

→ Suppose voltages applied are V_A, V_B, V_C & the charges/meter length is q_1, q_2, q_3 respectively.

→ Potential of conductor 'A' w.r.t infinite conductor plane.

$$V_{AN} = \frac{q_1}{2\pi\epsilon_0}\int_r^{\infty}\frac{1}{x}dx + \frac{q_2}{2\pi\epsilon_0}\int_{d_1}^{\infty}\frac{1}{x}dx + \frac{q_3}{2\pi\epsilon_0}\int_{d_3}^{\infty}\frac{1}{x}dx$$



$$= \frac{1}{2\pi\epsilon_0}\left[q_1[\log_e x]_r^{\infty} + q_2[\log_e x]_{d_1}^{\infty} + q_3[\log_e x]_{d_3}^{\infty}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1\log_e\infty - q_1\log_e r + q_2\log_e\infty - q_2\log_e d_1 + q_3\log_e\infty - q_3\log_e d_3\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[-q_1\log_e r - q_2\log_e d_1 - q_3\log_e d_3 + (q_1+q_2+q_3)\log_e\infty\right]$$

$$V_{AN} = \frac{1}{2\pi\epsilon_0}\left[q_1\log_e\frac{1}{r} + q_2\log_e\frac{1}{d_1} + q_3\log_e\frac{1}{d_3}\right] \quad \text{---} (1)$$

$(q_1+q_2+q_3=0)$

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Similarly;

$$V_{BN} = \frac{1}{2\pi\epsilon_0}\left[q_2 \log_e \frac{1}{r} + q_1 \log_e \frac{1}{d_1} + q_3 \log_e \frac{1}{d_2}\right] \text{ volts} \quad —(2)$$

and;

$$V_{CN} = \frac{1}{2\pi\epsilon_0}\left[q_3 \log_e \frac{1}{r} + q_1 \log_e \frac{1}{d_3} + q_2 \log_e \frac{1}{d_2}\right] \text{ volts} \quad —(3)$$

Substituting $q_3 = -(q_1+q_2)$ in eq (1) & (2).

$$V_{AN} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{1}{r} + q_2 \log_e \frac{1}{d_1} - (q_1+q_2) \log_e \frac{1}{d_3}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1\left(\log_e \frac{1}{r} - \cancel{q_1} \log_e \frac{1}{d_3}\right) + q_2\left(\log_e \frac{1}{d_1} - \log_e \frac{1}{d_3}\right)\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{d_3}{r} + q_2 \log_e \frac{d_3}{d_1}\right] \text{ volts.} \quad —(4)$$

&

$$V_{BN} = \frac{1}{2\pi\epsilon_0}\left[q_2 \log_e \frac{1}{r} + q_1 \log_e \frac{1}{d_1} - (q_1+q_2) \log_e \frac{1}{d_2}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1\left(\log_e \frac{1}{d_1} - \log_e \frac{1}{d_2}\right) + q_2\left(\log_e \frac{1}{r} - \log_e \frac{1}{d_2}\right)\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left(q_1 \log_e \frac{d_2}{d_1} + q_2 \log_e \frac{d_2}{r}\right) \text{ volts.} \quad —(5)$$

→ Multiply eq (4) by $\log_e \frac{d_2}{r}$ & eq (5) by $\log_e \frac{d_3}{d_1}$ we get.

$$V_{AN} \log_e \frac{d_2}{r} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{d_3}{r} \cdot \log_e \frac{d_2}{r} + q_2 \log_e \frac{d_3}{d_1} \cdot \log_e \frac{d_2}{r}\right]$$

&

$$V_{BN} \log_e \frac{d_3}{d_1} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{d_2}{d_1} \cdot \log_e \frac{d_3}{d_1} + q_2 \log_e \frac{d_2}{r} \cdot \log_e \frac{d_3}{d_1}\right]$$

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→ Subtracting above two equations we get;

$$V_{AN} \log_e \frac{d}{r} - V_{BN} \log_e \frac{d_3}{d_1} = \frac{q_1}{2\pi\epsilon_0}\left[\log_e \frac{d_3}{r} \cdot \log_e \frac{d_2}{r} - \log_e \frac{d_2}{d_1} \cdot \log_e \frac{d_3}{d_1}\right]$$

or

$$q_1 = 2\pi\epsilon_0 \left[\frac{V_{AN} \log_e \frac{d}{r} - V_{BN} \log_e \frac{d_3}{d_1}}{\log_e \frac{d_3}{r} \cdot \log_e \frac{d_2}{r} - \log_e \frac{d_2}{d_1} \cdot \log_e \frac{d_3}{d_1}}\right]$$

$$\rightarrow C_{AN} = \frac{q_1}{V_{AN}} = 2\pi\epsilon_0 \left[\frac{\log_e \frac{d}{r} - \frac{V_{BN}}{V_{AN}} \log_e \frac{d_3}{d_1}}{\log_e \frac{d_3}{r} \cdot \log_e \frac{d_2}{r} - \log_e \frac{d_2}{d_1} \cdot \log_e \frac{d_3}{d_1}}\right]$$

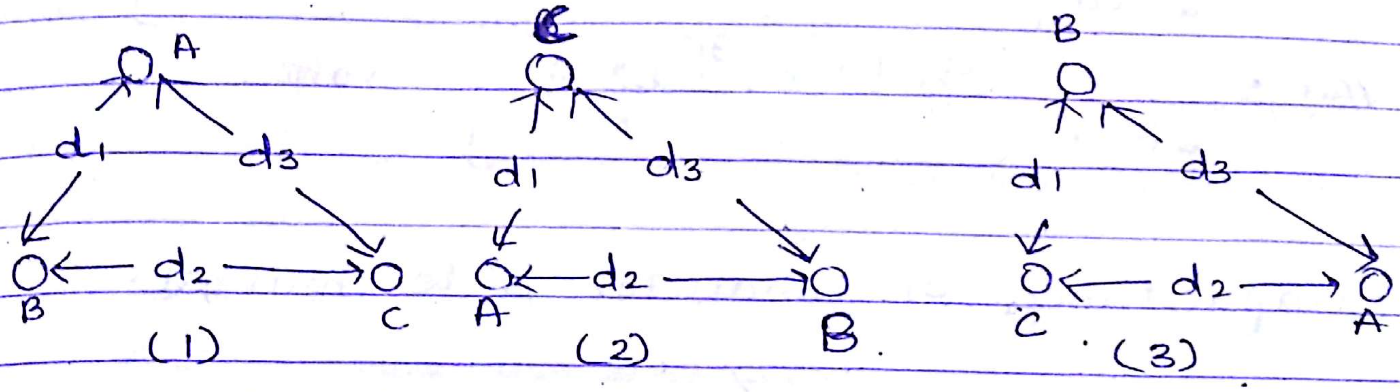
Similarly;

$$\rightarrow C_{BN} = \frac{q_2}{V_{BN}} = 2\pi\epsilon_0 \left[\frac{\log_e \frac{d_3}{r} - \frac{V_{CN}}{V_{BN}} \log_e \frac{d_1}{d_2}}{\log_e \frac{d_1}{r} \cdot \log_e \frac{d_3}{r} - \log_e \frac{d_3}{d_2} \cdot \log_e \frac{d_1}{d_2}}\right]$$

$$\rightarrow C_{CN} = \frac{Q_3}{V_{CN}} = 2\pi\epsilon_0 \left[\frac{\log_e \frac{d_1}{r} - \frac{V_{AN}}{V_{CN}} \log_e \frac{d_2}{d_3}}{\log_e \frac{d_2}{r} \cdot \log_e \frac{d_1}{r} - \log_e \frac{d_1}{d_3} \cdot \log_e \frac{d_2}{d_3}}\right]$$

F/m.

* Unsymmetrical line with transposed conductors:
- Assuming that charge per unit length is same in every part of transposed cycle.



- Voltage of conductor A $= \dfrac{V_{AN1} + V_{AN2} + V_{AN3}}{3}$.

$$-V_{AN1} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{1}{r} + q_2 \log_e \frac{1}{d_1} + q_3 \log_e \frac{1}{d_3}\right].$$

$$V_{AN2} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{1}{r} + q_2 \log_e \frac{1}{d_2} + q_3 \log_e \frac{1}{d_1}\right].$$

$$V_{AN3} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{1}{r} + q_2 \log_e \frac{1}{d_3} + q_3 \log_e \frac{1}{d_2}\right]$$

-Avg value of voltage in conductor A $= \dfrac{V_{AN1} + V_{AN2} + V_{AN3}}{3}$

$$= \frac{1}{6\pi\epsilon_0}\left[3q_1 \log_e \frac{1}{r} + q_2 \log_e \frac{1}{d_1 d_2 d_3} + q_3 \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$= \frac{1}{6\pi\epsilon_0}\left[3q_1 \log_e \frac{1}{r} + (q_2+q_3) \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{1}{r} - \frac{q_1}{3} \log_e \frac{1}{d_1 d_2 d_3}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{1}{r} + q_1 \log_e \sqrt[3]{d_1 d_2 d_3}\right]$$

$$V_{AN} = \frac{1}{2\pi\epsilon_0}\left[q_1 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \text{ volts.}$$

→ Capacitance of conductor 'A' to neutral.

$$C_{AN} = \frac{q_1}{V_{AN}} = \frac{\cancel{q_1}}{\frac{1}{2\pi\epsilon_0}\left[\cancel{q_1} \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right]}.$$

$$C_{AN} = \frac{2\pi\epsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} \text{ F/m.}$$

→ Similarly

$$C_{AN} = C_{BN} = C_{CN} = \frac{2\pi\epsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} \text{ F/m.}$$

* Equilateral spaced line:

→ $d_1 = d_2 = d_3 = d$.

$$C_{AN} = C_{BN} = C_{CN} = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m.}$$

* Capacitance of Double circuit 3ϕ overhead lines

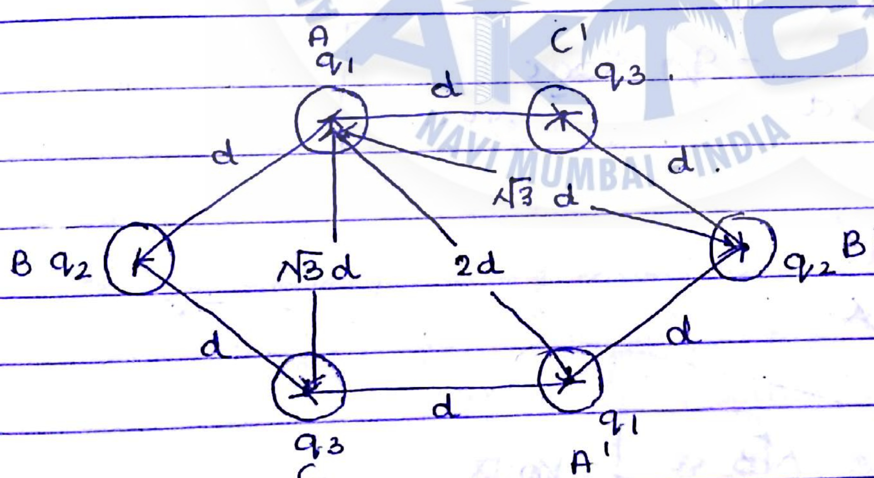
→ For calculation of inductance, determination of (GMR) of conductor is necessary because of internal flux linkage of conductor.

→ But in case of calculations of capacitance, since all charges reside on the surface of the conductor, actual radius of the conductor is used.

* Symmetrically spaced line :

→ Consider 3ϕ double circuit connected in parallel. conductors A, B, C forming one circuit & conductors A', B', C' forming another circuit.

→ let charge over conductors A, B, C be q_1, q_2, q_3 coulombs/meter length. Then charge over conductors A', B', C' will obviously be q_1, q_2, q_3 coulombs/meter length & $q_1+q_2+q_3=0$.



→ Potential of conductor 'A' with respect to infinite plane.

$$\cancel{V_{AN} = \frac{1}{2\pi\epsilon_0}\left[q_1\int_r^\infty \frac{1}{x}\,dx + q_2\int_d^\infty \frac{1}{x}\,dx +\right.}$$

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$$V_{AN} = \frac{1}{2\pi\epsilon_0}\left[q_1\left[\int_r^\infty \frac{1}{x}dx + \int_{2d}^\infty \frac{1}{x}dx\right] + q_2\left[\int_d^\infty \frac{1}{x}dx + \int_{\sqrt{3}d}^\infty \frac{1}{x}dx\right] + q_3\left[\int_{\sqrt{3}d}^\infty \frac{1}{x}dx + \int_d^\infty \frac{1}{x}dx\right]\right\}$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1(\log_e\infty - \log_e r + \log_e\infty - \log_e 2d) + q_2(\log_e\infty - \log_e d + \log_e\infty - \log_e\sqrt{3}d) + q_3(\log_e\infty - \log_e\sqrt{3}d + \log_e\infty - \log_e d)\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[2q_1\log_e\infty - q_1\log_e r - q_1\log_e 2d + 2q_2\log_e\infty - q_2\log_e d - q_2\log_e\sqrt{3}d + 2q_3\log_e\infty - q_3\log_e\sqrt{3}d - q_3\log_e d\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[2(q_1+q_2+q_3)\log_e\infty + q_1\log_e\frac{1}{r2d} + q_2\log_e\frac{1}{\sqrt{3}d^2} + q_3\log_e\frac{1}{\sqrt{3}d^2}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[2(q_1+q_2+q_3)\log_e\infty + q_1\log_e\frac{1}{2rd} + (q_2+q_3)\log_e\frac{1}{\sqrt{3}d^2}\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[q_1\log_e\frac{1}{2rd} - q_1\log_e\frac{1}{\sqrt{3}d^2}\right]$$

$$= \frac{q_1}{2\pi\epsilon_0}\log_e\frac{\sqrt{3}d^{\cancel{2}}}{2r\cancel{d}}$$

$$\boxed{V_{AN} = \frac{q_1}{2\pi\epsilon_0}\log_e\frac{\sqrt{3}d}{2r}} \text{ volts.}$$

→ Capacitance of conductor A to neutral.

$$C_{AN} = \frac{q_1}{V_{AN}} = \frac{2\pi\epsilon_0}{\log_e\frac{\sqrt{3}d}{2r}} \text{ F/m.}$$

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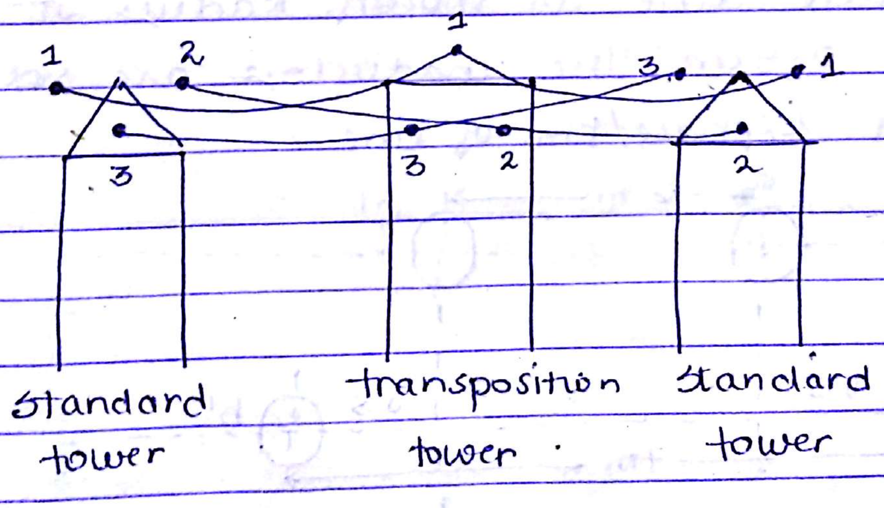
→ $C_{AN} = C_{BN} = C_{CN} = C_N = \dfrac{2\pi\epsilon_0}{\log_e \frac{\sqrt{3}d}{2r}}$ F/m.

→ Capacitance/phase $= 2C_N = \dfrac{4\pi\epsilon_0}{\log_e \frac{\sqrt{3}d}{2r}}$

* Flat vertically spaced line ⇒ S.S.
* Transpositioning of conductors :→
→ L & C will be different in case of irregularly spaced conductors.
→ 'Apparant R' is affected because of affect in mutual 'L'. Thus all three parameters are affected.
→ Due to unsymmetrical spacing, the magnetic feild external to the conductor is not zero. Thereby causing induced voltages in neibouring ckts particularly telephone lines & causes interference.
→ So we change the position of 3 phases on the line supports twice over the length of the line. This is transpositioning and its done such that each of the three possible arrangements of conductors exists for 1/3rd of line.



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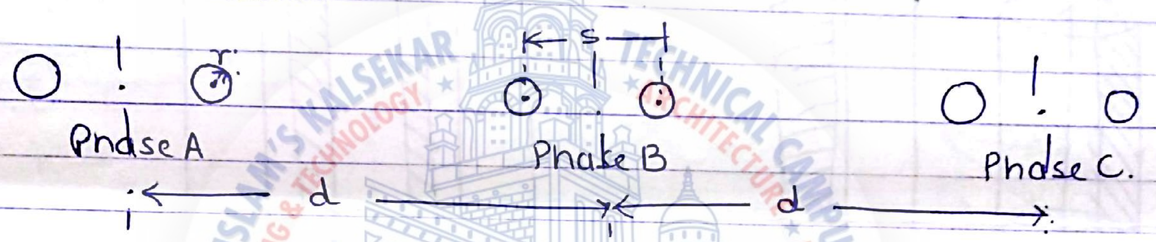
Bundled Conductor:

Demand of electric supply is ↑. To meet them power stations of large capacity are set up near sources of large power generation (thermal, hydro, neuclear).

- Therefore power stations are located far from load centers & its imperative to transmitt bulk power at moderate η which can be obtained by high voltage Xmission (Adv: ↑ η, ↓ drop, ↓ conductor material)
- EHV systems are generally adopted & 230kV & above voltage falls under that category.
- But if vg level is increased beyond 300kV we have the problem of corona & corona loss. & increased interference with neighbouring comm ckts.
- ∴ for lines above 400kV bundled conductors are used to avoid the above.



- Made up of 2 or more conductors/phase each called as sub-conductor
- Spacing between sub-conductor is less as compared to the spacing between phases
- Difference between bundle conductor & composite conductor is that the spacing between the sub-conductors is 0.2-0.6m whereas the wires of composite conductor touch each other.

- <u>Adv of Bundle Conductor</u>:

(1) Transmits bulk power with reduced losses, η ↑.

(2) Bundle conductor have greater capacitance to neutral, ∴ higher charging current which improves p.f.

(3). Because of bundling GMR has increased, L ↓, X ↓.

(4) Surge impedence of the line is given by $Z_0 = \sqrt{L/C}$
L ↓, C ↑ ∴ ↓ surge Z loading.

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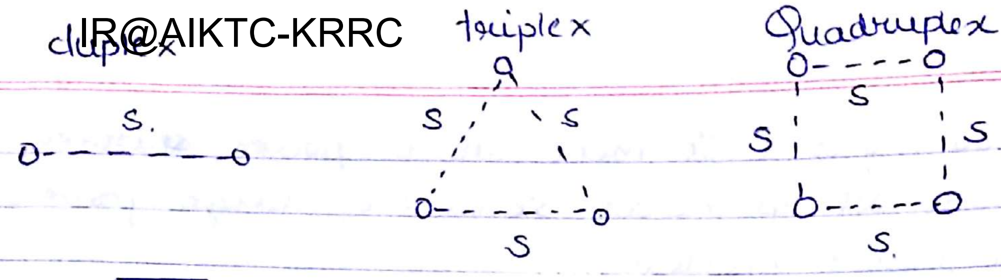
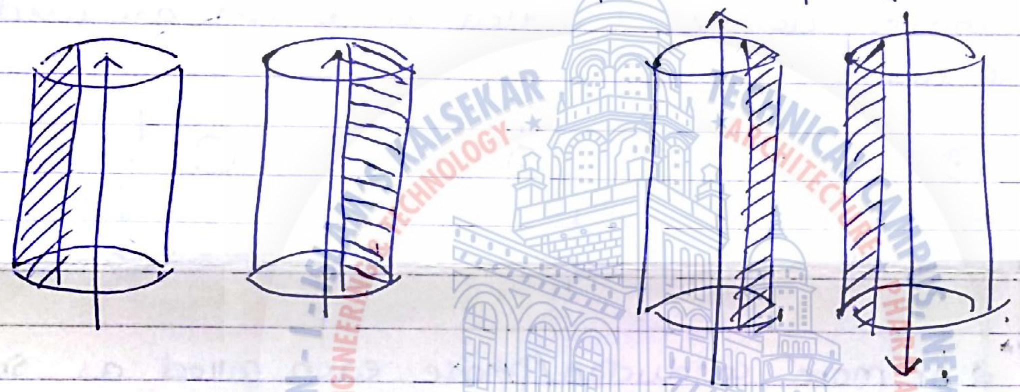


$D_s = \sqrt{r' s}$ → duplex arrangement.
$D_s = \sqrt[3]{r' s^2}$ → triplex.
$D_s = 1.09 \sqrt[4]{r' s^3}$ → quadruplex

* Proximity effect:

- The inductance & therefore the current distribution is affected by the presence of other conductors in the vicinity. This is known as proximity effect.



- Depends on
  - Conductor size
  - frequency.
  - Resistivity
  - Relative $\boxed{\text{permeability}}$ → allow passage of magnetic field.

* Capacitance of Xmission lines:

- Any two conductor seperated by an insulating medium constitute a capacitor.
- In overhead line two conductors form two plates of capacitor & air in between behaves as di-electric medium.
- Thus overhead lines has capacitance throughout the length of the line.
- When alternating p.d is applied across it, it draws charging current (leads) even if there is no load connected.

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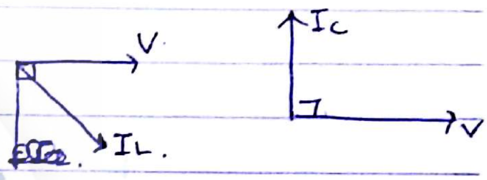
...rength of charging current depends on. (i) voltge.
(ii) C
(iii) f.

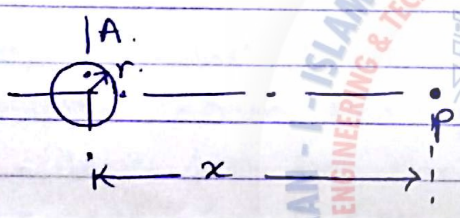
$$I_c = 2\pi f C V = \frac{V}{X_c}$$

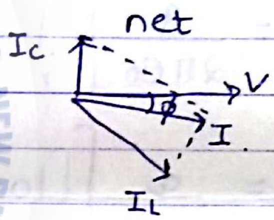
- If 'C' is high, 'I_c' will be high which will compensate the lagging component of current drawn (load current)
- Net current will be less. Hence less current will be drawn
- Which results in – ↓ losses, ↑ η.
  – ↓ IR drop, ↑ vg regulatn.
  – $\boxed{\text{improved p.f.}}$ →

load (inductive) Capacitor



✱ Potential at Charged Single Conductor:





→ Electric feild intensity (E) at point P $= \dfrac{q}{2\pi \epsilon_0 \epsilon_r x}$ volts/m. (q → charge)

$\phi \to \downarrow$; p.f → ↑.

– $\epsilon_r = 1$ (air)

$$\therefore E = \frac{q}{2\pi \epsilon_0 x} \text{ volts/m.}$$

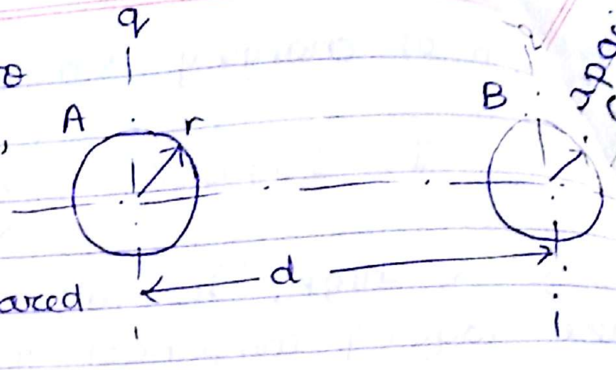
$$\rightarrow V_A = \int_r^\infty \frac{q}{2\pi \epsilon_0 x}\, dx.$$

- Electric charge causes an electric feild around it which extends upto infinity
- If charge is introduced in this feild, it will be attracted or repelled according to nature of charge.

Potential at any point in electric feild is work done in moving a unit +ve charge from infinity to that point.

* Capacitance of 1φ overhead line.

- Consider 1φ overhead line with two parallel conductors, each of radius r, placed at distance d meters in air
- It is assumed that the distance between conductor is large as compared to its radii



→ P.D between conductor 'A' & neutral plane at ∞.

$$V_{AN} = \int_r^{\infty} \frac{q}{2\pi\epsilon_0 x} dx + \int_d^{\infty} \frac{-q}{2\pi\epsilon_0 x} dx$$

infinite plane is 'x' distance away from conductor

$$= \frac{q}{2\pi\epsilon_0}\left[\int_r^{\infty} \frac{1}{x} dx - \int_d^{\infty} \frac{1}{x} dx\right]$$

$$= \frac{q}{2\pi\epsilon_0}\left\{[\log_e x]_r^{\infty} - [\log_e x]_d^{\infty}\right\}$$

$$= \frac{q}{2\pi\epsilon_0}\left[\cancel{\log_e \infty} - \log_e r - \cancel{\log_e \infty} + \log_e d\right]$$

$$\boxed{V_{AN} = \frac{q}{2\pi\epsilon_0} \log_e \frac{d}{r}}$$

→ P.D between conductor 'B' and the neutral plane at ∞.

$$V_{BN} = \int_r^{\infty} \frac{-q}{2\pi\epsilon_0 x} dx + \int_d^{\infty} \frac{q}{2\pi\epsilon_0 x} dx$$

$$\boxed{V_{BN} = \frac{-q}{2\pi\epsilon_0} \log_e \frac{d}{r}}$$

→ P.D Between conductors A & B.

$$V_{AB} = V_{AN} - V_{BN} = \frac{q}{2\pi\epsilon_0}\left[\log_e \frac{d}{r} + \log_e \frac{d}{r}\right]$$

$$\boxed{V_{AB} = \frac{q}{\pi\epsilon_0} \log_e \frac{d}{r}} \text{ volts.}$$

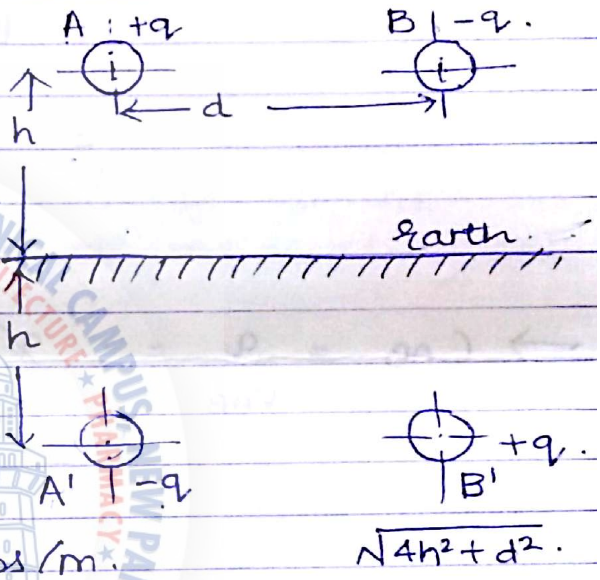
Capacitance of line $C_{AB} = \frac{q}{V_{AB}} = \frac{q}{\frac{q}{\pi \epsilon_0} \log_e \frac{d}{r}}$

(∵ Capacitance between two conductors)

$$\boxed{C = \frac{\pi \epsilon_0}{\log_e d/r}} \quad \text{Farad/meter}$$

→ Capacitance for each conductor or (Cn) Capacitance to neutral $= 2\, C_{AB}$ farad/meter.

* Effect of Earth on Capacitance of 1φ overhead lines:



- Assume A' and B' to be the image conductors of A & B.
- let the height of the conductor be 'h' meter above the ground & +q coulomb/m be the charge on A, −q coulomb/m be the charge on B.
- The earth is considered to be at zero potential which is possible only if there is image conductor A' having a charge of −q coulombs/m at depth of 'h' meters below earth.
- Similarly there is conductor B' below the earth having a charge of +q coulombs/m length.

$$\rightarrow V_{AN} = \int_r^\infty \frac{q}{2\pi \epsilon_0 x} dx + \int_d^\infty \frac{-q}{2\pi\epsilon_0 x} dx + \int_{2h}^\infty \frac{-q}{2\pi \epsilon_0 x} dx + \int_{\sqrt{4h^2+d^2}}^\infty \frac{q}{2\pi \epsilon_0 x} dx$$

$$= \frac{q}{2\pi\epsilon_0}\left[\int_r^\infty \frac{1}{x} dx - \int_d^\infty \frac{1}{x} dx - \int_{2h}^\infty \frac{1}{x} dx + \int_{\sqrt{4h^2+d^2}}^\infty \frac{1}{x} dx.\right]$$

$$= \frac{q}{2\pi\epsilon_0}\left[[\log_e x]_r^\infty - [\log_e x]_d^\infty - [\log_e x]_{2h}^\infty + [\log_e x]_{\sqrt{4h^2+d^2}}^\infty\right]$$

$$= \frac{q}{2\pi\epsilon_0}\left[\cancel{\log_e \infty} - \log_e r - \cancel{\log_e \infty} + \log_e d - \cancel{\log_e \infty} + \log_e 2h + \cancel{\log_e \infty} - \log_e \sqrt{4h^2+d^2}\right]$$

$$= \frac{q}{2\pi\epsilon_0}\left[\log_e \frac{d}{r} + \log_e \frac{2h}{\sqrt{4h^2+d^2}}\right]$$

$$= \frac{q}{2\pi\epsilon_0}\left[\log_e \frac{d\,2h}{r\sqrt{4h^2+d^2}}\right].$$

$$\rightarrow V_{BN} = \frac{-q}{2\pi\epsilon_0}\log_e \frac{2dh}{r\sqrt{4h^2+d^2}}.$$

$$\rightarrow V_{AB} = V_{AN} - V_{BN} = \frac{q}{2\pi\epsilon_0}\left[\log_e \frac{2dh}{r\sqrt{4h^2+d^2}} + \log_e \frac{2dh}{r\sqrt{4h^2+d^2}}\right]$$

$$= \frac{q}{\cancel{2}\pi\epsilon_0}\,\cancel{2}\left[\log_e \frac{2dh}{r\sqrt{4h^2+d^2}}\right]$$

$$= \frac{q}{\pi\epsilon_0}\log_e \frac{d}{r\sqrt{\frac{4h^2}{4h^2}+\frac{d^2}{4h^2}}}.$$ Dividing by 2h.

$$= \frac{q}{\pi\epsilon_0}\log_e \frac{d}{r\sqrt{1+\frac{d^2}{4h^2}}}.$$ volts.

$$\rightarrow C_{AB} = \frac{q}{V_{AB}} = \frac{\cancel{q}}{\frac{\cancel{q}}{\pi\epsilon_0}\log_e \frac{d}{r\sqrt{1+\frac{d^2}{4h^2}}}} = \frac{\pi\epsilon_0}{\log_e \frac{d}{r\sqrt{1+d^2/4h^2}}}$$ F/m.

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Components of transmission line R, L, C, $\boxed{\text{Shunt Conductance}}$ (leakage over insulators).
Performance of Xmission line depends on these parameters & study of these parameters is imp for the electrical design of Xmission lines.

* Line Resistance:

- Opposition to the flow of 'I'
- Causes I^2R losses
- $R = \frac{\rho l}{a}$ — $\rho \rightarrow$ Resistivity of material (depends on conductor material); $l \rightarrow$ length; $a \rightarrow$ X section area.

* Skin Effect:

- Definition
- It depends on;
  (i) Type of Material (ii) freq (iii) Diameter of conductor (iv) Shape of conductor.

- Skin effect is much smaller for stranded conductors.
- Stranded conductors are used for transmission & distribution
- Hollow conductors are used for bus-bars

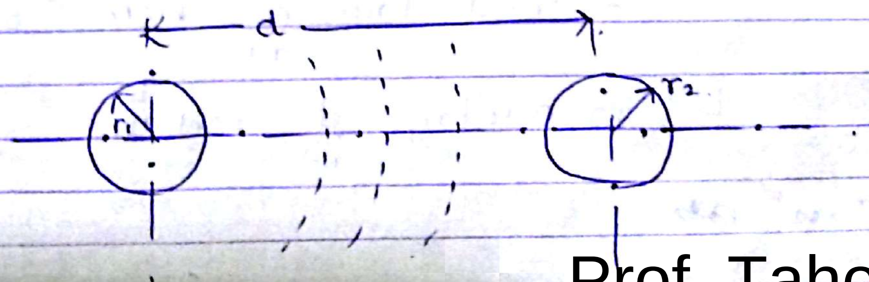
* Line Inductance

- Definition. • $L = \psi / I$.

* Inductance of 1ϕ 2 wire line:

Assumptions

- Consider 1ϕ line consisting of two ‖ conductors A & B of radii r_1 & r_2 spaced 'd' meters apart. A & B carry same current ($I_A = I_B$) but opposite directions as one forms the return path of other.
- It is assumed that all flux produced by current in conductor 'A' links all current upto the center of conductor 'B' & that flux beyond the center of conductor B does not link any current.



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- We know external flux linkages for conductor A is given by.

$$\boxed{\psi_{A\,ext} = 2\times10^{-7} I \log_e \frac{d}{r_1}} \quad \text{Wb turn/meter.}$$

- Internal flux linkages of conductor A.

$$\boxed{\psi_{A\,int} = \frac{1}{2} I \times 10^{-7} \quad \text{Wb turn/meter}}$$

- Total flux linkages $= \psi_{A\,ext} + \psi_{A\,int} = 2\times10^{-7} I \log_e \frac{d}{r_1} + \frac{1}{2} I \times 10^{-7}$

$$= \left(0.5 + 2\log_e \frac{d}{r_1}\right) I \times 10^{-7} \quad \text{Wb turns/m.}$$

- Total inductance of conductor A;

$$L_A = \frac{\psi_A}{I} = \left(0.5 + 2\log_e \frac{d}{r_1}\right)\times10^{-7} \text{ Henry/m.}$$

$$= 2\times10^{-7}\left(0.25 + \log_e \frac{d}{r_1}\right) \text{H/m.}$$

$$= 2\times10^{-7}\left(\log_e e^{1/4} + \log_e \frac{d}{r_1}\right) \text{H/m.}$$

$\boxed{\log A + \log B = \log(A \cdot B)}$

$$= 2\times10^{-7} \log_e\left(\frac{d}{r_1 e^{-1/4}}\right) \text{H/m.}$$

- The product $(r_1 e^{-1/4})$ is 'Geometric Mean Radius' (GMR) of the conductor. If GMR is r_1', then

$$r_1' = 0.7788\, r_1$$

$$\therefore \quad L_A = 2\times10^{-7} \log_e \frac{d}{r_1'} \text{ H/m.}$$

$$\therefore \quad L_B = 2\times10^{-7} \log_e \frac{d}{r_2'} \text{ H/m.}$$

- Total inductance $(L_A + L_B) = 2\times10^{-7}\left(\log_e \frac{d}{r_1'} + \log_e \frac{d}{r_2'}\right)$

$$L = 2\times10^{-7}\, 2\log_e \frac{d}{r'} \quad (\because r_1' = r_2' = r')$$

$$L = 4\times10^{-7} \log_e \frac{d}{r'} \text{ H/m. or}$$

$$L = 0.4 \log_e \frac{d}{r'} \text{ mH/km.}$$

$1\text{H} \rightarrow 10^{-3}\text{mH}$
$4\times10^{-7}\text{ --} \rightarrow ?\text{mH}$

$1\text{km} \rightarrow 1000\text{m}$
$?\text{km} \rightarrow \text{-- m.}$

$\frac{4\times10^{-7} \times \cancel{10^{2}} \times \cancel{10^{3}}}{10^{-3}\times10^{-3}}$

$\frac{4\times10^{-7}}{?}, 0.4$

$\frac{\times10^{-3}}{\frac{1}{1000}}$

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