MODULE 2

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* Analysis of simple Rectifier circuits:

(lec 3)

• Assumptions & justifications:

- AC voltage has no impedance & delivers constant voltage of sinusoidal waveform & constant freq.

(phase shifted by 120°)

- If poly phase source it delivers balance voltages

- Transformers have no leakage impedance or exciting admittance (∵ Practically we should go for large leakage 'Z'. For analysis, here 'Z' = 0)

- The dc load has infinite inductance (ie dc current is constant & ripple free) (O/P voltage is not ripple free)

- Valve is ideal (ie offers zero resistance during conduction & infinite resistance during non conducting state.

• Some definitions

- The Volt-Ampere rating of the valve is taken as the product of average current & peak inverse voltage.

- Peak inverse voltage (PIV) is the peak voltage that appears across the valve during non conduction state.

- Rating (VA) of the transformer is the product of rms voltage & rms current.

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- Pulse no of a converter is the number of pulsations (cycle of ripples) of dc voltage per cycle of ac voltage.

- A group of valves in which only one valve conducts at a time (neglecting overlap) is known as commutation group (q)

- Vdo is the average dc voltage the terminal of converter when delay angle is zero.

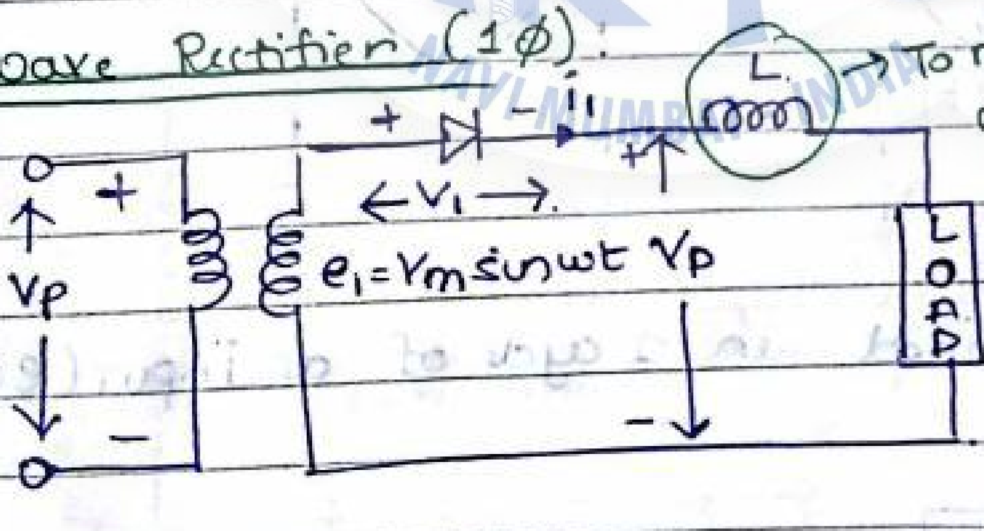
- Number of series valves is represented by 'S'

- Number of parallel valves is represented by 'r'

- Then;

$$p = q \times s \times r$$

• Half wave Rectifier (1ϕ)



→ To make dc current ripplefree but doesn't affect much because we get only half cycle o/p. (∴ dc current cannot be constant)

- It is the simplest rectifier

- Current is inherently intermittent, so dc current cannot be constant.

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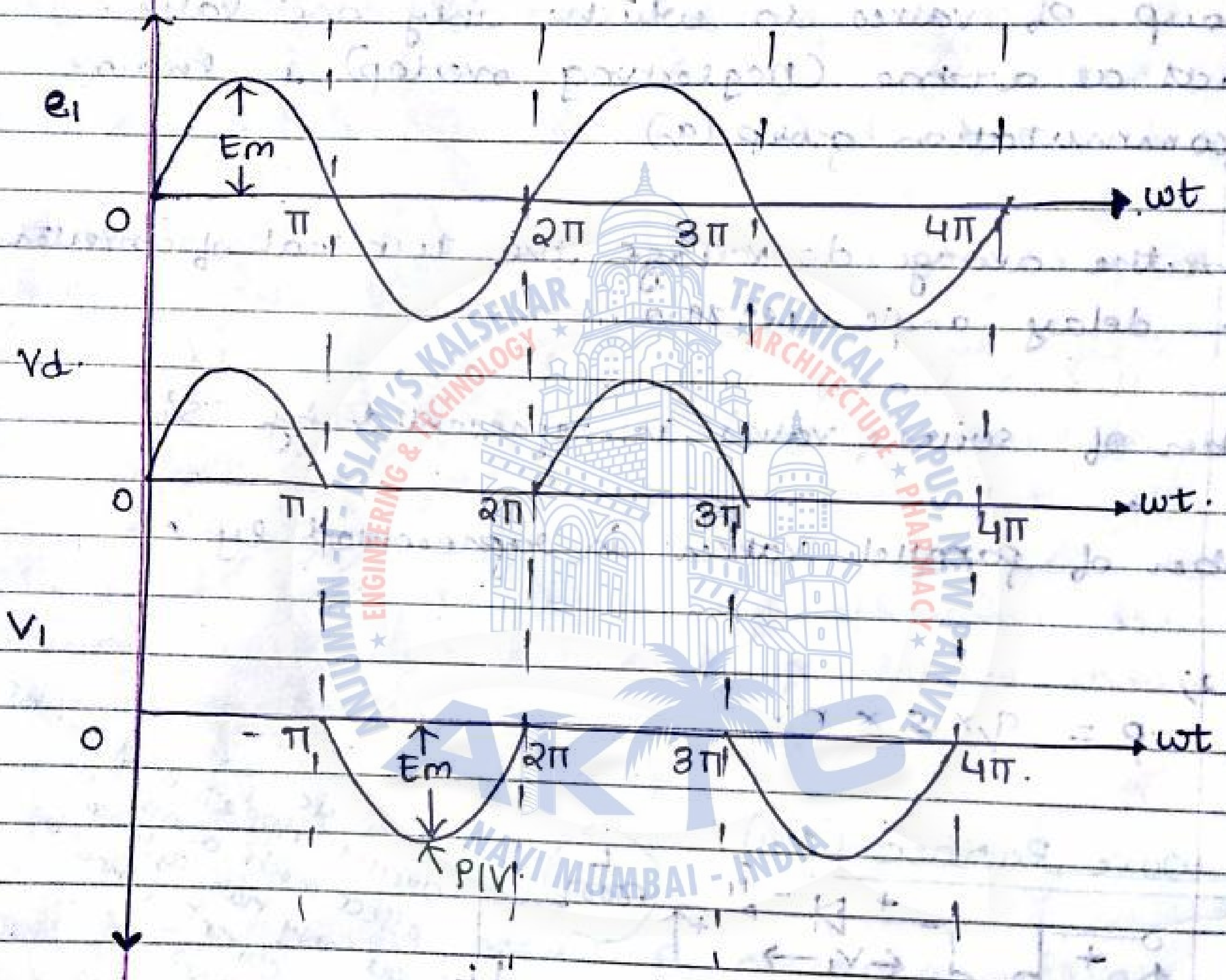
- DC current & voltage pulsate at the same freq as the ac voltage & current.

- Useful for small power applications



- No of pulses (of Vd) obtained in 1 cycle of ac input(e_1) is just 'one'

$$\boxed{\begin{aligned} V_1 &= e_1 - V_d \\ e_1 &= E_m \cos \omega t \end{aligned}}$$

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- V_{do} (Average dc voltage in one cycle)

$$V_{do} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \; d(\omega t)$$

We cant take $(0-2\pi)$ because +ve half is present but −ve half is zero. so considering $(0-\pi)$ & dividing by 2π.

$$= \frac{1}{2\pi}(V_m)\left(-\cos\omega t\right)_0^{\pi} = \frac{V_m}{\pi}$$

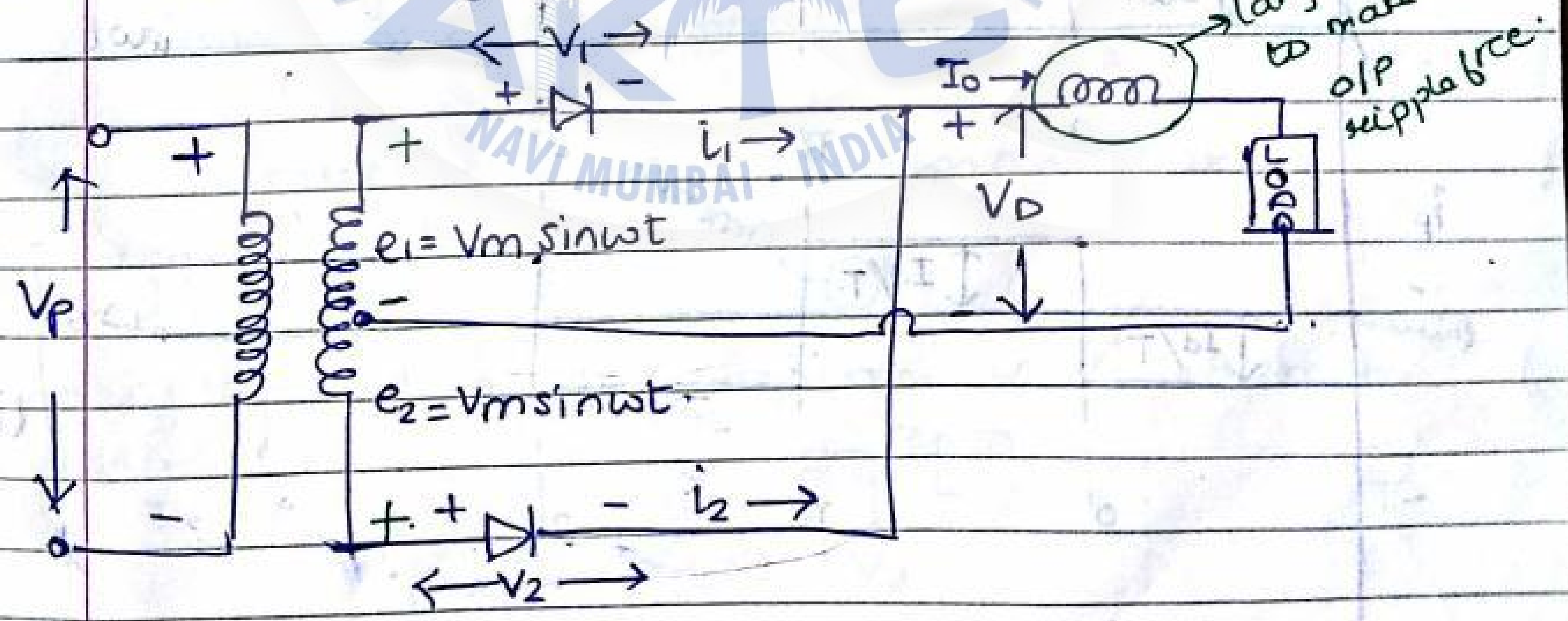
- PIV = V_m (or E_m)

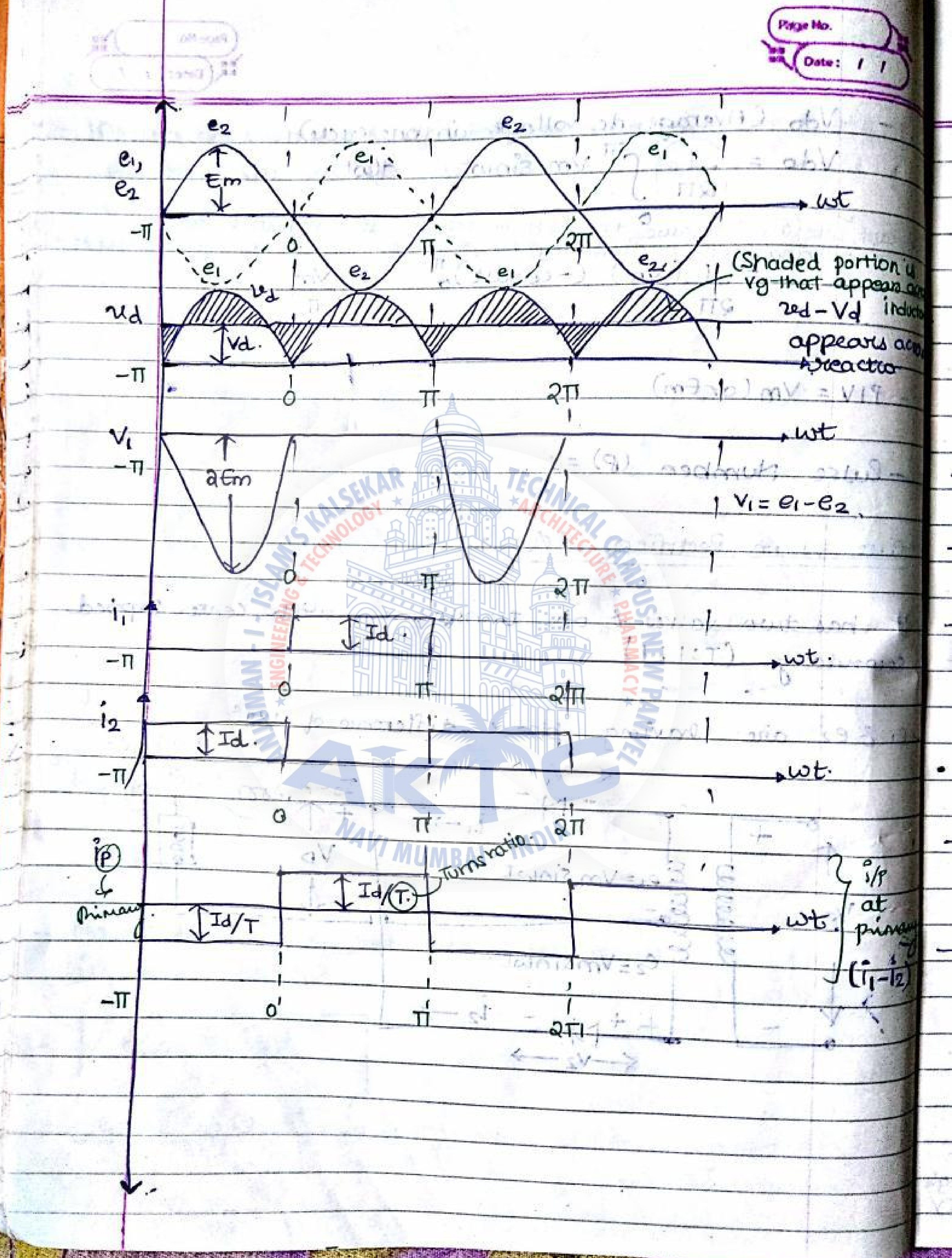
- Pulse Number (P) = 1

- **Full Wave Rectifier (1ϕ)**

- It has two valves & one transformer with center tapped secondary (T:1:1)

- e_1 & e_2 are having phase difference of 180°.

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e_1, e_2
e_2
Em
e_1
ωt
$-\pi$
0
π
2π
v_d
Vd
(Shaded portion is vg that appears across $v_d - V_d$ inductor appears across reactor
V_1
2Em
$V_1 = e_1 - e_2$
i_1
Id
i_2
Id
ip
primary
Id/T
Id/T
Turns ratio
i/p at primary $(i_1 - i_2)$

$e_1 - V_1 - V_D - V_2 = 0$ $\therefore e_1 - V_D = 2x$

$V_1 = e_1 - V_D - V_2$

$e_1 - V_D = V_1/V_2$

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→ $V_{do} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d(\omega t)$

$= \frac{V_m}{\pi}(-\cos\omega t)_0^{\pi} = \frac{2V_m}{\pi}$

We are taking the average of one pulse only ie one half cycle. cuz after that the waveform is continues ie its cyclic.

→ PIV = $2V_m$

→ Pulse Number (P) = 2. ($\because$ 2 pulses in 1 cycle)

→ Average current = $I_d/2$.

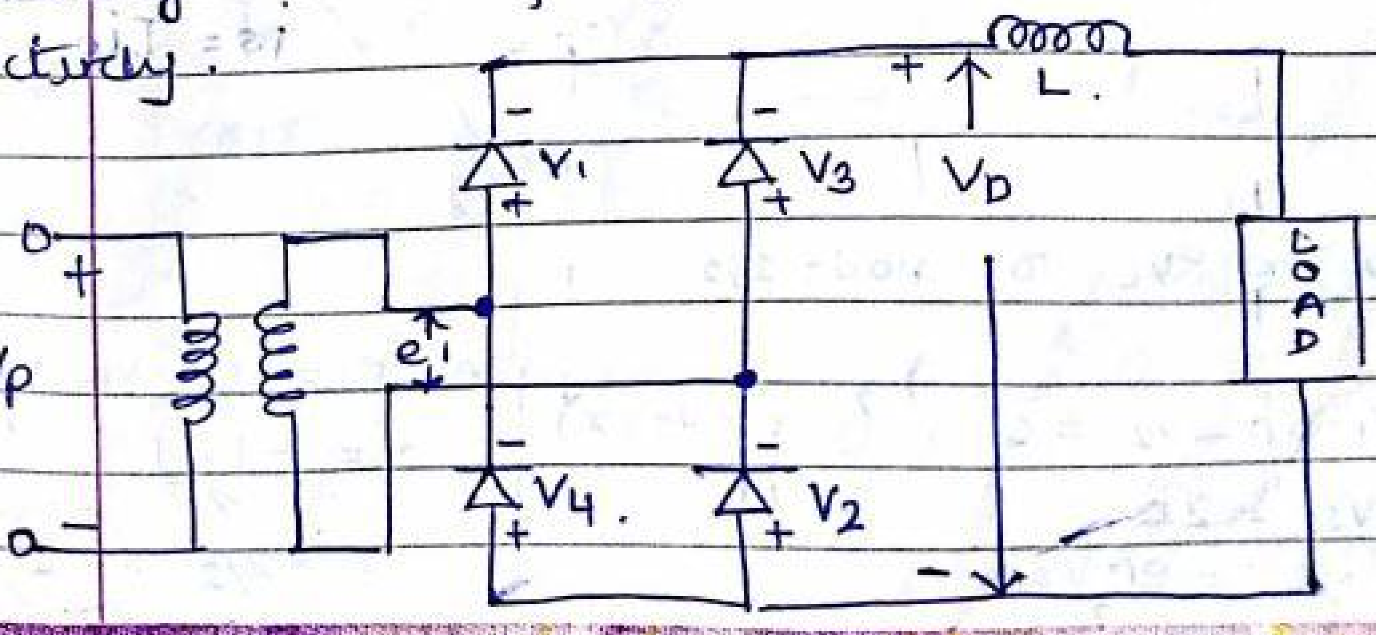
→ Valve Rating = $V_m I_d$.

→ Transformer is not utilized properly so we go for the bridge rectifier ckt.

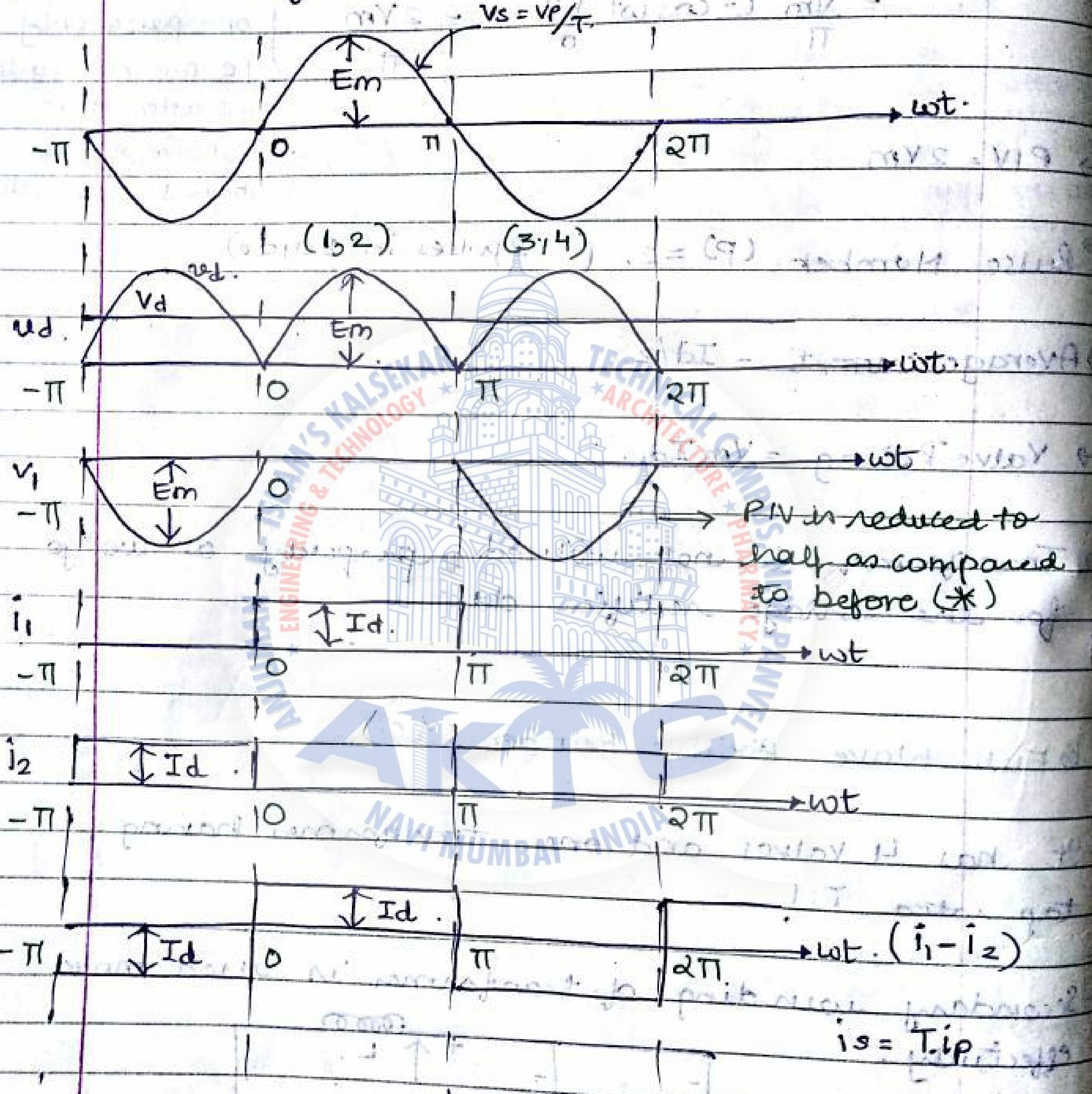
• B Full Wave Bridge Rectifier (1ϕ).

- It has 4 valves and one transformer having tap ratio T:1.

- Secondary winding of transformer is used more effectively.



transformer is utilized twice in one cycle. that is why utilization of transformer is more.



* On applying KVL to diode 1,2.

$e_1 - V_1 - V_D - V_2 = 0$ $\quad (\because V_1 = V_2 = x)$

$e_1 - V_D = 2x$

$\therefore x = \frac{e_1 - V_D}{2}$

let $e_1 = -1$, $V_D = 1$

$\therefore x = \frac{-1-1}{2}$

$= -2/2 = -1$

hence single PIV

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$-1+1$
$-1-1$ $\frac{2}{2}$: 1

→ $V_{do} = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t)$

$= \frac{1}{\pi} V_m (-\cos\omega t)_0^{\pi} = \frac{2V_m}{\pi}$

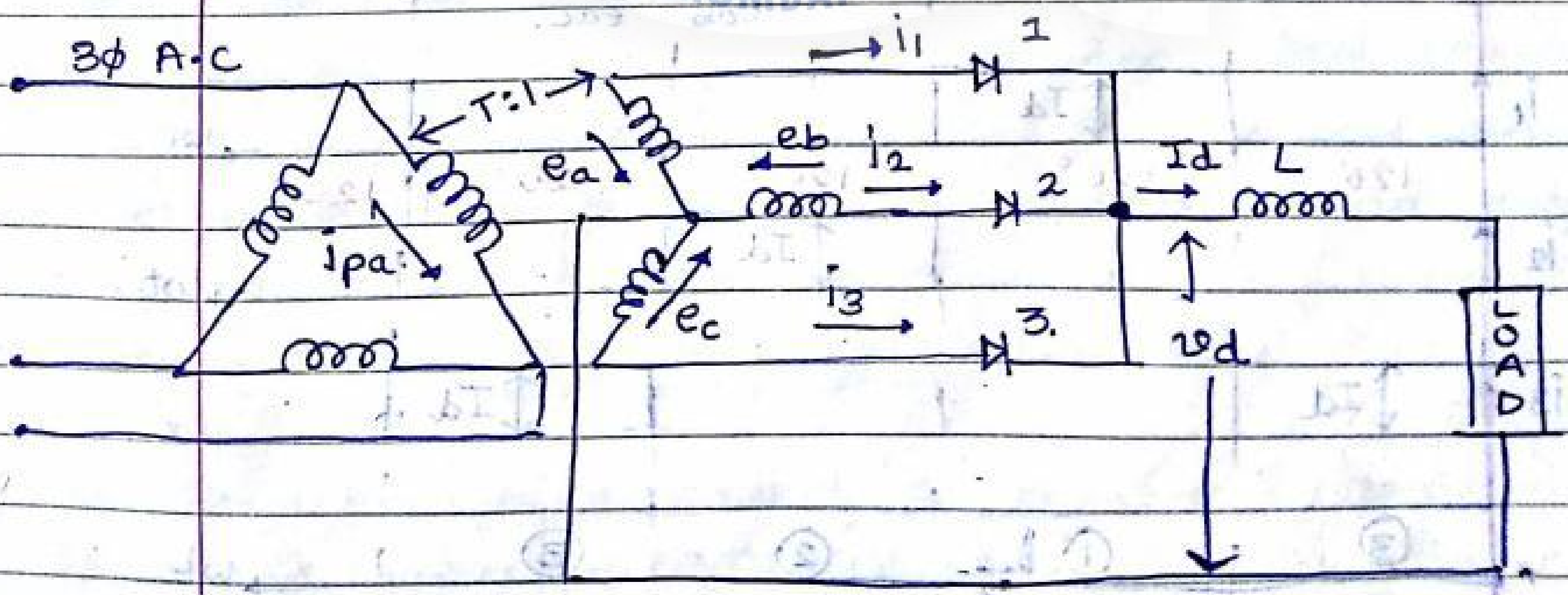
- $\boxed{PIV = V_m}$

- Pulse Number (P) = 2
- Average Current = $I_d/2$
- Valve Rating = $0.5 V_m I_d$ } Power rating reduced by half & hence cheaper.

- Four Valves are used.

- It is used for high voltage applications where valve PIV is a limiting factor.

• 3ϕ Rectifiers (one way)



nce
single
PIV

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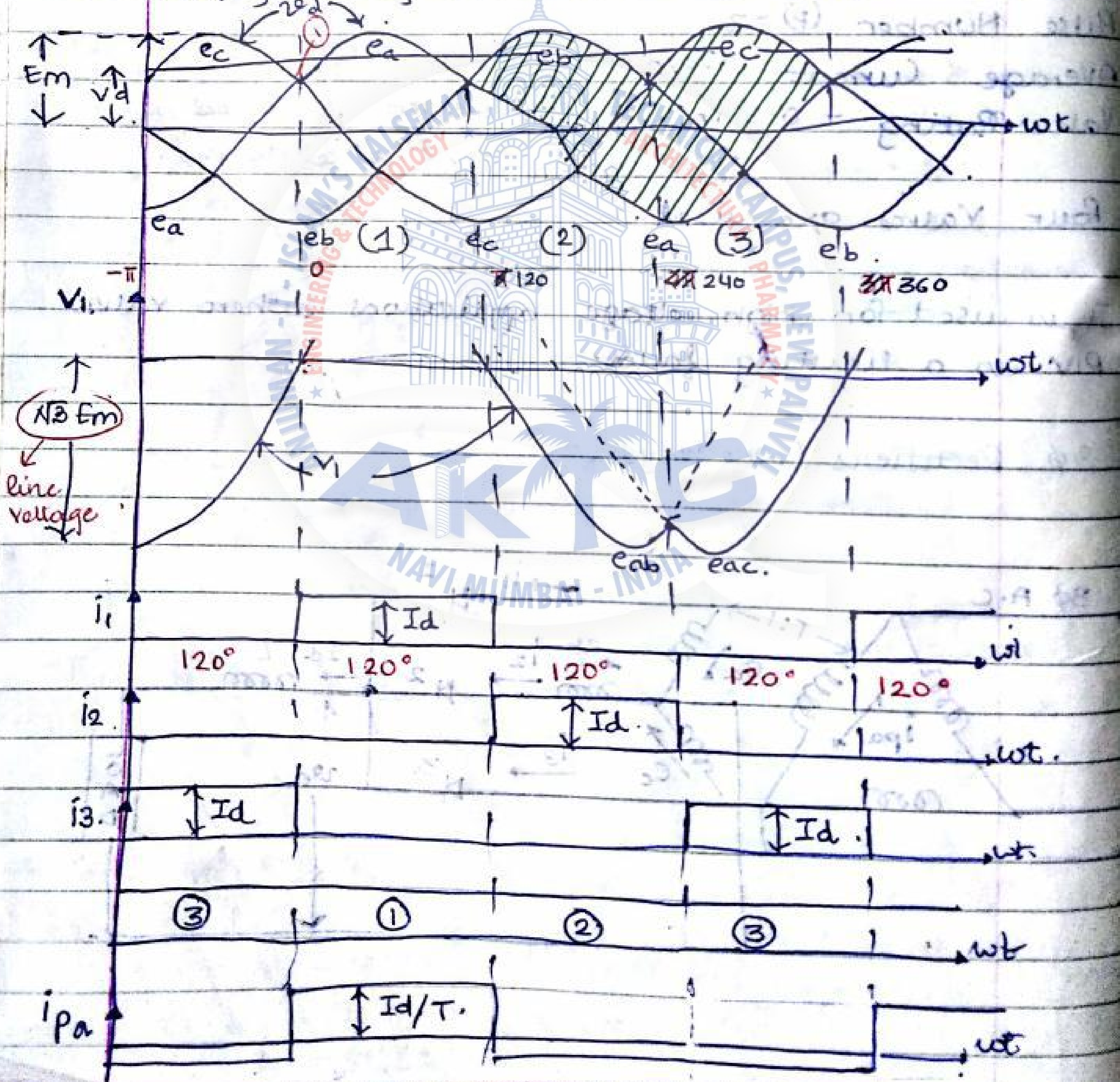
- There is a continuous conduction of valves (valve 1-2 ... and so on.

- It is assumed 'dc' o/p 'I' is ripple free & the dc o/p voltage is v_d which is instantaneous value.

- In order to obtain average value of v_d ie (V_d) we simply integrate the instantaneous value v_d.



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(1) At that instant $e_a = e_c$ & beyond point on e_a, $e_a > e_c$ & valve 1 can be fired because the condition for any valve to ignite/conduct is that if it is forward biased.

- When valve 3 is conducting the voltage across valve 1 i.e. V_1 is

3 phases in one cycle of V_1.

$$\begin{cases} V_1 = e_a - e_c & \rightarrow \text{when it is off } (\because \text{ valve 3 is conducting}) \\ V_1 = 0 & \rightarrow \text{when it is conducting} \\ V_1 = e_a - e_b & \rightarrow \text{when it is off } (\because \text{ valve 2 is conducting}) \end{cases}$$

- commutation voltage is the voltage at which valve is going to be fired.

- Hence the commutation voltage for valve (1) when valve 3 is conducting is $(e_a - e_c)$ & should be +ve for firing of valve 1.

- One cycle has 3 pulses. $(PIV = \sqrt{3}\, E_m)$

$(VA \text{ rating} = V_{peak} \cdot I_{av} = \sqrt{3} V_m \frac{I_d}{3})$

- The problem is however about the transformer which is not utilized properly. In the primary side only 120° current is being conducted in one cycle. Remaining time the 2 phases are idle.
($\because$ Xmer is not symmetrically loaded)

- Thus there is appearance of 'dc' current in the secondary winding of transformer which is again a disadvantage.

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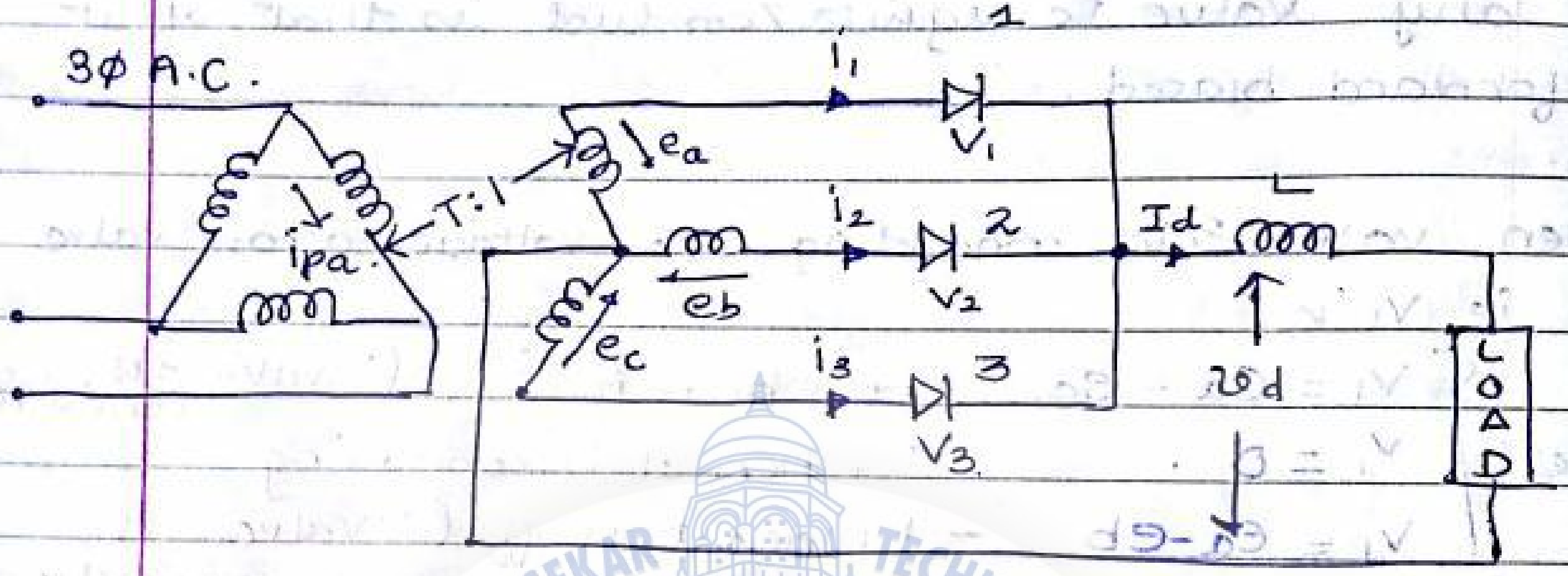
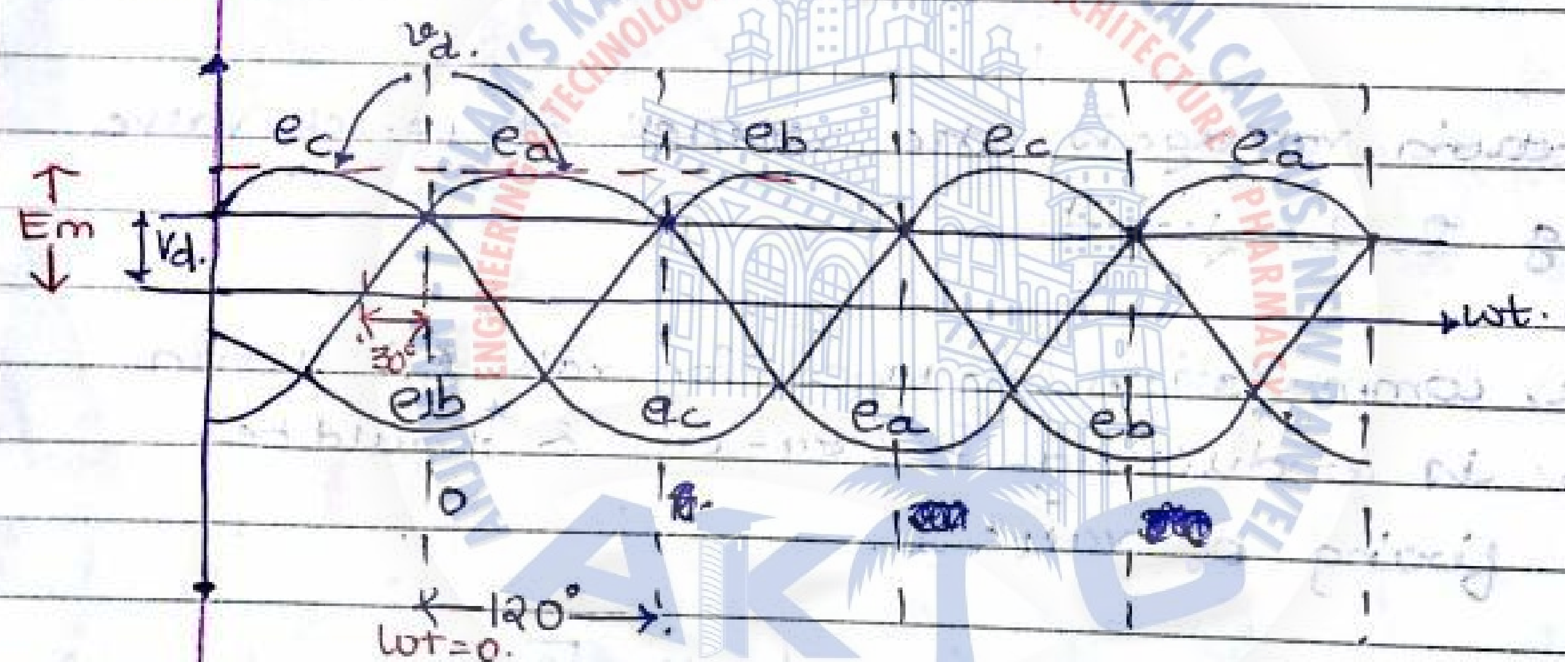
* Three phase Rectifier circuit

(lec 5)

⇒ Consider the previous circuit.





- $e_a = E_m \sin(\omega t + 30^\circ)$

- $$V_{do} = \frac{1}{(2\pi/3)} \int_0^{120^\circ} E_m \sin(\omega t + 30^\circ)\, d(\omega t)$$

120°

- To convert the sin term into cos term the following is done. It is done to make integration simple

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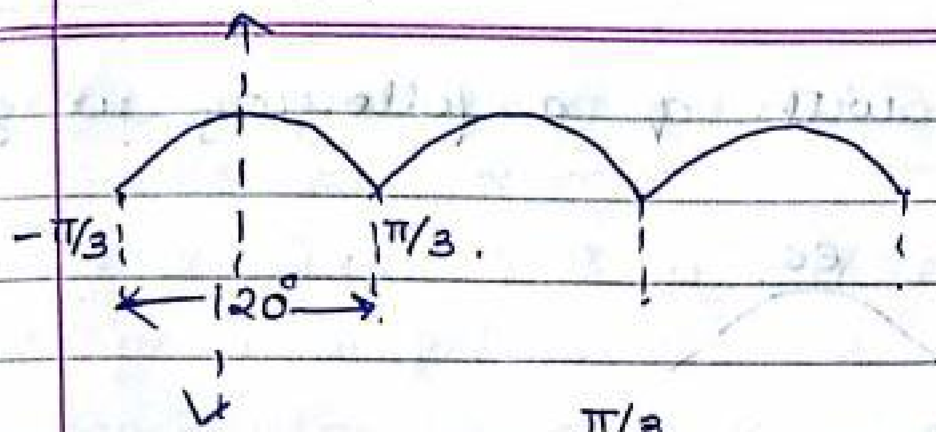
$\omega t = 0$.



$$\therefore V_{do} = \frac{1}{2\pi/3} \int_{-\pi/3}^{\pi/3} E_m (\cos \omega t)\, d(\omega t)$$

$120°$

$$= \frac{1}{2\pi/3} (E_m) (\sin \omega t)_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{\frac{2\pi}{3}} (E_m)(\sqrt{3})$$

$$\boxed{V_{do} = \frac{3\sqrt{3}E_m}{2\pi}}$$

- PIV $= \sqrt{3} E_m$

- Pulse Number $(P) = 3$.

- Valve 1 voltage $v_1 = 0 \quad 0 \leq \omega t \leq 120°$

$= e_a - e_b \quad 120° \leq \omega t \leq 240°$

$= e_a - e_c \quad 240° \leq \omega t \leq 360°$

commutation group

- To calculate $P = q \times r \times s$, we have;

one parallel path of valves

$\boxed{q = 3}$ $(3 \times 1 \times 1)$

one series path of valves.

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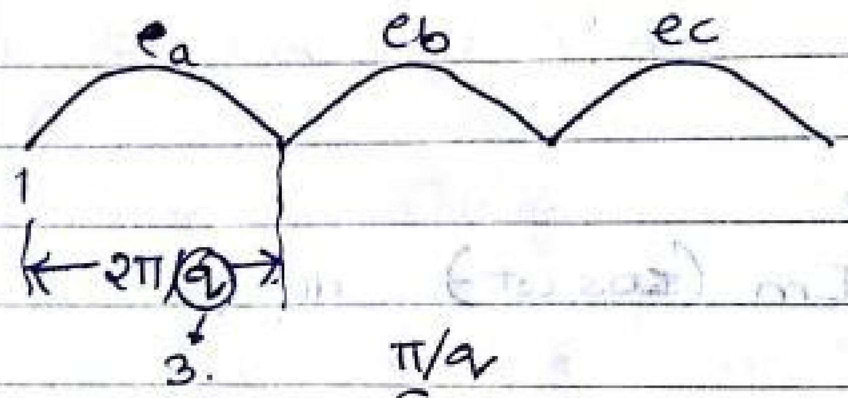
- Now replacing '3' with q as following we get



$$V_{do} = \frac{1}{2\pi/q} \int_{-\pi/q}^{\pi/q} E_m \cos\omega t \, d\omega t.$$

$$\boxed{V_{do} = \frac{S\,q}{2\pi} E_m \sin\left(\pi/q\right)}$$

[s is added because there is one valve for a phase, if there are two valves, the ~~voltage~~ voltage is doubled s = 2, so on...] ie 'S' no of series valves].

- It is worthwhile to mention that when '1' conducts 2 & 3 will be off. Now as '1' gets turned off, '2' will go 'on'. This however does not take place instantaneously & there is a short period for which '1' & '2' will both be 'ON' which results in short circuited causing maloperation of the converter ckt.

- But here we are considering the ideal analysis, where we assume the current is instantaneously taken up by valve 2 as valve one turns off.

→ If q is even, PIV occurs when the valve with a phase displacement of 180° is conducting & will be equal to PIV = 2Em

= PIV = 2Em.

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- PIV in general terms

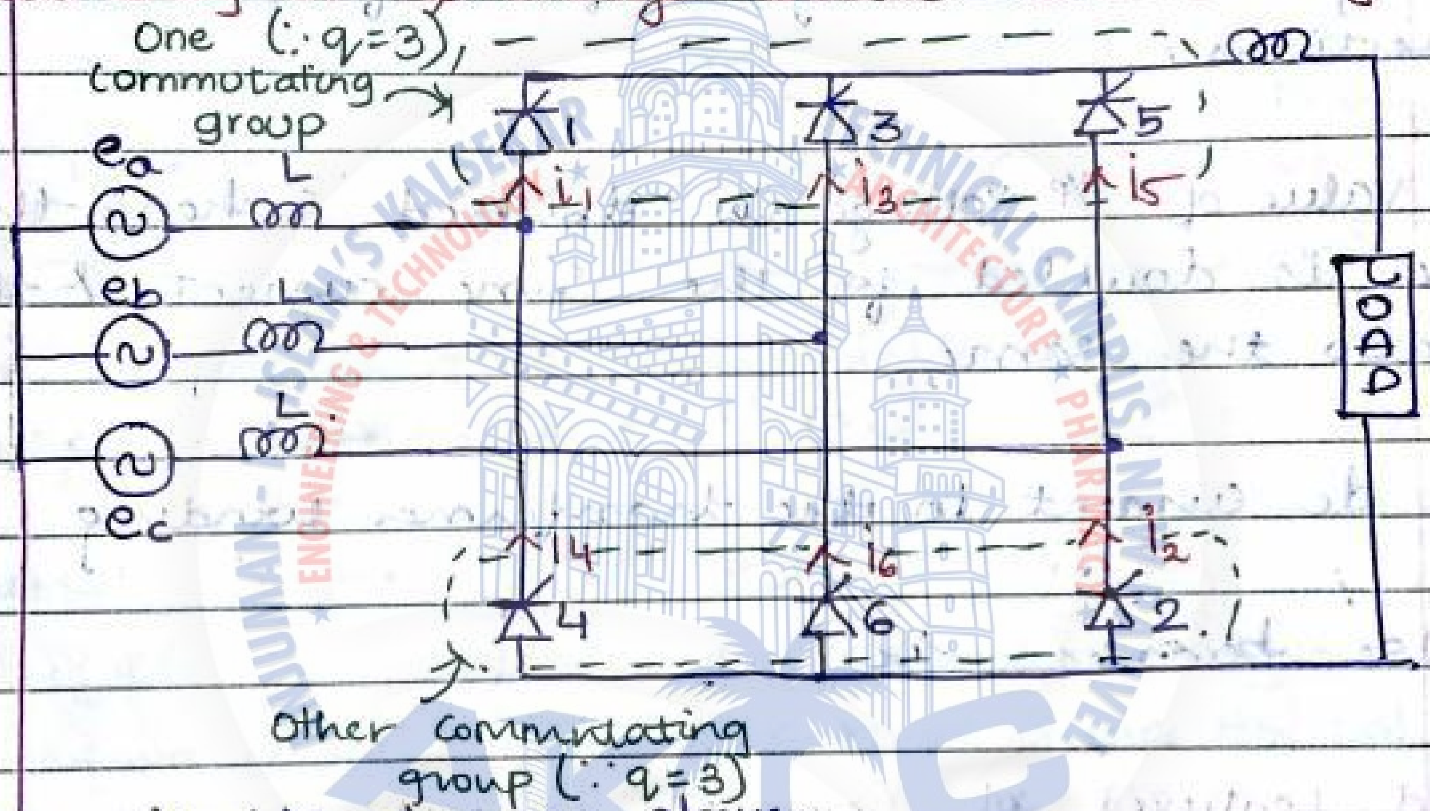
$PIV = 2E_m \longrightarrow q$ is even

$2E_m \cos \pi/2q \longrightarrow q$ is odd.

→ If q = odd, PIV occurs when the valve with a phase displacement of $\pi \pm \pi/q$ is conducting & will be equal to $PIV = 2E_m\cos(\pi/2q)$

- commutation group is chosen such that only one valve is conducting

✱ 3Ø two way or 3Ø Bridge Rectifier (Gratz Bridge)



- For the circuit shown above,

$q=3$ (one valve of the commutating group will be conducting at a time)

- For the circuit shown,

$S=2$ (two valves/path are conducting at a time)

therefore o/p voltage will be double compared to previous case ie the one way rectifier ckt.

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- This is a connection of two one way rectifier ckt

- In the 3ϕ, one way ckt, if the three valves are reversed, the circuit operates as before except the directions of dc voltage & current are reversed.

- In the bridge converter, the same transformer is feeding two one way rectifiers of opposite connections.

- The value of o/p voltage is doubled & hence the power is doubled for the same current & the PIV is also the same

- No dc current in the transformer winding

- Pulse becomes six

*Desired features of Converter circuit

(1) High pulse number (P) [i.e. to reduce harmonics on ac & dc side. P cannot be ↑ a lot, else it makes ckt complex]
(2) PIV/Vdo should be as low as possible
(3) Vdo/E should be as high as possible
(4) Transformer utilization factor (TUF) should be near to unity

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(1) High Pulse Number (p)

Harmonic No = $np \pm 1$

- 6 pulse → 5, 7, 11, 13 ...

- 12 pulse → 5, 7 X, 11, 13 ...

5, 7 are cancelled by the λ-Δ transformer hence do not reflect on ac side.

- It is worthwile to mention that the lower order harmonics have the larger magnitude as compared to higher order harmonics.
- With the cancellation of 5, 7 (having ↑ magnitude), we just have to get rid of 11, 13 ... ie the higher order harmonics which are easier to filter out.

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We know; $PIV = 2E_m$ $(q = \text{even})$
$= 2E_m \cos(\pi/2q)$ $(q = \text{odd})$

$$V_{do} = \frac{sq\,E_m}{2\pi} \sin(\pi/q)$$

$$\therefore \frac{PIV}{V_{do}} \begin{cases} = \dfrac{2\pi}{sq \sin(\pi/q)} & (q = \text{even}) \\ = \dfrac{\pi}{sq \sin(\pi/2q)} & (q = \text{odd}) \end{cases}$$

② PIV/Vdo should be as low as possible :

- PIV decides the rating of the valves with less PIV the converter is less expensive.

- Vdo should be as max as possible & PIV low so that the net ratio is small.

③ Vdo/E should be as high as possible :

- $E = \frac{E_m}{\sqrt{2}}$ Erms → Impressed voltage in xmer Secondary

- $V_{do} = \frac{sq\,E_m}{2\pi} \sin(\pi/q)$

or ;

- $\frac{V_{do}}{E} = \frac{sq(\sqrt{2})}{2\pi} \sin(\pi/q)$

Vdo → o/p dc voltage

E → Voltage in 2ndary of xmer

- The above means that for a small value of 'E' the resulting 'Vdo' should be high, in other words the ratio of Vdo/E should be high.

- The ratio of Vdo/E is dependent on the two parameters i.e 's' & 'q'

- following combinations of q, r, s are possible to get '6' = pulse.

$P = q \times r \times s$

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Pulse	q	r	s
6	6	1	1
	3	2	1
	3	1	2
	2	3	1
	2	1	3

→ optimal configuration ie the Graetz Bridge, which gives all the desired features of converter circuit. (row q = 3, r = 1, s = 2)

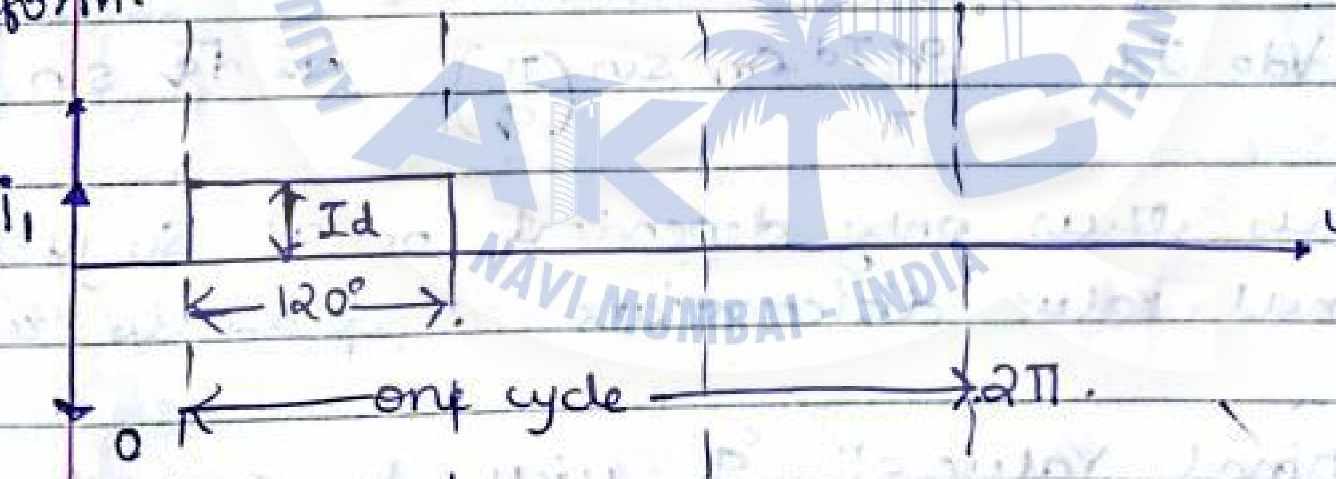
④ TUF should be unity.

(lec 6)

$$-\text{TUF} = \frac{\text{Rms secondary Power}}{\text{dc Power}} = \frac{V_{rms} \cdot I_{rms}}{V_{dc} I_{dc}} = 1$$

$$= \frac{P_t}{V_d \cdot I_d} = \frac{V_t \, I_t}{V_d \cdot I_d}$$

- Consider a 3ϕ Rectifier one way we have the following waveform.



we get

$$I_t^2 = \frac{1}{2\pi} \int_0^{2\pi/3} I_d^2 \, d(\omega t)$$

$$I_t^2 = \frac{I_d^2}{3}$$

$$\therefore \boxed{I_t = \frac{I_d}{\sqrt{3}}}$$

3 valves in commutating group ie 'q'

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- $\therefore$ $\boxed{I_t = \frac{I_d}{r\sqrt{q}}}$ → 'r' is the no of parallel valves

- $P_t = p \left(\frac{E_m}{\sqrt{2}}\right) I_t$ → rms value of secondary Xmer voltage

No of pulses.

$= p \frac{E_m}{\sqrt{2}} \frac{I_d}{r\sqrt{q}}$.

$\boxed{P_t = p \frac{E_m I_d}{r\sqrt{2q}}}$

- TUF (reciprocal is Xmer utilization efficiency) can be calculated using above eqs.

$$TUF = \frac{P_t}{V_{do} \cdot I_d} = \frac{p\, E_m I_d / r\sqrt{2q}}{\frac{q s I_d E_m}{\pi} \sin\left(\frac{\pi}{q}\right)} = \frac{\pi}{\sqrt{2}\sqrt{q} \sin\left(\frac{\pi}{q}\right)}.$$

- The TUF is thus only dependent on 'q' so just to get the optimal value differentiate & equate to zero.
- The optimal value of 'q' will be equal to 3.
- For 6 pulse converter the various values are presented in the table.

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For 6 pulse we have; (P = 6)

SI No	q	r	s	PIV/Vdo	Vdo/E	TUF
1	2	1	3	1.047	2.700	1.571
2	2	3	1	3.142	0.900	1.571
3	3	1	2	1.047	2.340	1.481
4	3	2	1	2.094	1.169	1.481
5	6	1	1	2.094	1.350	1.814

best option

- On comparing (3) & (1) we realize

(1) has ~~than~~ lower PIV/Vdo ~~than~~ same as (3).
(1) has higher Vdo/E than (3)
(1) has higher TUF than (3) (TUF should be closest to unity)

- It is preferable to use (3) because we have 3 ϕ & very easily we can identify the no of valves in a group. q can be three or multiple of three.

- The TUF in (3) can be made better by using improved configuration.

- The current rating of transformers can be further increased by a factor of $\sqrt{2}$ while decreasing the number of winding by factor 2.

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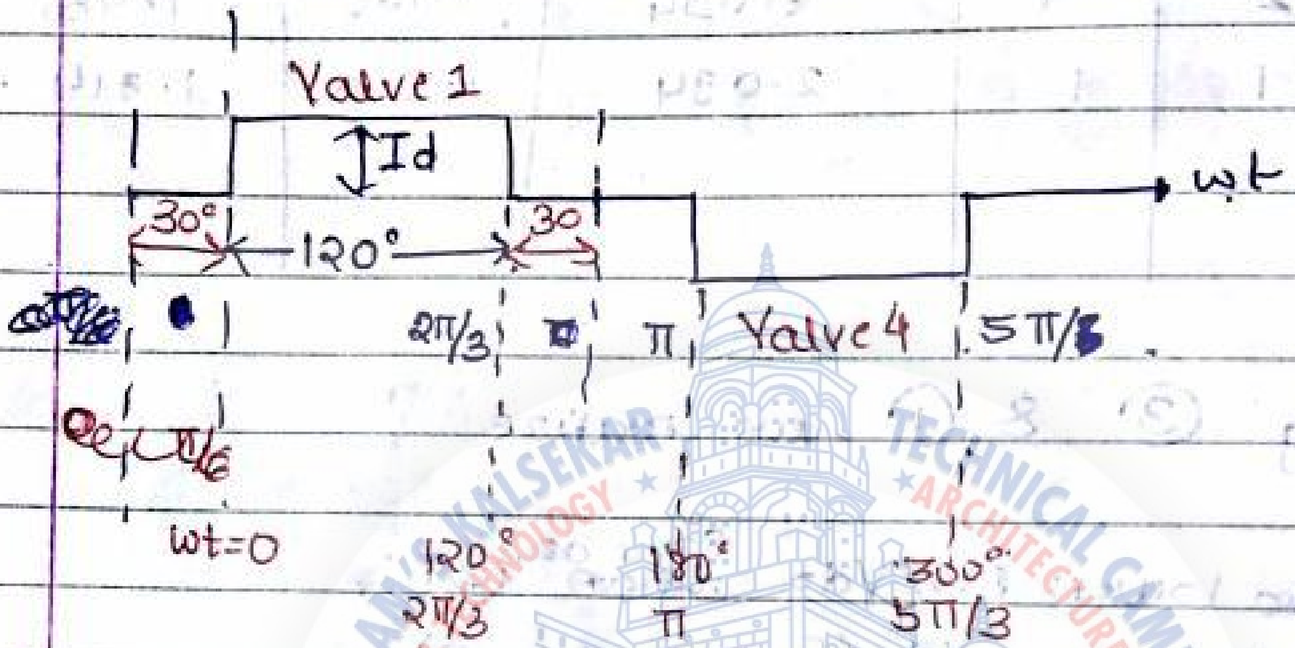
• Current Waveform of Graetz Bridge

- Consider the Graetz circuit. Valve '1' is conducting for 120°. The phase current 'Ia' of phase 'a' will apply a +ve current to valve '1' for 120° & then -ve current to valve '4' for 120° as shown below.



$$\therefore\ I_t^2 = \frac{1}{2\pi}\left[\int_0^{2\pi/3} I_d^2\, d(\omega t) + \int_{\pi}^{5\pi/3} I_d^2\, d(\omega t)\right]$$

$$I_t^2 = \frac{2 I_d^2}{3}$$

$$\therefore\ \boxed{I_t = \sqrt{\frac{2}{3}}\, I_d} \quad \text{or} \quad \boxed{I_t = \sqrt{2}\,\frac{I_d}{\sqrt{3}}}$$

improved by factor $\sqrt{2}$.

- For a 12 pulse converter, the same analysis can be performed and it is found that 2 six pulse converters connected in series by two transformers with phase displacement of 30° ie if one transformer is λλ other should be Star-Delta.

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- Different cases depend on the conduction of valves in a commutating group can be analysed seperately.

- If at all instants only one valve in the commutating group conducts there will be no overlap.
 → Two valves of same commutating group conducting for a short period simultaneously. This happens when current from one valve is transferred to other valve of same commutating group.

- For 12 pulse converter, the same analysis can be repeated by making the tabular format of q, r, s. & then choose the optimum option from the wide range of options. ← (Can be given as assignments to students)

- The sudden change of current from one valve to another is not instantaneous due to the presence of source inductance & the concept of overlap comes into picture.

- First we will analyse the ideal case with no overlap & ignition delay & check;
what will be the o/p vg, pulses, harmonics, current in each valve & how the currents are shared will be analysed deeply for this 6 pulse ckt

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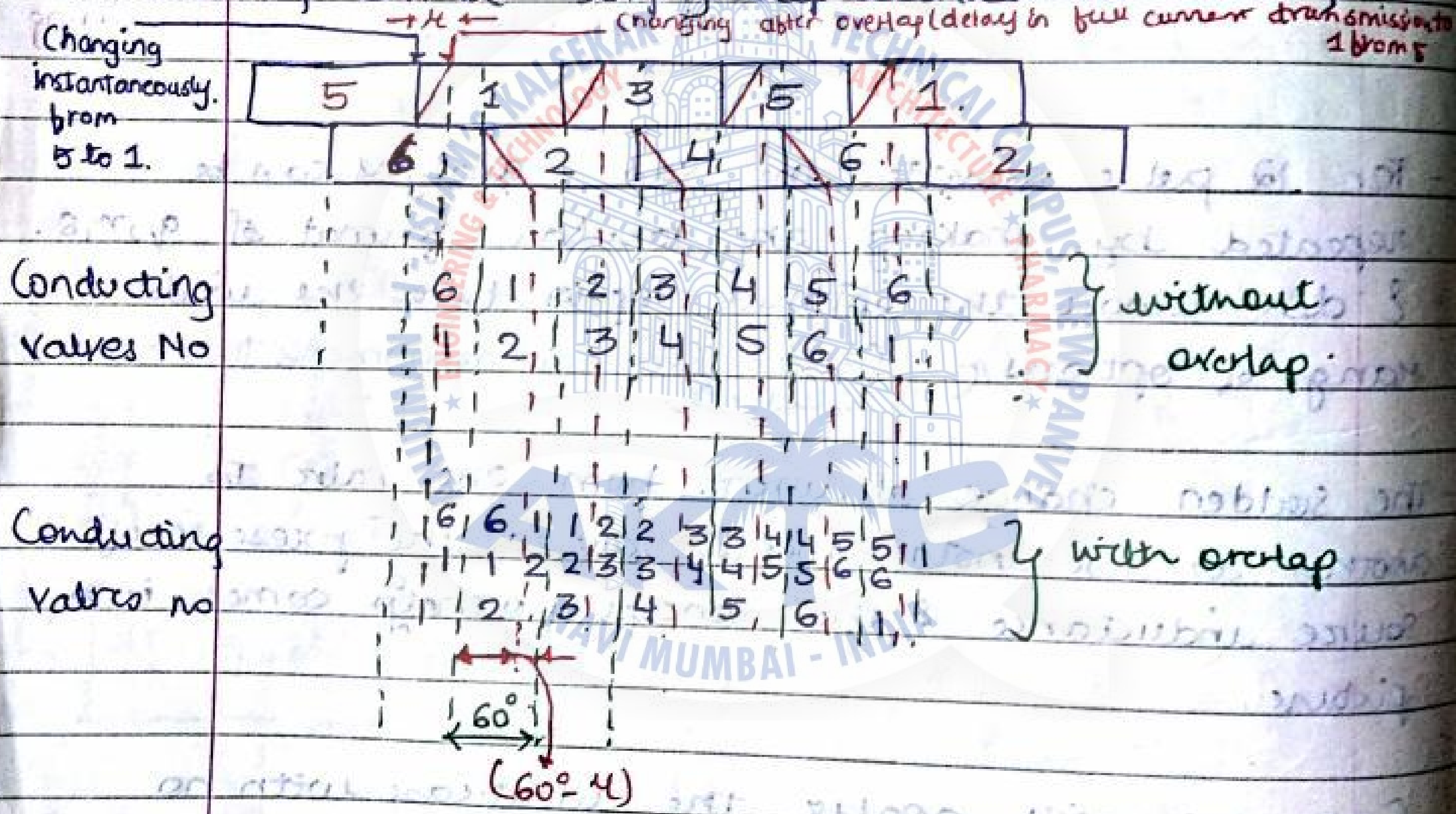
- <u>Conduction Sequence</u>.

Conducting valve	6	1	2	3	4	5
Next fired Valve	1	2	3	4	5	6
Conduction period	60°	60°	60°	60°	60°	60°

← 360° →

The overlap can be simply represented as under;



- If 'μ' value is more, there is possibility of getting 4 valve conductions. It behaves like short circuits

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- Here ~~three~~ four cases arise

Modes $\mu = 0 \longrightarrow$ 2 valves conduction

MODE 1 : $\boxed{\mu < 60^\circ \longrightarrow 2, 3 \text{ valves.}}$ → best option

MODE 2 : $\mu = 60^\circ \longrightarrow$ 3 valves.

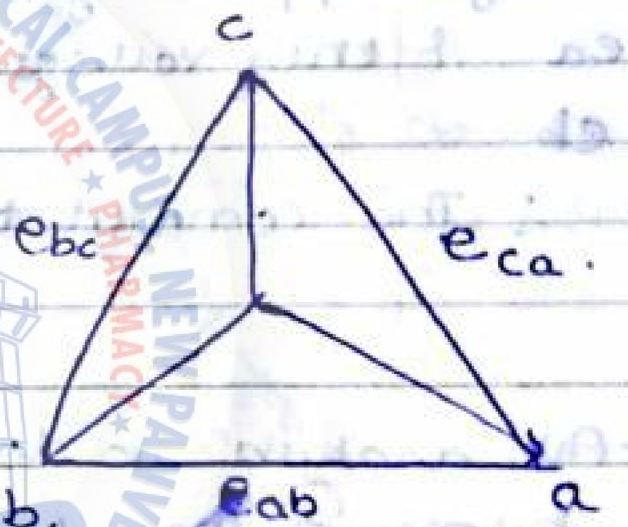
MODE 3 : $\boxed{\mu > 60^\circ \longrightarrow 3, 4 \text{ valves}}$ → undesirable, results in short circuit

. **3ϕ voltages :**

→ commutation voltage of valve '3' after '1' stops conducting

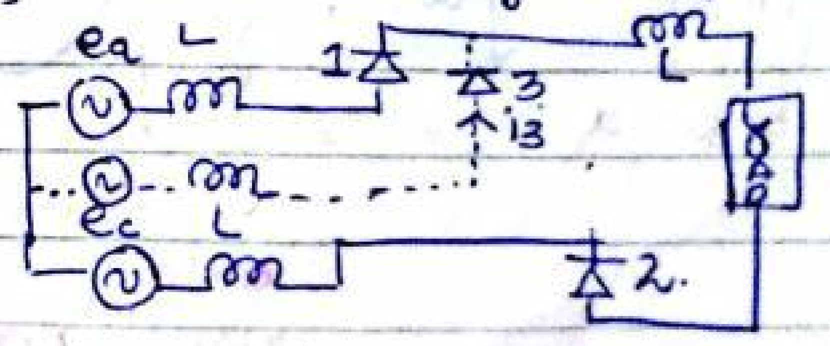
- Taking eba as reference voltage as shown in the fig, the other voltages can be written as;

$$e_{ba} = \sqrt{3}\, E_m \sin(\omega t)$$
$$e_a = E_m \sin(\omega t + 5\pi/6)$$
$$e_b = E_m \sin(\omega t + \pi/6)$$
$$e_c = E_m \sin(\omega t - \pi/2)$$



$$e_{cb} = e_c - e_b = \sqrt{3}\, E_m \sin(\omega t - 120^\circ)$$
$$e_{ac} = \sqrt{3}\, E_m \sin(\omega t + 120^\circ)$$
$$e_{bc} = \sqrt{3}\, E_m \sin(\omega t + 60^\circ)$$

- To understand the concept of commutation voltage consider the graetz bridge.
- Suppose '1' & '2' were conducting, now '1' will go off for '3' to conduct
- Voltage at cathode of '3' is e_a since '1' is now off & the voltage at anode of '3' is e_b so;



- $e_b - e_a$ should be '+ve' for valve '3' to conduct. & this voltage is commutation voltage.

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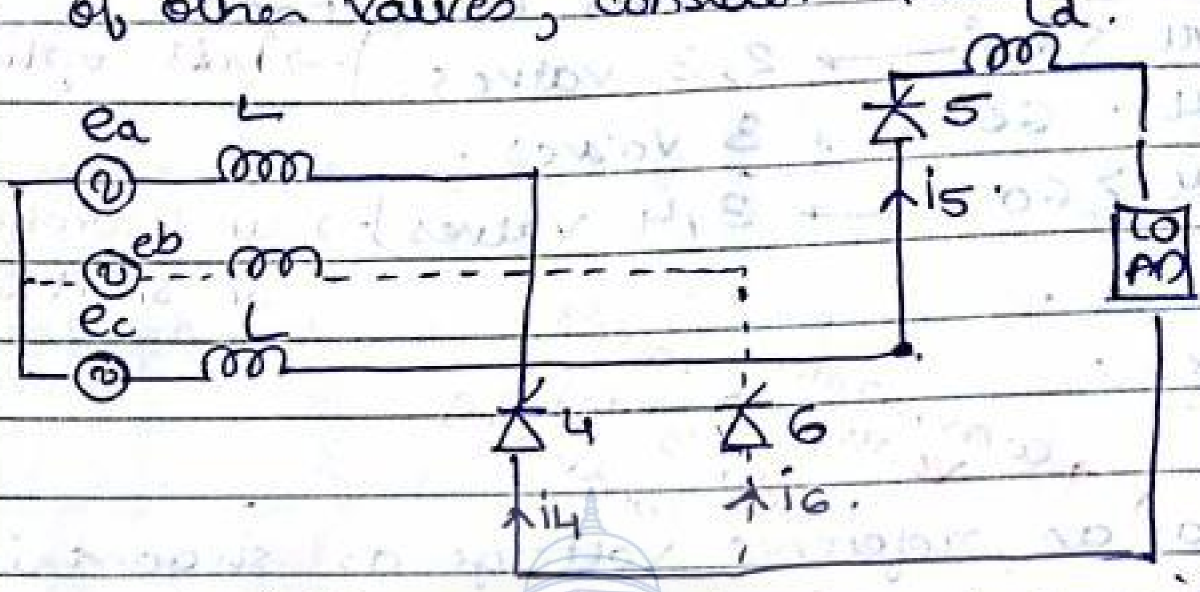
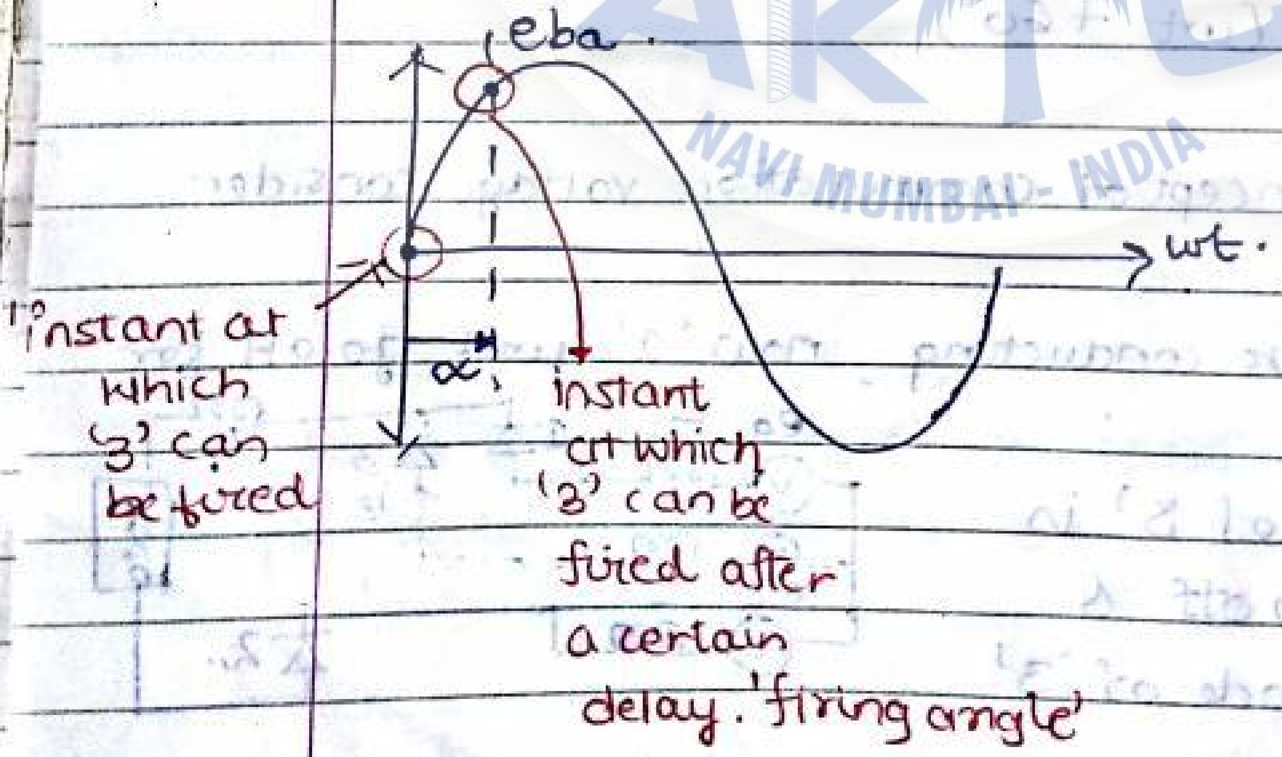
- Similarly we can calculate the commutation voltage of other valves, consider valve '6'



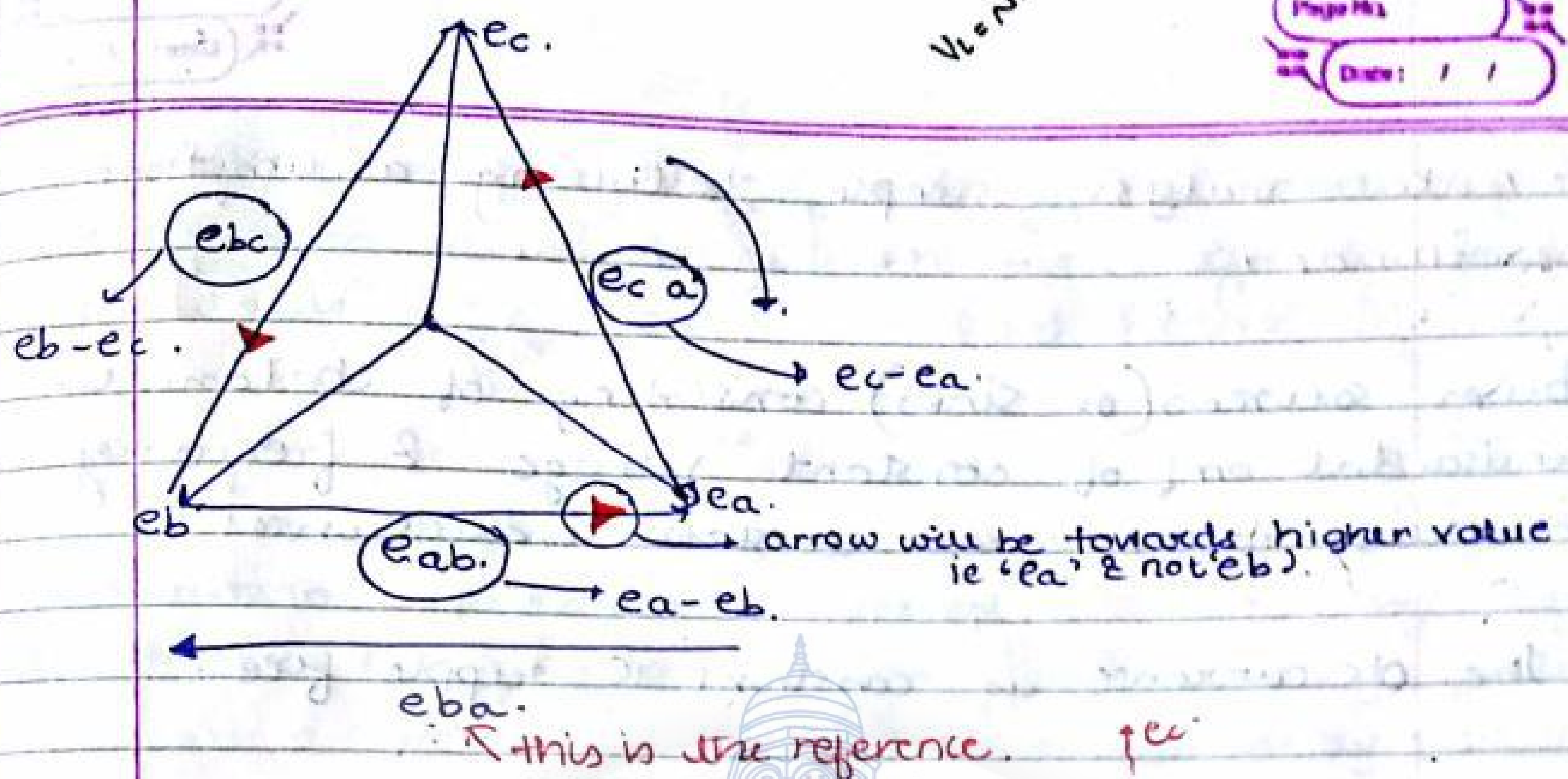
- Note; valves 4, & 5 were conducting before '6' was fired
- Voltage appearing at the Anode of '6' as '4' goes off is ea & the voltage appearing at the cathode of '6' is eb

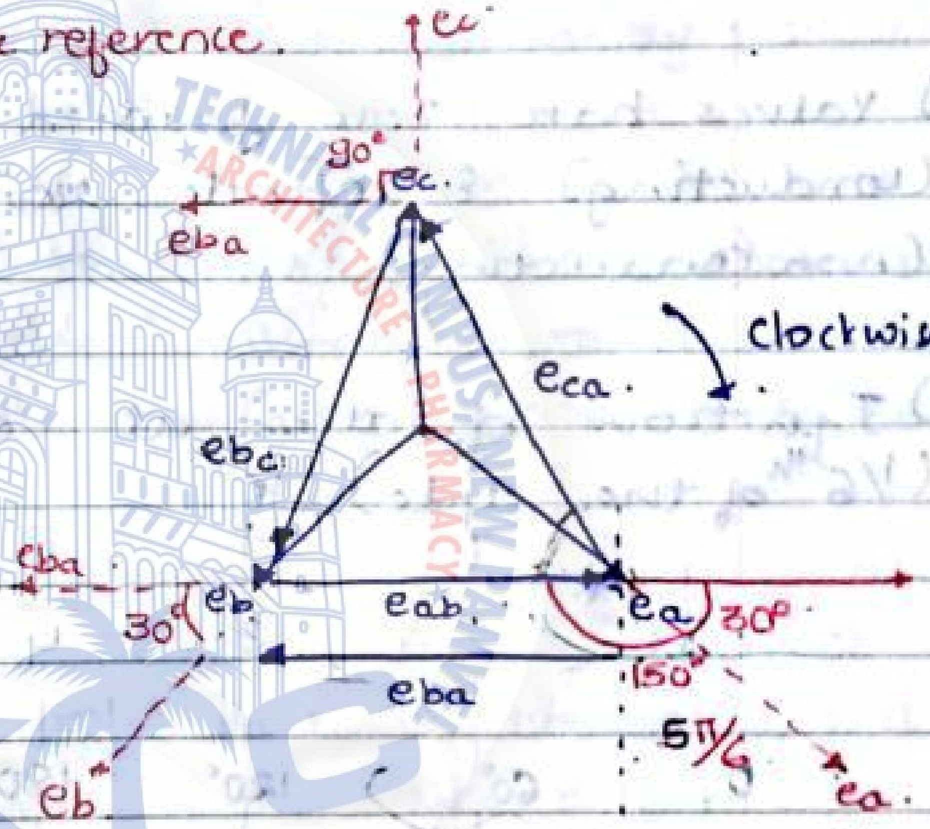
$\therefore$ The commutation voltage $= e_a - e_b$ [should be +ve to conduct '6'].

- All analysis is done assuming '1' & '2' were conducting before '3' was fired.



$V_L = \sqrt{3} V_{ph}$





line value ... line value

- $e_{ba} = \sqrt{3}\,E_m \sin\omega t$

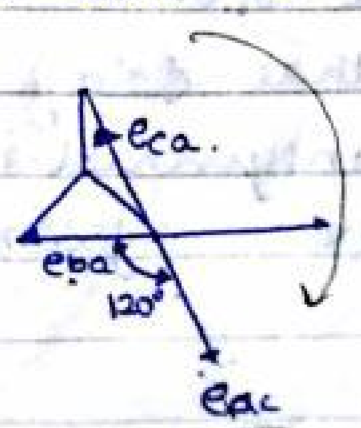
phase value ... phase value

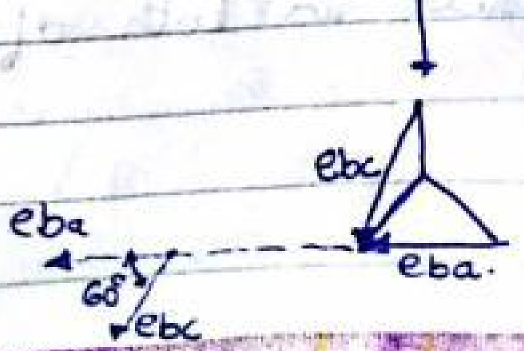
- $e_a = E_m \sin(\omega t + 5\pi/6)$
- $e_b = E_m \sin(\omega t + \pi/6)$
- $e_c = E_m \sin(\omega t - \pi/2)$

- Angle between e_a & e_{ba} is found & the sign of angle is taken '+ve' because in clockwise direction 'e_a' leads 'e_{ba}'.

Similarly

- $e_{ac} = \sqrt{3}\,E_m \sin(\omega t + 2\pi/3)$
- $e_{bc} = \sqrt{3}\,E_m \sin(\omega t + \pi/3)$





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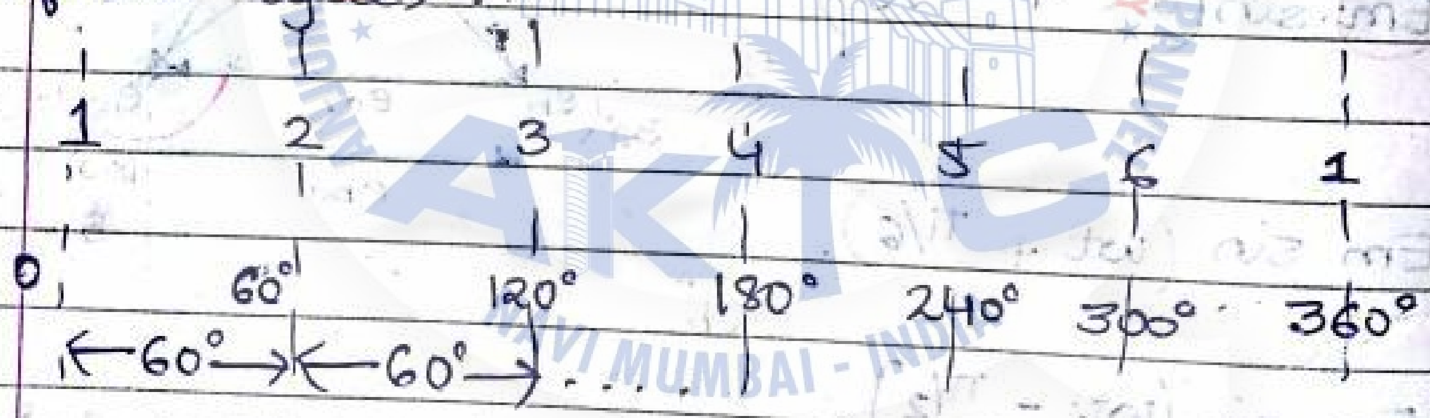
• To make analysis simple following assumptions are made.

(1) Power sources (or sinks) consisting of balanced sinusoidal emf of constant voltage & frequency in series with equal lossless inductances.

(2) The dc current is constant ie ripple free.

(3) Valves have zero forward resistance when on (conducting) & infinite resistance when off (non conducting).

(4) Ignition of valves at equal intervals of 60° (1/6th of the cycle) ie



- We assume a symmetrical conduction pattern which appears across o/p.
- There is a possibility that you fire 2 after 80° instead of 60°
- But that delay will cause the o/p pattern to be unsymmetrical & result in lots of harmonics to the system.

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(5) A valve can only be fired if the gate signal appearing across it is +ve & the gate signal is available.

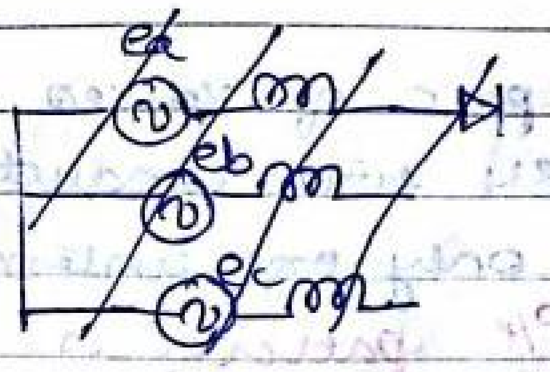
· Overlap angle (u):

- The duration when the current is shared by by the conducting valves in a commutating group is called overlap angle & is measured by the overlap (commutation) angle 'u'.

- Let us consider that the valve 1, & 2 are conducting voltage appearing at the cathode of valve 3 is e_a. Since valve 1 is conducting & the anode will have voltage e_b.

- The valve '3' will only conduct if $e_b > e_a$ ie when voltage e_{ba} is '+ve'. This is known as the commutation voltage of valve 3. Valve 3 can now be fired by any gate pulse with any delay angle 'α' which is $\alpha°$ after the zero crossing of the commutation voltage of valve 3.

• Valve 2,3 conduction case:



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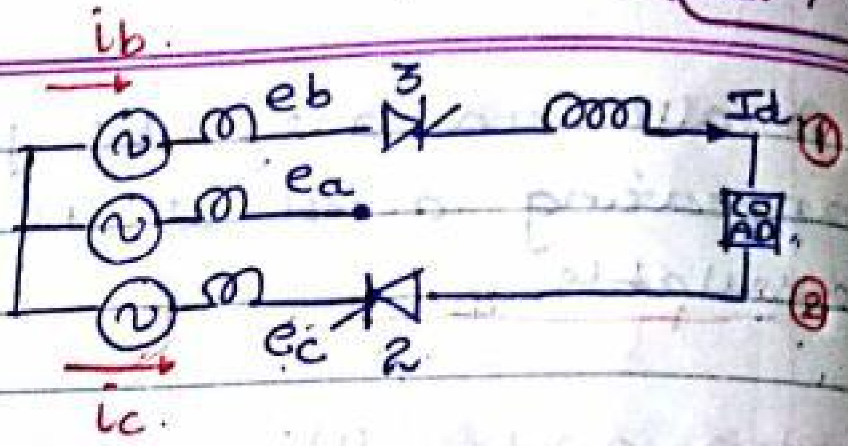
- On observation of fig;



$i_a = 0$ [∵ Phase A O.C]

$i_b = I_d$.

$i_c = -I_d$ → Here we want to know current in valve phase 'C' which has conventional direction as shown. But Id flows in reverse so; $\boxed{i_c = -I_d}$

- Currents in valves:

$i_1 = 0$ → Note: Here we want to know the current flowing in valve 2 which is from anode to Cathode so its '+ve'

$i_2 = +I_d$.

$i_3 = I_d$.

$i_4 = 0$

$i_5 = 0$

$i_6 = 0$

- O/P voltage (without overlap)

O/P Voltage = $e_b - e_c = e_{bc}$

- we need to be careful here; the commutating voltage of '3' is e_{ba}. But O/P voltage is e_{bc}, so care is to be taken about the difference

- Since the pair of valves 2,3 will conduct for 60° & valves 3&4 will conduct for next 60° for the dc O/P voltage only one interval can be considered (because O/P pattern is symmetrical)

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• O/P Voltage (with overlap)

- If the delay angle is α_0, the instantaneous DC voltage e_{bc} will appear across the DC terminal from α to $60+\alpha$, the average dc voltage will be given by;

$$V_d = \frac{1}{(\pi/3)} \int_{\alpha}^{(\pi/3+\alpha)} e_{bc} \, d(\omega t)$$

from phasor

$$= \frac{3}{\pi} \int_{\alpha}^{(\alpha+\pi/3)} \left(\sqrt{3}\, E_m \sin(\omega t + \pi/3)\right) d(\omega t)$$

$$= \frac{3\sqrt{3}\,E_m}{\pi} \Big[-\cos(\omega t + \pi/3) \Big]_{\alpha}^{(\pi/3+\alpha)}$$

$$= \frac{3\sqrt{3}\,E_m}{\pi} \left[-\cos\left(\frac{2\pi}{3}+\alpha\right) + \cos\left(\alpha + \pi/3\right) \right]$$

$$-\cos B + \cos A = -2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$$

$$= \frac{3\sqrt{3}\,E_m}{\pi} \left[2\underbrace{\sin\left(\frac{\pi}{2}+\alpha\right)}_{\cos\alpha} \cdot \underbrace{\sin\left(\frac{\pi}{6}\right)}_{1/2} \right]$$

$$V_d = \frac{3\sqrt{3}\,E_m}{\pi} \cos\alpha$$

$$\boxed{V_d = V_{do} \cos\alpha}$$

Avg dc voltage appearing across the dc link.

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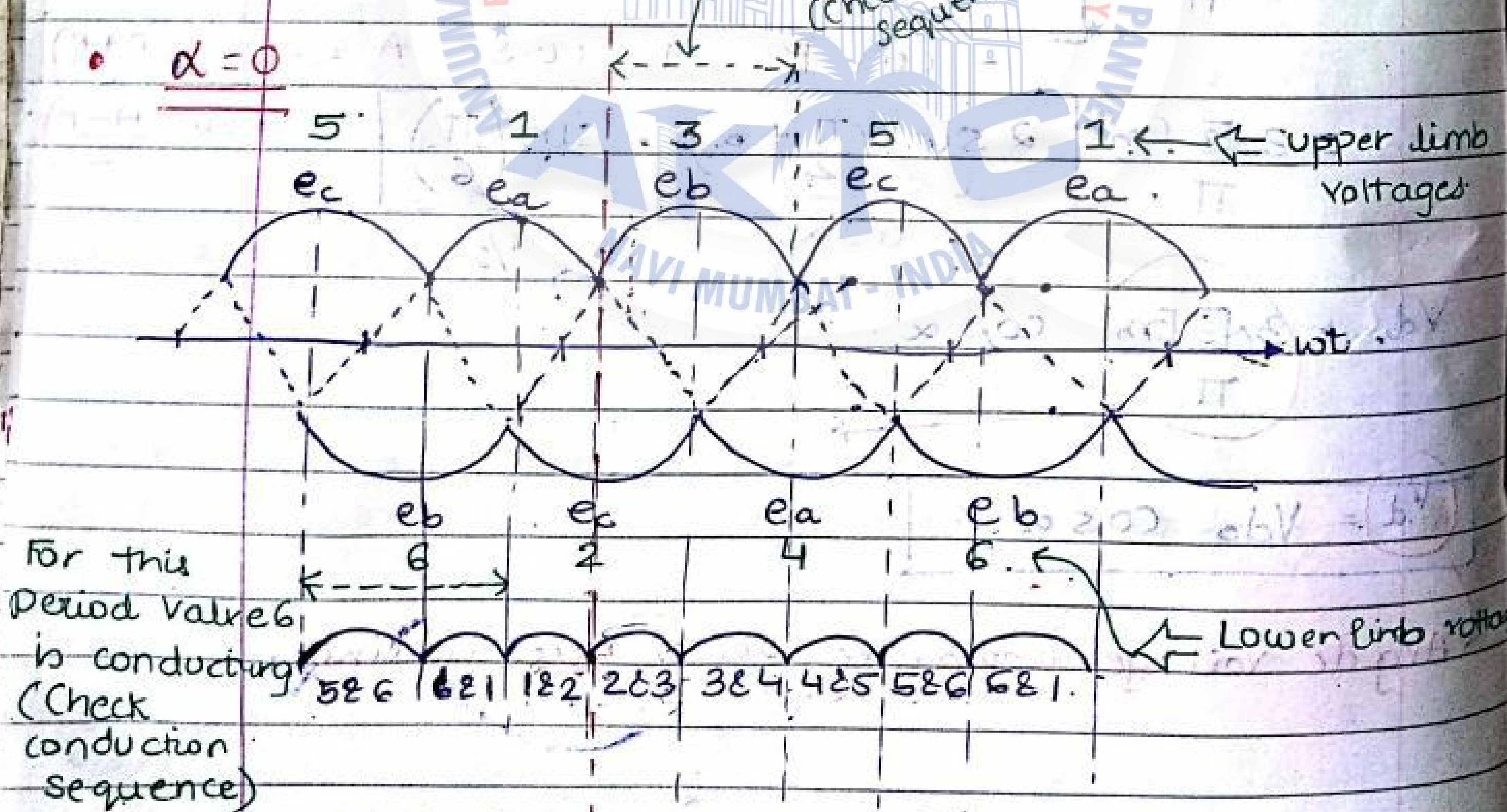
- This eq indicates that the avg dc voltage across the bridge will vary with firing angle α.

- The dc o/p voltage will be max when $\alpha = 0°$ & zero at $\alpha = 90°$

- When the voltage across the bridge is +ve, it acts as a rectifier & when it is negative it is known as inversion operation of bridge.

- $(0 - 90°) \rightarrow$ no overlap — o/p vg '+ve'
$90°$ onwards $\rightarrow$ inversion o/p $\rightarrow$ o/p vg '-ve'

• $\alpha = 0$



Conduction sequence & o/p vg waveform.

- Although delay angle 'α' can vary from 0° – 180°. delay angle cannot be less than certain minimum limit (say 5°) in order to ensure firing of all series connected thyristors.

- Similarly the upper limit of delay angle is also restricted due to turn of time of valve.

- Delay angle is not allowed to go beyond (180° – γ) where γ is extinction angle.

- It is the minimum margin angle which is typically 15°.

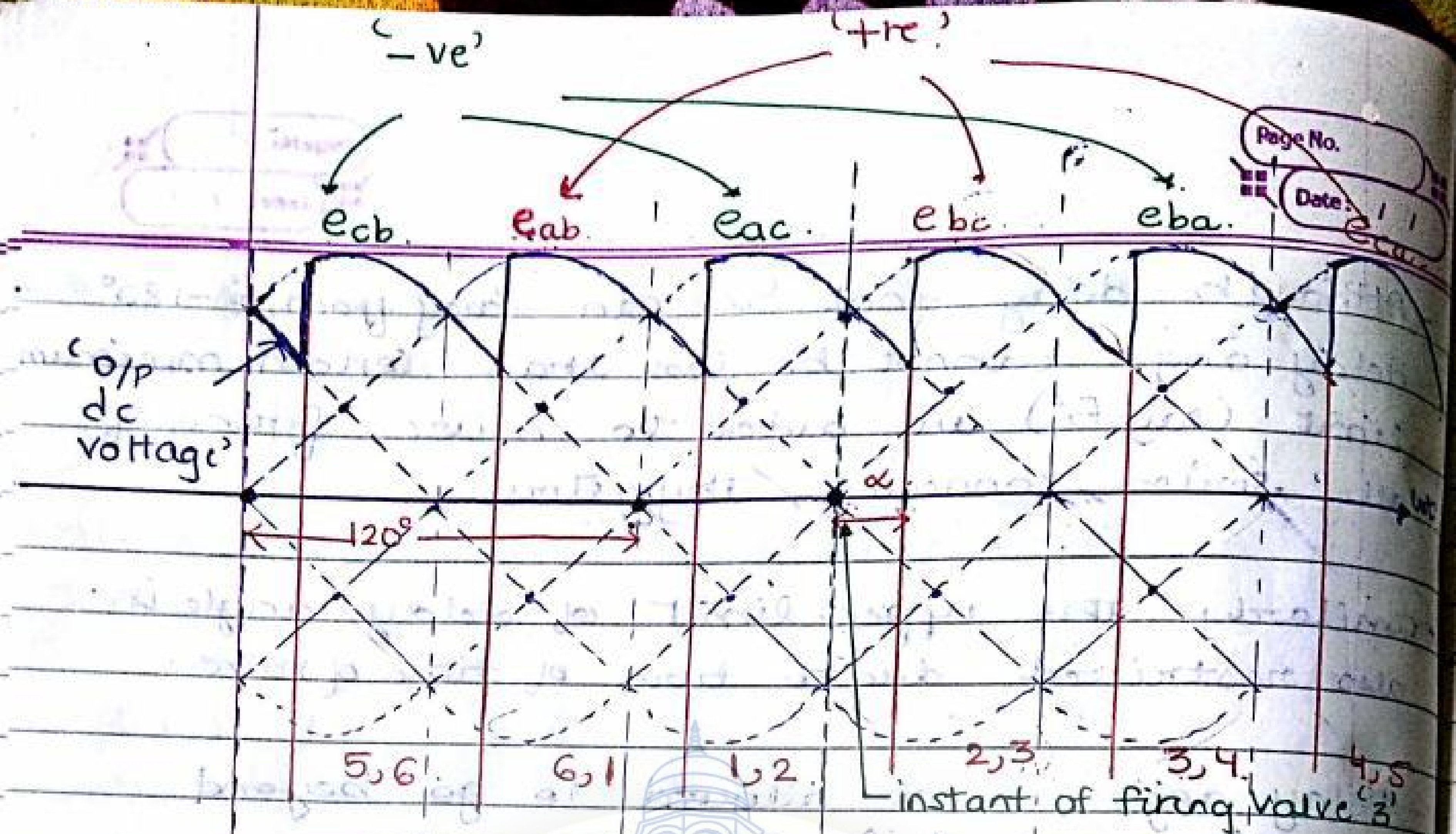
- However in normal operation of inverter it is not allowed to go below 15°. The typical values of γ is 15° – 20°

· Effect of firing angle on voltage waveform. ($\alpha < 90°$)

- The line voltages $\underbrace{e_{ab}, e_{bc}, e_{ca}}_{+ve}$ and $\underbrace{e_{ac}, e_{cb}, e_{ba}}_{-ve}$.

can be plotted if only 'one' voltage is known.

- 1st e_{ab}, e_{bc}, e_{ca} are plotted with difference of 120°.
then opposite wave of e_{ab} is e_{ba}
- e_{bc} is e_{cb}
- e_{ca} is e_{ac}.

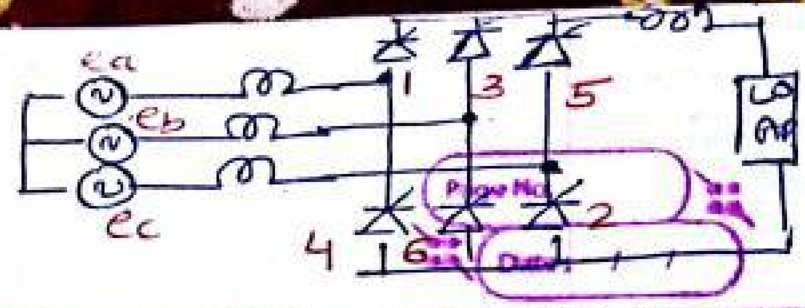


• AC current waveform:-

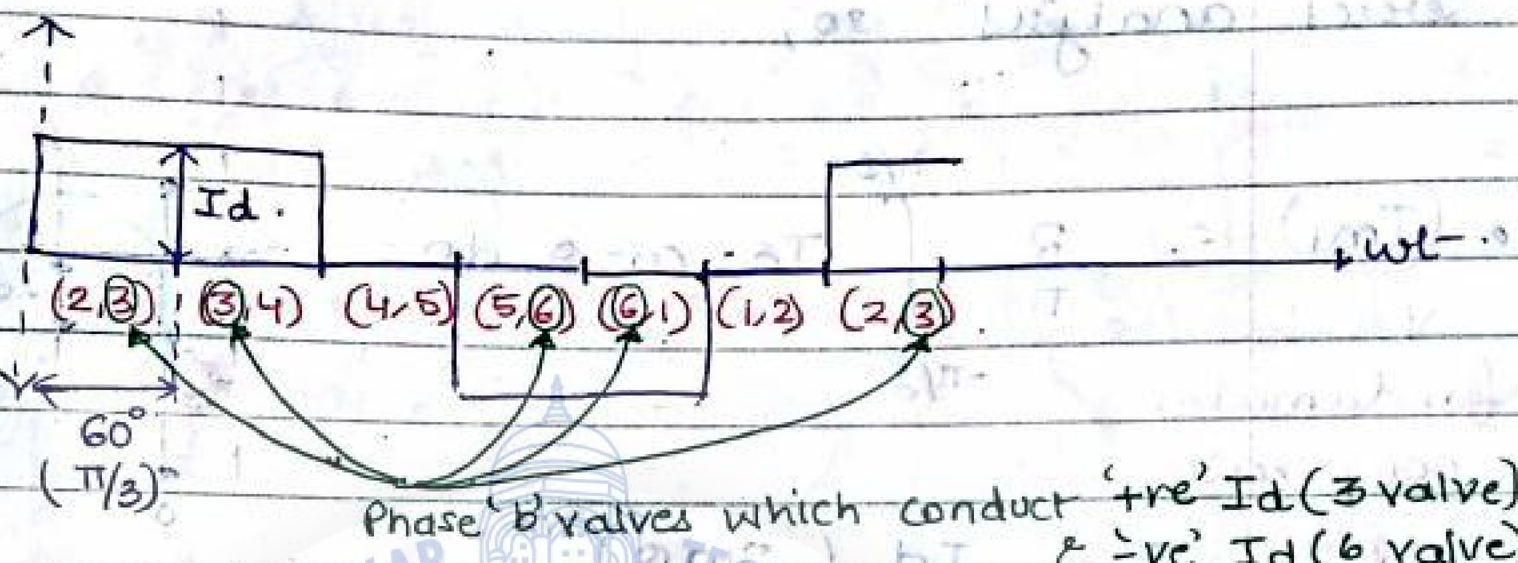
- If the reactance of smoothing reactor is very large the o/p DC current can be assumed as constant (ripple free)

- It means there are no harmonics in DC current the ac current which will flow in the 2ndary winding of Xmer is shown

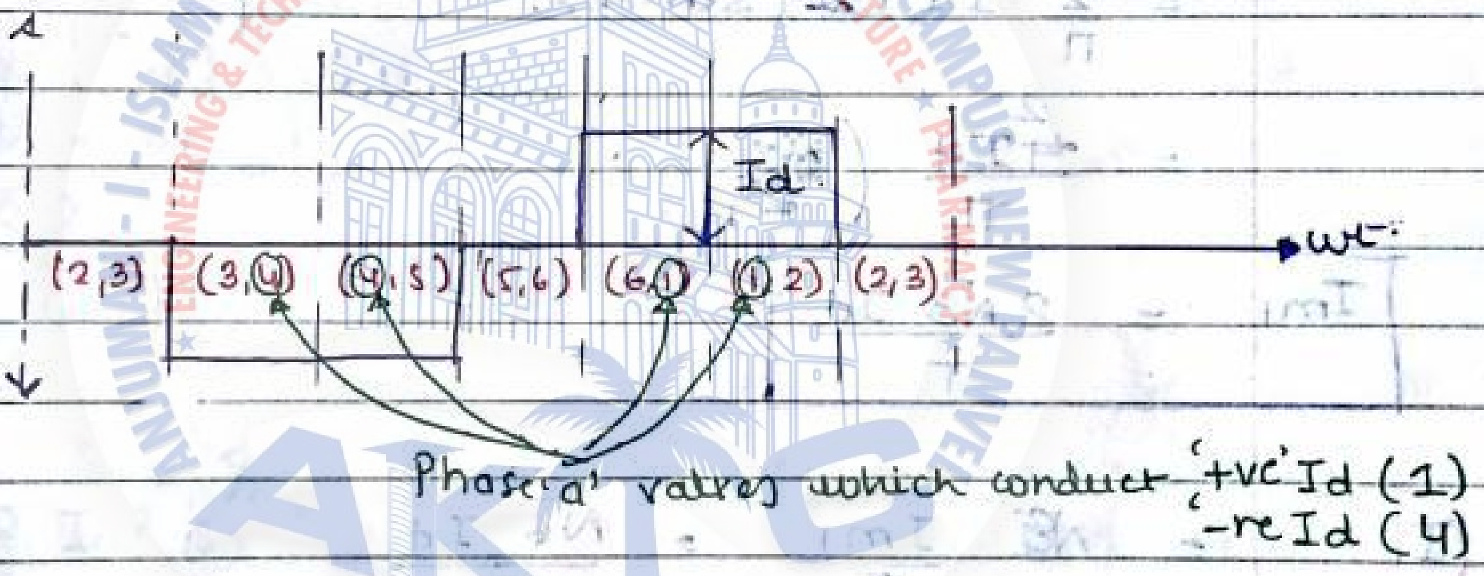
- Since '3' is about to conduct let us draw phase 'b' voltages.



• Phase B.



• Phase A.



- In a similar fashion 'phase c' current can be drawn.

- Now we need to calculate the fundamental component of the waveform shown above. (ie rms).
- It is not perfect sinusoidal, but looks like pulsating

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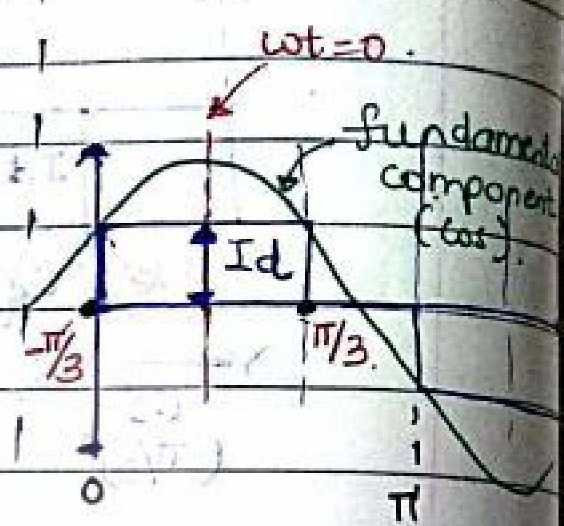
- fundamental component is calculated by fourier series analysis so;



- $I_{m1} = \frac{2}{\pi}\int_{-\pi/3}^{\pi/3} I_d \cos\theta \, d\theta$

fundamental component peak value $= \frac{2}{\pi} I_d \left(\sin\theta\right)_{-\pi/3}^{\pi/3}$

$= \frac{2}{\pi} I_d (2) \sin \pi/3.$

$= \frac{4 I_d}{\pi}\left(\frac{\sqrt{3}}{2}\right)$

$$\boxed{I_{m1} = \frac{2\sqrt{3}\, I_d}{\pi}}$$

- $I_1 = \frac{I_{m1}}{\sqrt{2}} = \frac{\sqrt{6}\, I_d}{\pi}$

rms value of fundamental component

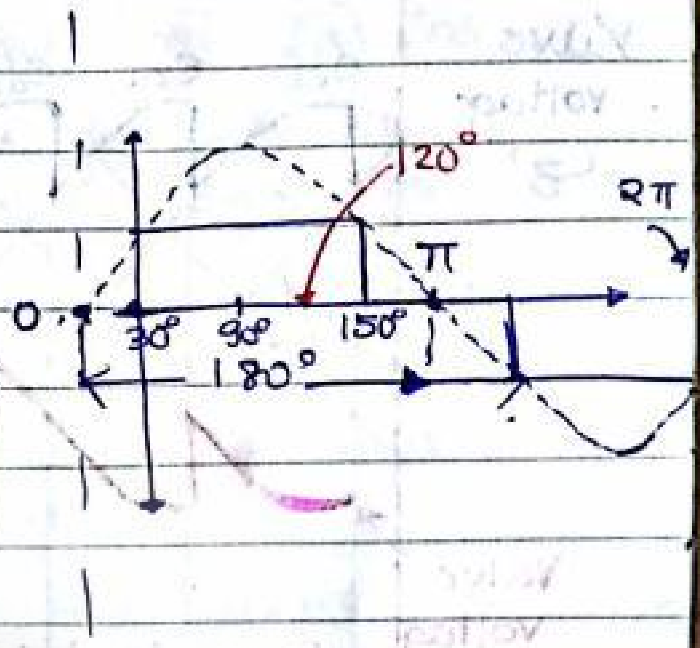
$$\boxed{I_1 = \frac{\sqrt{6}\, I_d}{\pi}}$$

⇒ Active Power is defined by the fundamental component flowing through the phase current & therefore I_1 is to be calculated.

Note : $I_h = \frac{I_1}{h}$, we know for 6 pulse $h = (np \pm 1)$; ie $\underbrace{5, 7}_{\text{less value}}$ $\underbrace{11, 13}_{}$

less value I_h more — I_h is less.

- $I_{Total} = I_1 + I_h$



$$\therefore I = \left[\frac{1}{2\pi}\int_0^{2\pi} I_d^2 \, d\theta\right]^{1/2}$$

$$= \left[\frac{1}{2\pi}\left(\int_0^{2\pi/3} I_d^2 \, d\theta + 0 + \int_{\pi}^{5\pi/3} I_d^2 \, d\theta\right)\right]^{1/2}$$

$2\pi/3 \to 120°$, $\pi \to 180°$

$$\therefore I = \left[\frac{1}{2\pi}(2)\int_0^{2\pi/3} I_d^2 \cdot d\theta\right]^{1/2}$$

$$= \left[\frac{1}{\cancel{2\pi}}(\cancel{2})\, I_d^2 \left(\frac{2\cancel{\pi}}{3} - 0\right)\right]^{1/2}$$

$$= \left[\frac{2}{3} I_d^2\right]^{1/2}$$

$$\therefore \boxed{I = \sqrt{\frac{2}{3}}\, I_d} \to \text{Total } I \text{ including } I_1 \text{ \& } I_h.$$

(Draw valve voltage … valve – Assignment)

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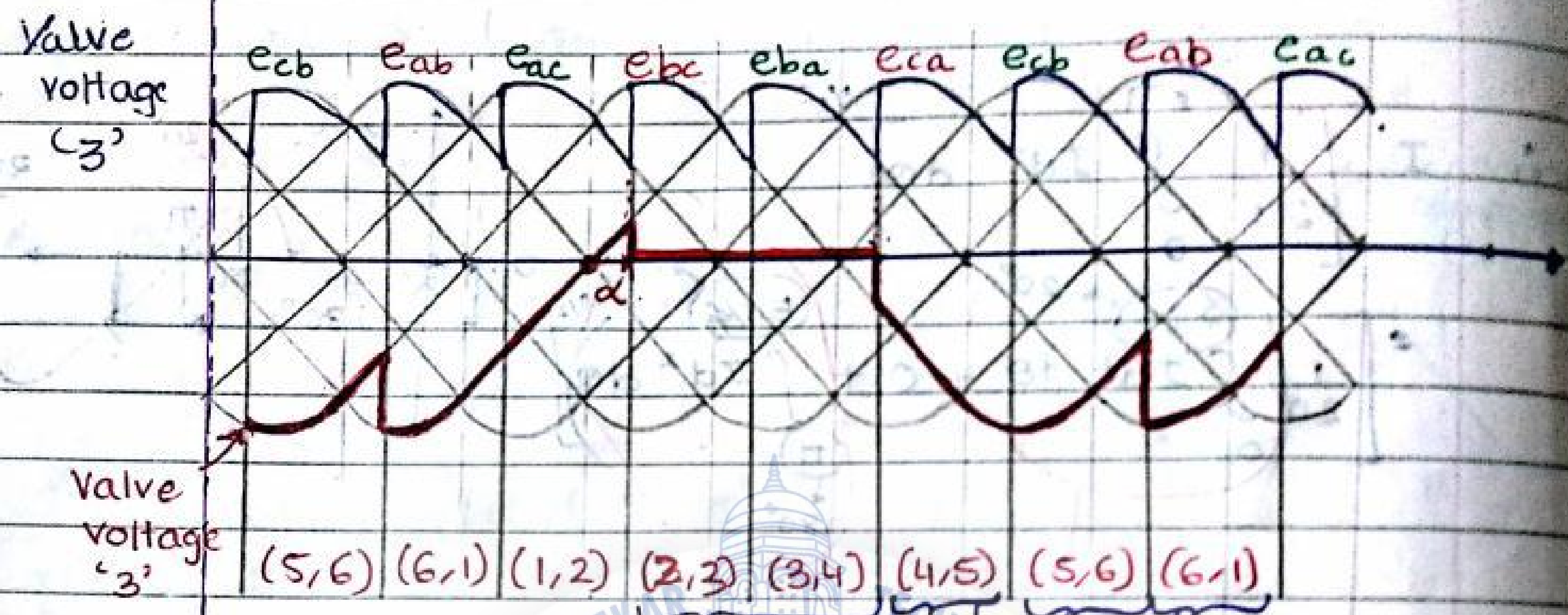
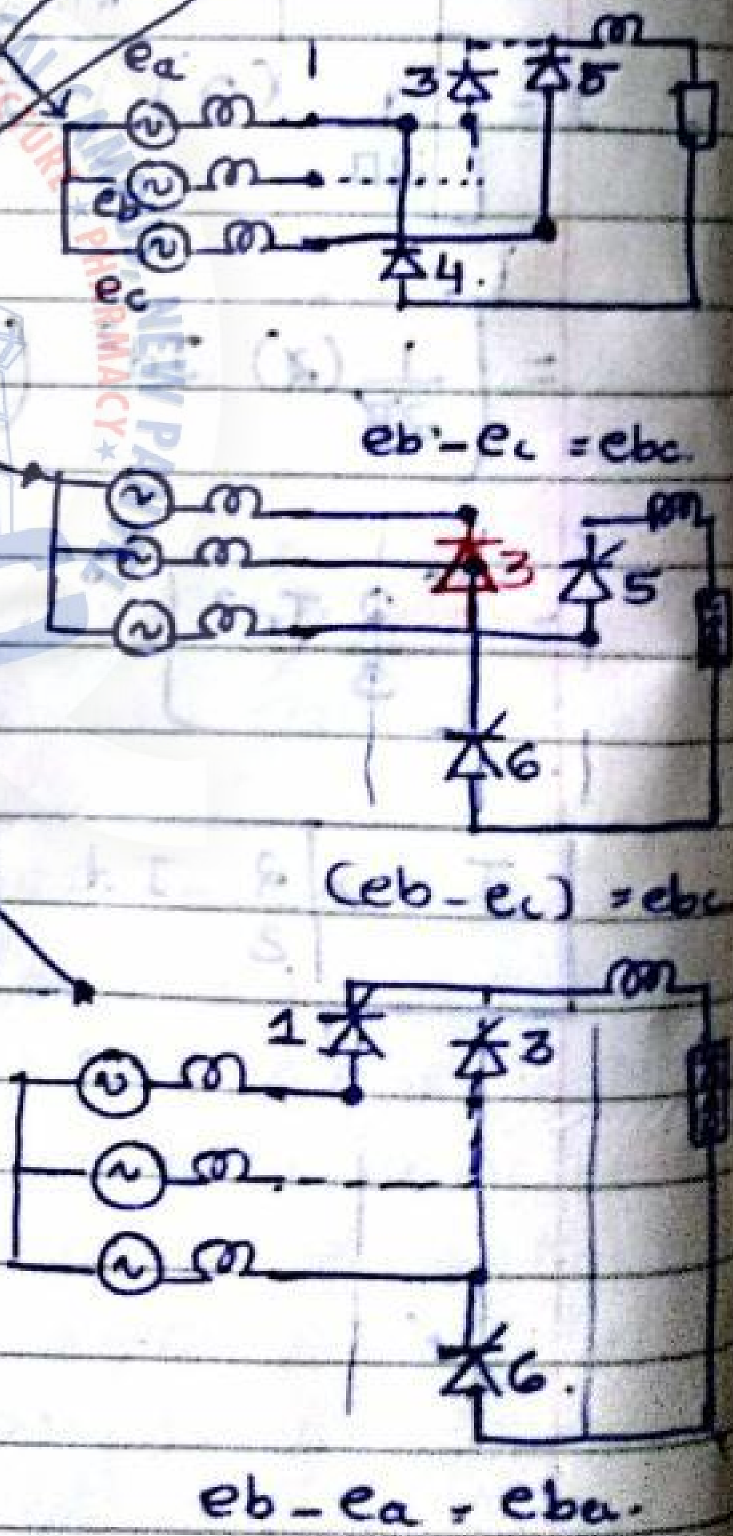
• Valve Voltages:





~~When volt~~

- In the rectification ckt shown above, the valve voltages are almoste '-ve'

- Rectifier ckt is therefore very stable ckt because if if the pulse is given by mistake the '-ve' voltage wont make a valve to conduct.

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• Power factor

→ $P_{ac} = \sqrt{3} E_{LL} I_1 \cos\phi = \sqrt{3} E_{LL} \left(\frac{\sqrt{6}}{\pi} I_d\right) \cos\phi$

fundamental component power

$$\boxed{P_{ac} = \frac{3\sqrt{2} E_{LL}}{\pi} I_d \cos\phi}$$

$\sqrt{2} E_{ph} = \frac{\sqrt{2} E_{LL}}{\sqrt{3}}$

→ $P_{dc} = V_d I_d = \frac{3\sqrt{3}}{\pi} E_m I_d \cos\alpha$

$E_{rms} = \frac{E_m}{\sqrt{2}}$

$$\boxed{P_{dc} = \frac{3\sqrt{2} E_{LL}}{\pi} I_d \cos\alpha}$$

→ In a lossless converter:

$$\boxed{P_{ac} = P_{dc}}$$

→ ∴ on equating P_{ac} & P_{dc}

$$\boxed{\cos\phi = \cos\alpha}$$

- This means the delay angle is directly proportional to P.F.
- If α of O/P terminal is reduced. The P.F of ac becomes very less.
- There is a limit put on α to prevent the above.
- α is therefore not delayed too much.
- 'α' is varied to control the dc power because the current in dc link 'I_d' is constant & V_d is changed to change 'P_{dc}'.

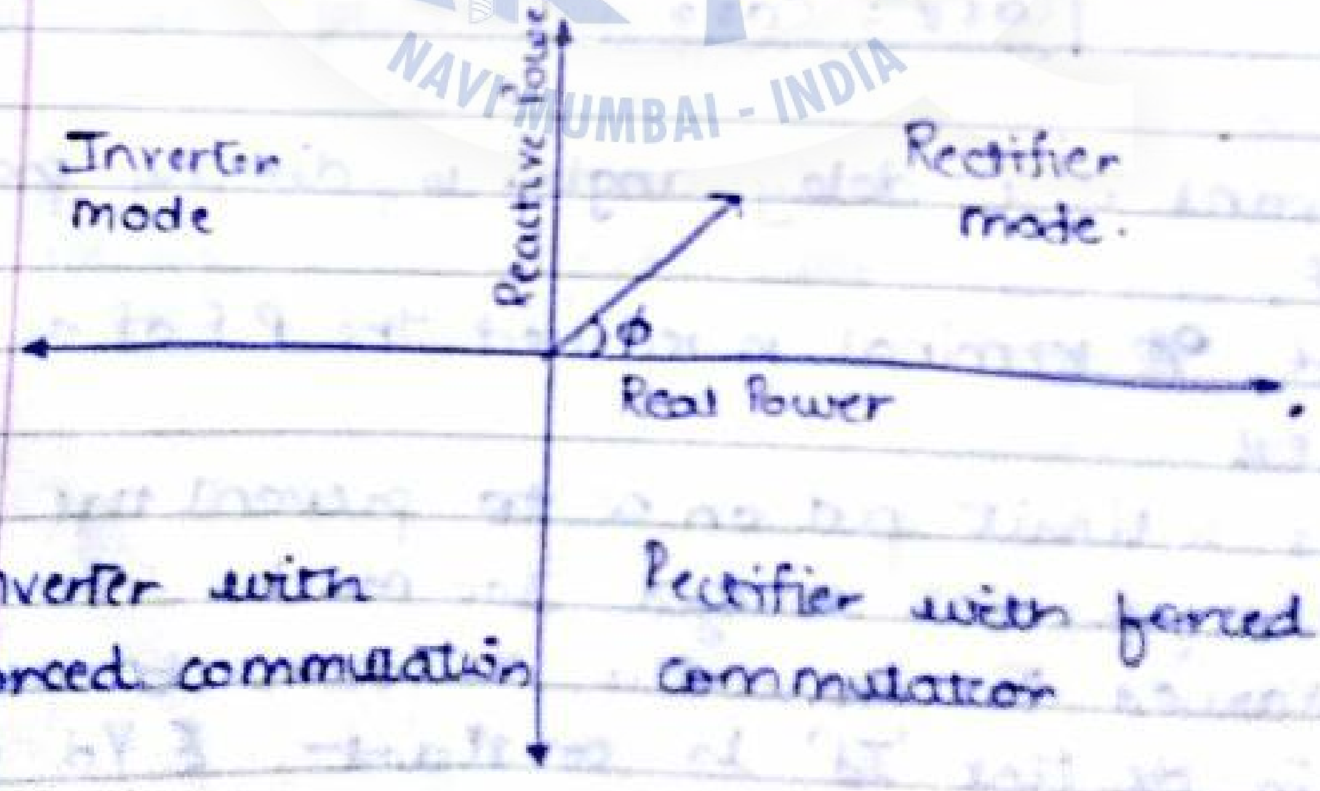
- α has a lower limit called α_{min} & also can't be increased beyond certain value because the reactive power requirement will be excessive ie α_{max}.

- Therefore the controller has to keep up with the various limits making control complex.

- If α becomes more than 90°, the $\cos\alpha$ term will be '+ve' but lagging, the Vd becomes '-ve' ∴ Pdc becomes '-ve'

- 

from above it's clear that Rectifier & inverter requires huge amount of 'Q'.

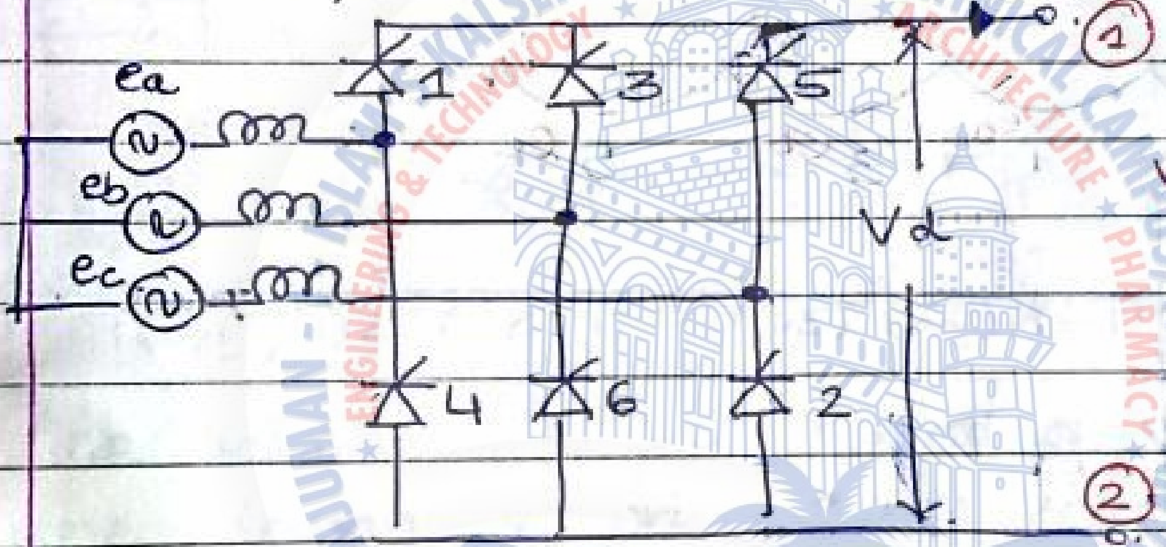
- 

- $\boxed{\cos\alpha = \cos\phi}$ holds true only in ideal case with no overlap.

- Extra commutation ckts/valves can be employed to change the reactive power requirement & cause the p.f. to lead.

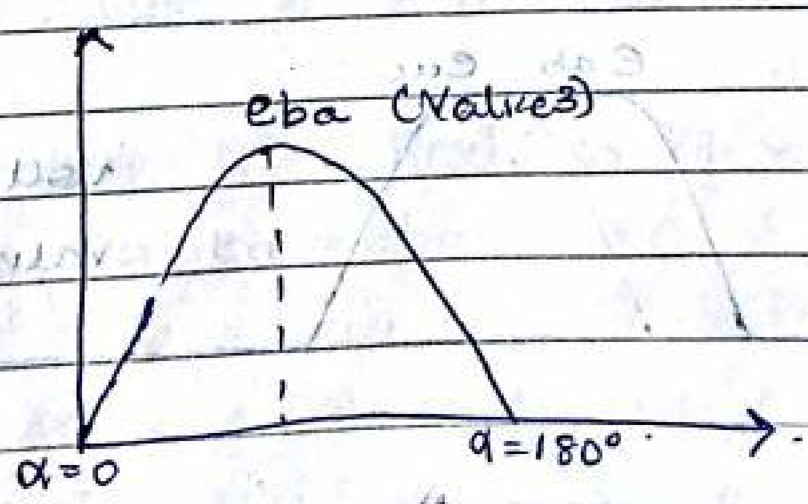
* Inverter Operation

- Consider a 6 pulse Rectifier ckt, Id.



$V_{①} - V_{②}$ = instantaneous voltage across link.

- In the inverter operation we have to delay the firing angle.
- The commutation voltage of valve eba as shown below;



- In the rectification mode, if $\alpha = 90°$, we get the zero voltage $\alpha < 90°$ we get '+ve' o/p across ① & ②.

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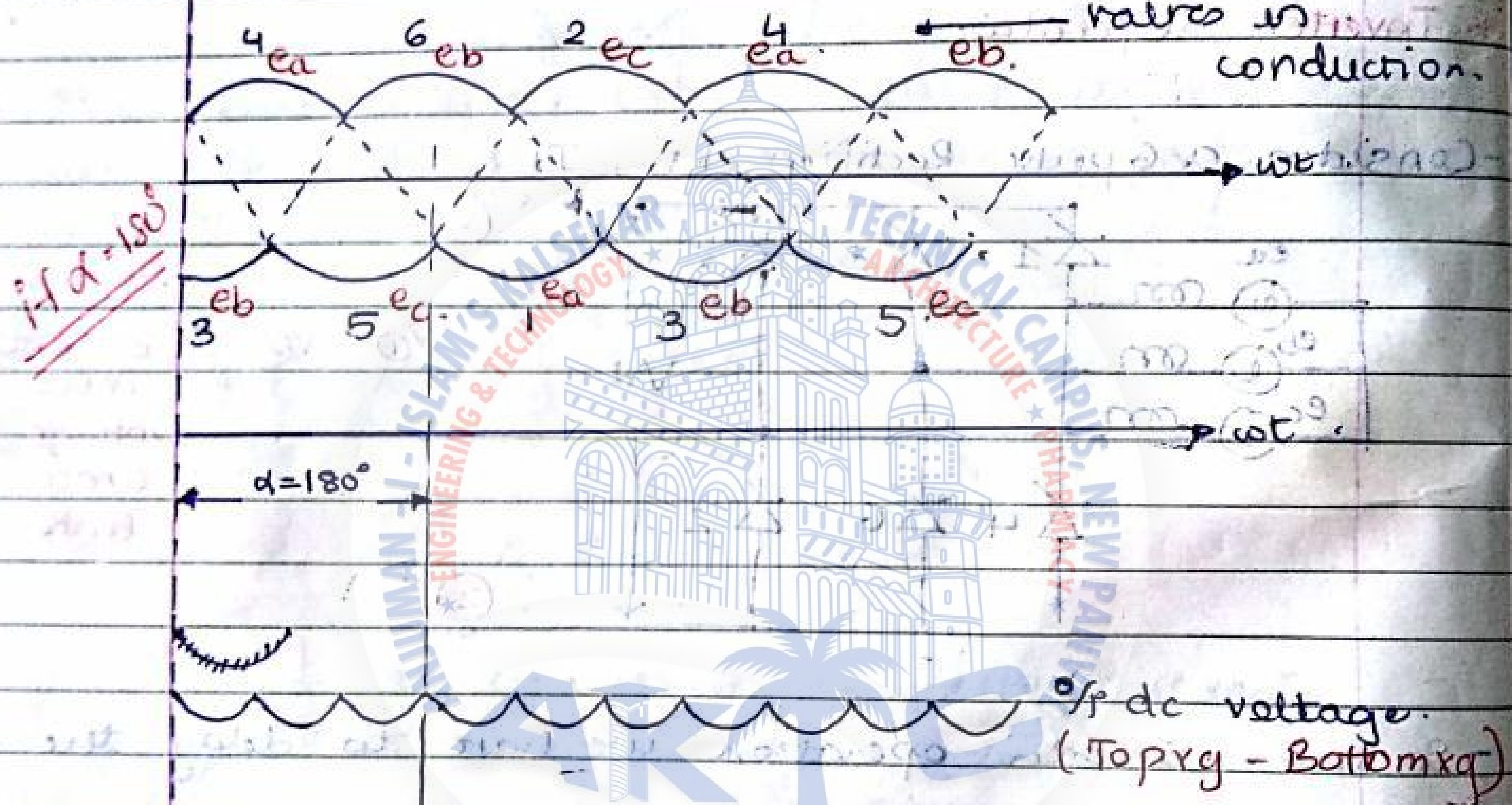
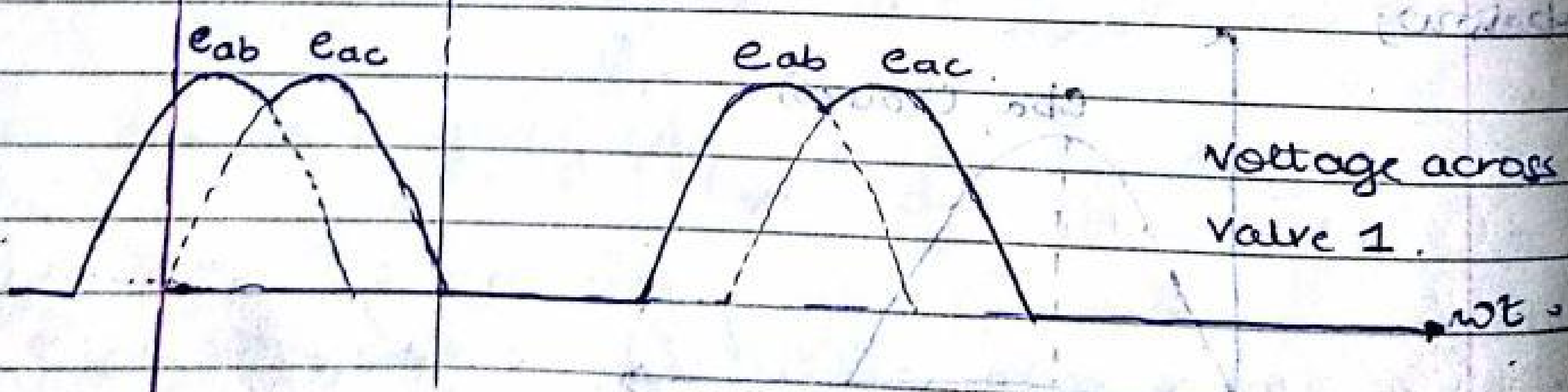
and when $\alpha > 90°$, voltage will be '-ve' across ① & ②

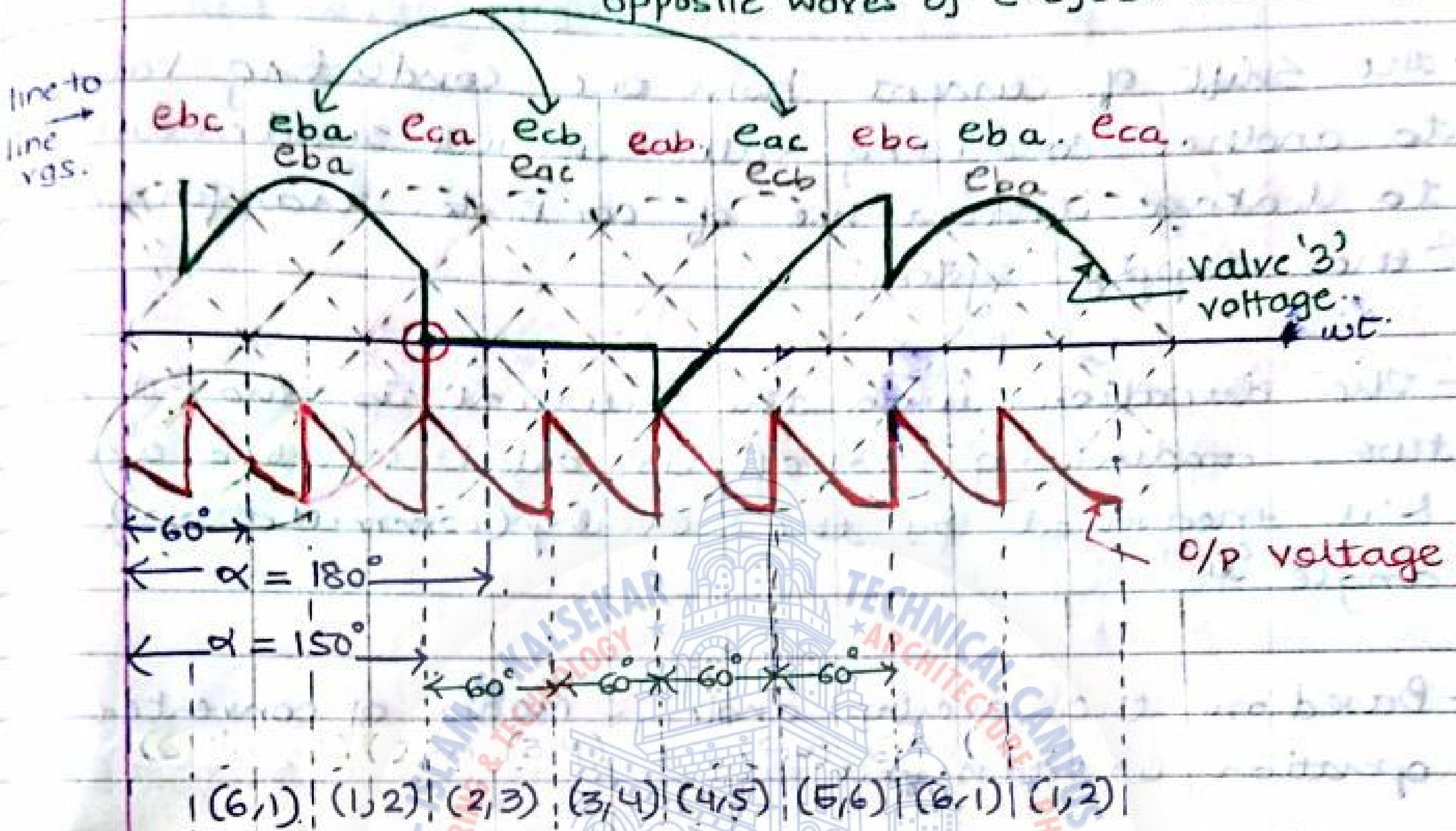
- So for inverter operation $\alpha > \pi/2 (90°)$. In ideal case if $\alpha = 180°$ (although there is a min angle required before 180° called (γ) extinction angle in practical application) we get the following voltages.





- Voltages in inverter mode.

• O/P voltage in case of inversion ($\alpha = 150°$)



- Here 'eba' is focussed ie commutating voltage of valve '3'. let the instant at which valve '3' be fired be ($\alpha = 150°$)
- when '3' is fired; '1' & '2' are conducting
  ∴ voltage across 3^{link} = eb − ec = ebc.
  ∴ the waveform of ebc will be followed as o/p across valve '3' link.

- After 60°, valve '4' is fired, so o/p voltage becomes;
  eb − ea = eba.

- After 60°, valve '5' is fired; so o/p voltage becomes.
  ec − ea = eca. So on & so forth.

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- Converter analysis with Overlap:

- The shift of current from one conducting valve to another conducting valve is not sudden due to leakage inductance of converter transformer & the supply system.

- The duration when the current is shared by two conducting valves is called overlap angle & is measured by the overlap (commutation) angle 'u'

- Based on the overlap angle, 3 modes of converter operation is classified as

(1) Mode 1 : 2 and 3 valve conduction ($u < 60°$)
where;
- two valve conduct for $(60° - u)$
- three valve conducts for u;

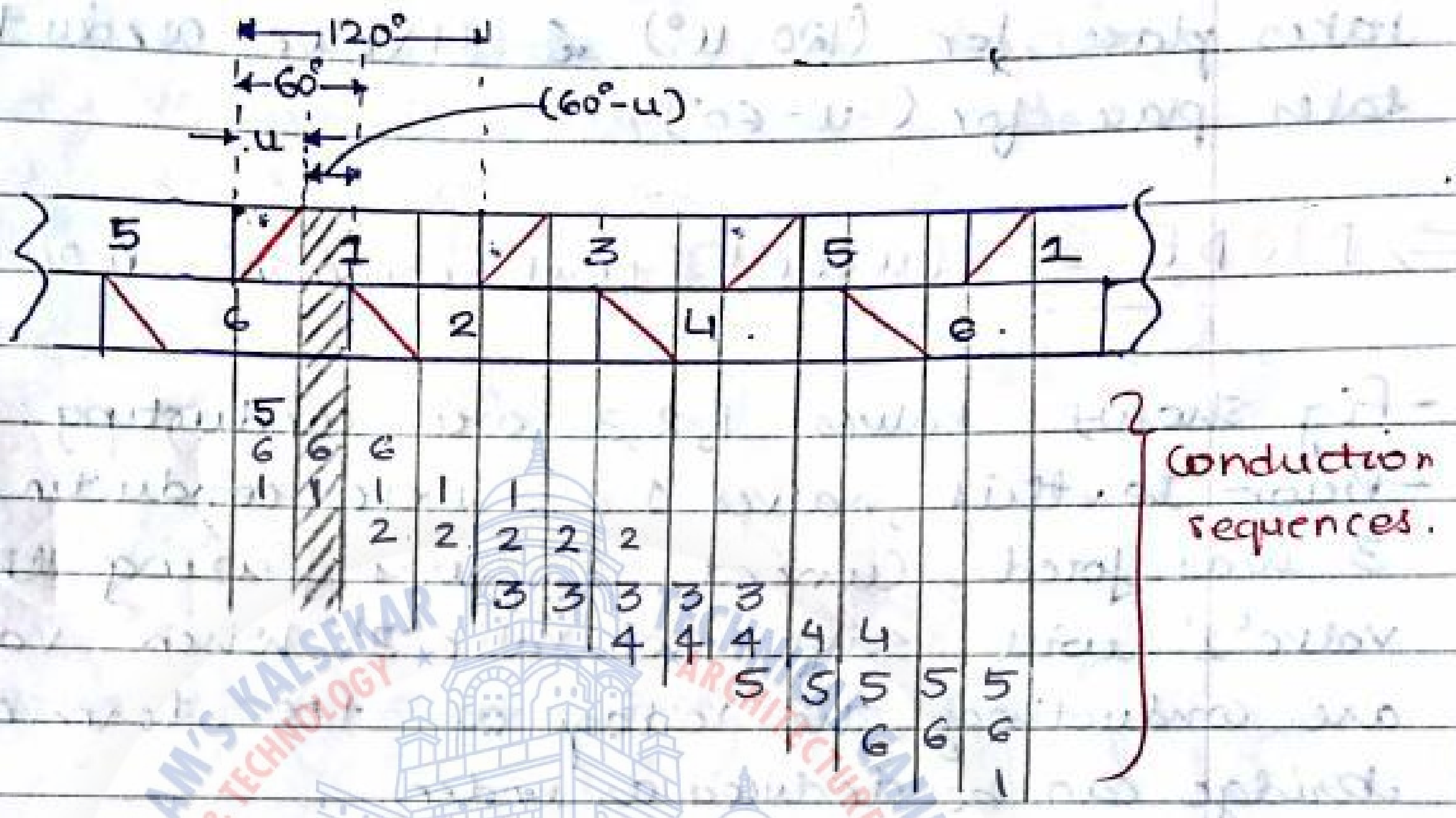
very common mode.

(2) Mode 2 : 3 valve conduction ($u = 60°$)
- where : 3 valve conduct during each interval for 60°.

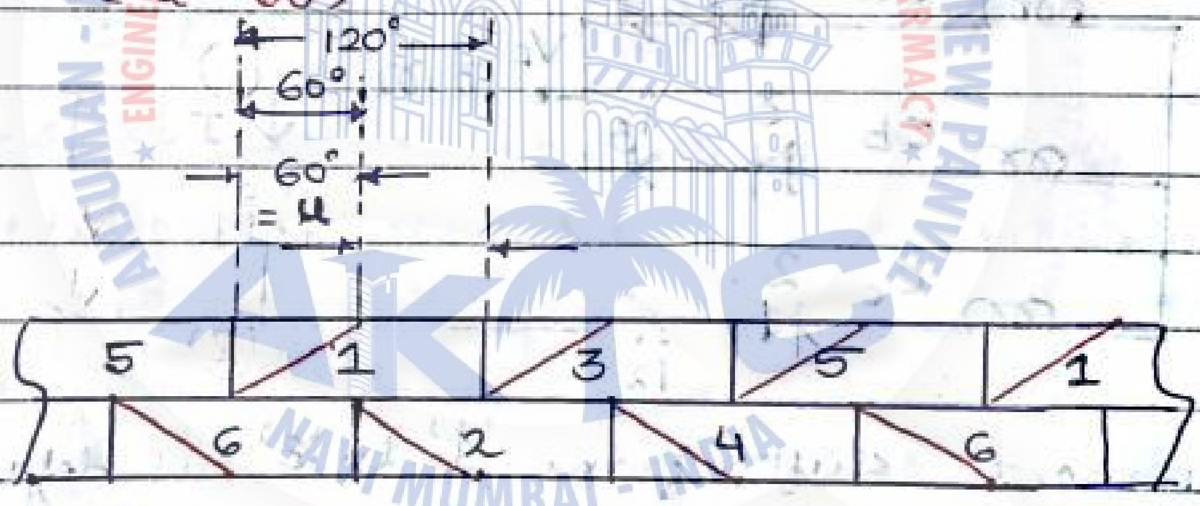
(3) Mode 3 : 3 and 4 valve conduction ($u > 60°$)
- where 3 valves conduct for $(120° - u)$
4 valves conduct for $(u - 60°)$

Not so common happens during faults.

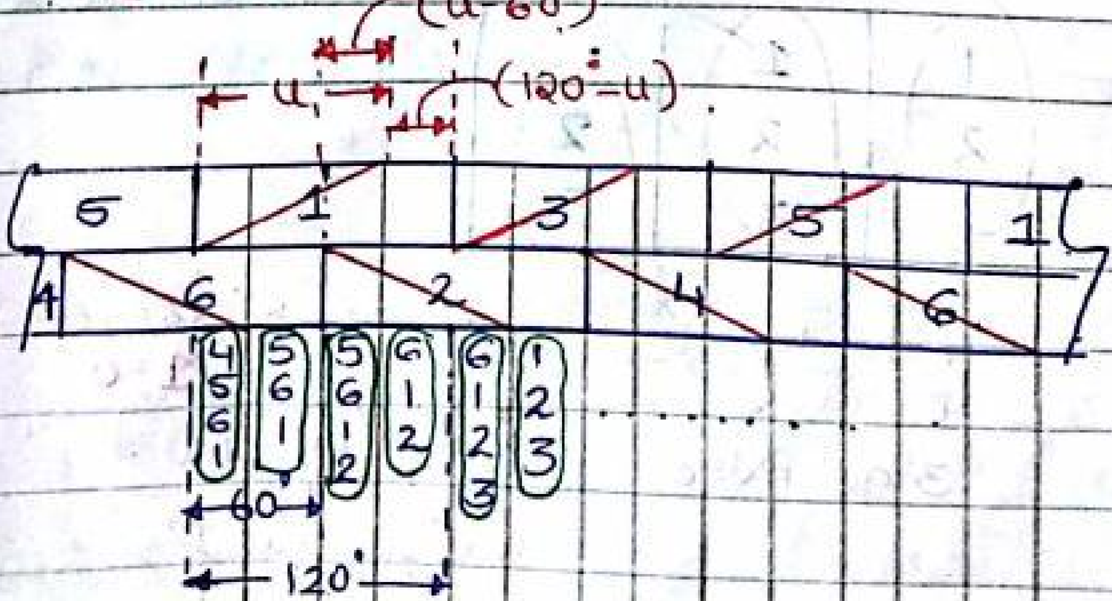
- Mode 1 ($u < 60°$)



- Mode 2 ($u = 60°$)



- Mode 3 ($u > 60°$)



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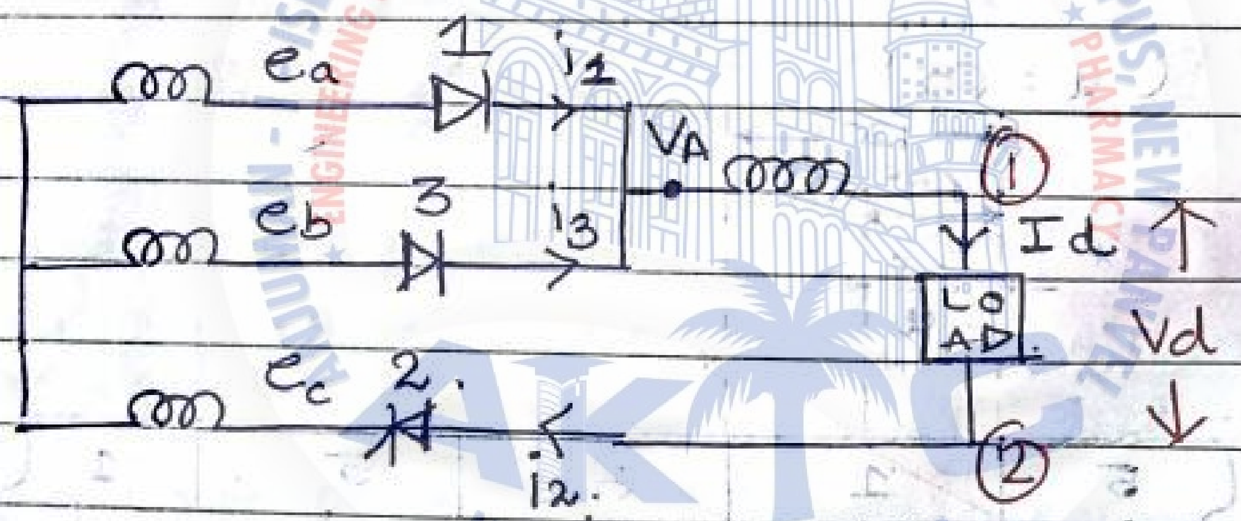
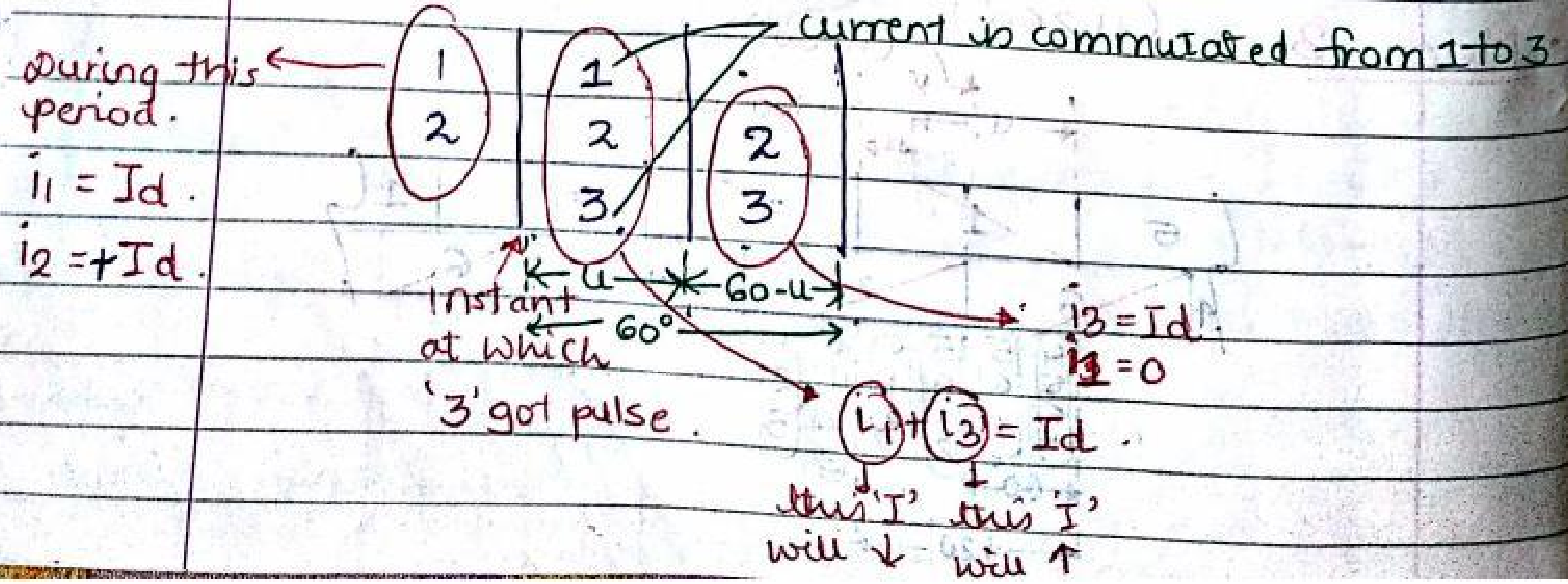
- from the above it is clear that 3 valve conduction takes place for $(120-u^\circ)$ & 4 valve conduction takes place for $(u-60^\circ)$.

⇒ MODE 1 $(u<60^\circ)$ [Most common mode of operation]

- Fig shows valves 1,2,3 are conducting.
- Prior to this, valves 1 & 2 were conducting & valve 3 was fired. Current i_1 was flowing through valve '1' will shift to valve '3'. When valves '1' & '3' are conducting, the voltage at the terminal of bridge can be deduced as under:



- The current shift from '1' to '3' is delayed by angle 'u' which is $<60^\circ$.
- The conduction pattern is shown below.



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- Now as seen phase 'A' is short circuited with phase 'B' as '1' & '3' both are conducting therefore V_A can be deduced as under

$$e_a - L\frac{di_1}{dt} = V_A \quad \text{--- (1)}$$

$$e_b - L\frac{di_3}{dt} = V_A \quad \text{--- (2)}$$

adding (1) & (2)

$i_1 + i_3 = I_d$.

$$(e_a+e_b) - \left(L\frac{di_1}{dt} + L\frac{di_3}{dt}\right) = 2V_A$$

$$(e_a+e_b) - \left(2L\frac{dI_d}{dt}\right) = 2V_A$$

I_d = constant

$$(e_a+e_b) - (0) = 2V_A$$

$$\therefore \boxed{V_A = \frac{e_a+e_b}{2}}$$

$$\therefore \boxed{V_A = \frac{e_a+e_b}{2} = \frac{-e_c}{2}}$$

$e_a+e_b+e_c = 0$

$\therefore e_a+e_b = -e_c$

- The o/p voltage V_d is;

vg at terminal (1)

Voltage at terminal (2)

$$V_d = \begin{cases} \dfrac{e_b+e_a}{2} - e_c = -\dfrac{3}{2}e_c & 0 \le \omega t \le u \\ e_b - e_c = e_{bc} & u \le \omega t \le 60-u \end{cases}$$

when '1' is off vg is e_b at (1)

~~tot~~ vg at (2) is e_c [∵ check conduction pattern]

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- The average dc voltage can be obtained by taking average over period of 60° thus;

$$V_d = \frac{1}{(\pi/3)}\left[\int_{0+(\alpha)}^{u+\alpha} -\frac{3}{2} e_c\, d\omega t + \int_{u(+\alpha)}^{60\cancel{(u)}+(\alpha)} e_{bc}\, d\omega t\right]$$

60°

won't be considered because upper limit = 60+α, lower limit = α, on substraction 60+α−α = 60°

(+α) is added if '3' is fired after a certain delay angle.

$V_c = \sqrt{3}V_L$

~~e_c~~ $e_c = E_m \sin(\omega t + \pi/2)$
$e_c = -E_m \cos \omega t$

$$= \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+u}\left(\frac{-3}{2}e_c\right)d(\omega t) + \int_{\alpha+u}^{60+\alpha}(e_{bc})\,d(\omega t)\right]$$

phase value.

$$= \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+u}\frac{3}{2}(E_m \cos\omega t\, d(\omega t)) + \int_{\alpha+u}^{60+\alpha}\sqrt{3}E_m \sin(\omega t + 60)\, d\omega t\right]$$

$$= \frac{3}{\pi}\left[\frac{3E_m}{2}(\sin(\alpha+u) - \sin\alpha) + \sqrt{3}E_m(\cos(\alpha+u+60°) - \cos(\alpha-120))\right]$$

$$= \frac{3}{\pi}E_m\left[\frac{3}{2}\sin(\alpha+u) - \frac{3}{2}\sin\alpha + \frac{\sqrt{3}}{2}\cos(\alpha+u) - \frac{3}{2}\sin(\alpha+u) + \frac{\sqrt{3}}{2}\cos\alpha + \frac{3}{2}\sin\alpha\right]$$

$$= \frac{3}{\pi}E_m\left[\frac{\sqrt{3}}{2}\cos(\alpha+u) + \frac{\sqrt{3}}{2}\cos\alpha\right]$$

$$V_d = \frac{3\sqrt{3}\,E_m}{2\pi}\left[\cos\alpha + \cos(\alpha+u)\right]$$

$$\boxed{V_d = \frac{V_{do}}{2}\left[\cos\alpha + \cos(\alpha+u)\right]}$$

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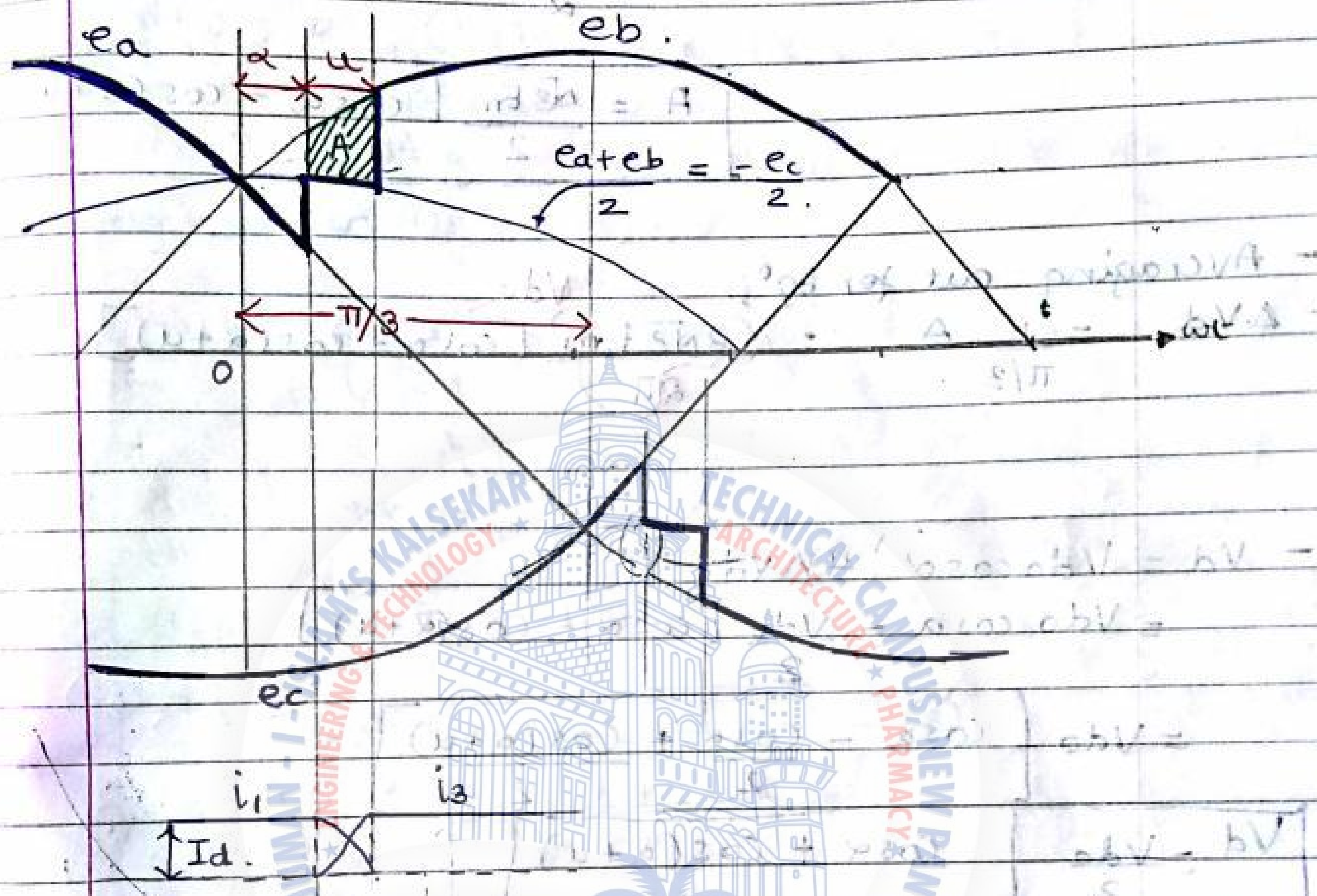
The commutation from valve 1 to valve 3 is shown;



- From the above it is clear that the presence of overlap 'u' reduces the o/p vg.

· Average dc voltage (Alternate Method)

- When $u = 0$;

$$V_d = V_{do}\cos\alpha.$$

upper waveform, lower waveform

$$- \text{Area } A = \int_{\alpha}^{\alpha+u} \left(e_b - \frac{e_a+e_b}{2} \right) d(\omega t)$$

$$= \int_{\alpha}^{\alpha+u} \frac{e_b - e_a}{2} \, d(\omega t).$$

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$$-\frac{\sqrt{3}E_m}{2}\cos\omega t\Big|_{\alpha}^{\alpha+u}$$

$$= \int_{\alpha}^{\alpha+u} \frac{e_{ba}}{2}\, d(\omega t) = \int_{\alpha}^{\alpha+u} \frac{\sqrt{3}\,E_m}{2} \sin\omega t \; d\omega t$$

$$\boxed{A = \frac{\sqrt{3}E_m}{2}\left[\cos\alpha - \cos(\alpha+u)\right]}$$

- Averaging out for 60°;

$$-\Delta V_d = \frac{1}{\pi/3} A = \frac{\overset{V_{do}}{3\sqrt{3}E_m}}{2\pi}\left[\cos\alpha - \cos(\alpha+u)\right]$$

$$- V_d = V_{do}\cos\alpha - \Delta V_d$$

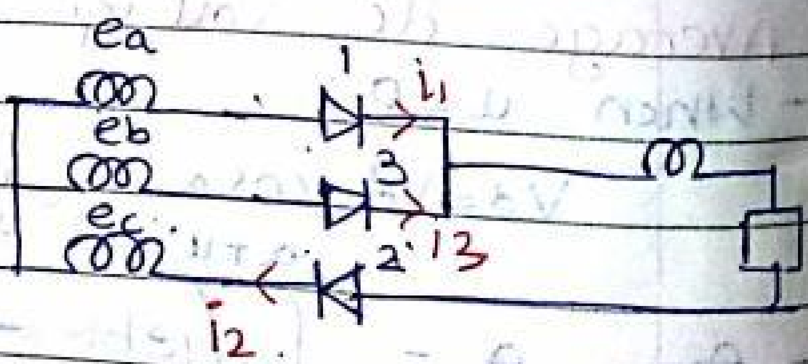
$$= V_{do}\cos\alpha - \frac{V_{do}}{2}\left[\cos\alpha - \cos(\alpha+u)\right]$$

$$= V_{do}\left[\cos\alpha - \frac{\cos\alpha}{2} + \frac{\cos(\alpha+u)}{2}\right]$$

$$\boxed{V_d = \frac{V_{do}}{2}\left[\cos\alpha + \cos(\alpha+u)\right]}$$

• The Valve current can be deduced as under:

We have;



$$e_a - L\frac{di_1}{dt} = e_b - L\frac{di_3}{dt}$$

$$\therefore L\frac{di_3}{dt} - L\frac{di_1}{dt} = e_b - e_a \quad \text{——} (1)$$

we also know;

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$i_1 + i_3 = I_d$. $\xrightarrow{}$ constant

or $\frac{di_1}{dt} + \frac{di_3}{dt} = \frac{dI_d}{dt} \rightarrow 0$.

$\frac{di}{dt} = -\frac{di_3}{dt}$ —— (2)

- putting (2) in (1).

$L\frac{di_3}{dt} - L\left(-\frac{di_3}{dt}\right) = e_b - e_a$

$\therefore 2L\frac{di_3}{dt} = e_{ba}$

$\therefore 2L\frac{di_3}{dt} = \sqrt{3}E_m \sin\omega t$

$\therefore \boxed{i_3 = -\frac{\sqrt{3}E_m}{2\omega L}\cos\omega t + A}$ —— (3)

Because we have $\int$ w.r.to 't' not 'ωt'

• When; $\omega t = \alpha$ (ie at the instant of firing '3')
$i_3 = 0$

$\therefore A = \frac{\sqrt{3}E_m}{2\omega L}\cos\alpha$ } from (3). —— (4)

Substituting (4) in (3).

$\therefore i_3 = \frac{\sqrt{3}E_m}{2\omega L}(\cos\alpha - \cos\omega t)$ $\quad \frac{\sqrt{3}E_m}{2\omega L} \rightarrow I_{s2}$.

$= I_{s2}(\cos\alpha - \cos\omega t) \qquad 0 \leq \omega t \leq u$.

– Now;

$$i_1 = I_d - i_3.$$

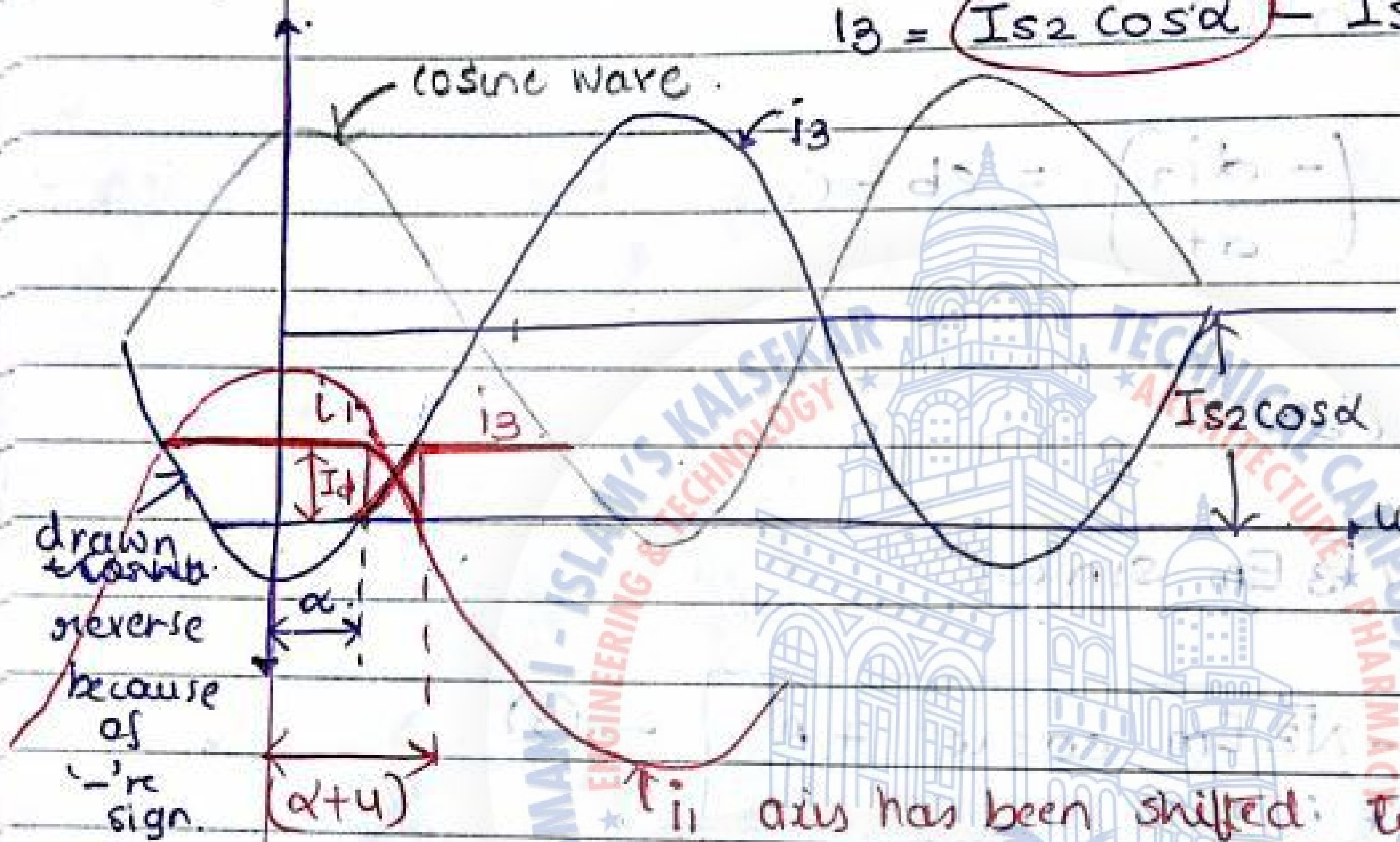
$$i_1 = I_d - I_{s2}(\cos\alpha - \cos\omega t) \qquad u \leq \omega t \leq \alpha + u$$

• Waveform of i_3 & i_1

↑ constant term add to i_3

$$i_3 = I_{s2}\cos\alpha - I_{s2}\cos\omega t$$

–(lec 9 (22:11))



drawn reverse because of '–'ve sign. axis has shifted up cuz of $I_{s2}\cos\alpha$ constant.

i_1 axis has been shifted to 'I_d' & '–ve' of i_3 is taken to obtain this waveshape [$\because i_1 = I_d - i_3$]

(lec 9 : 26:52)

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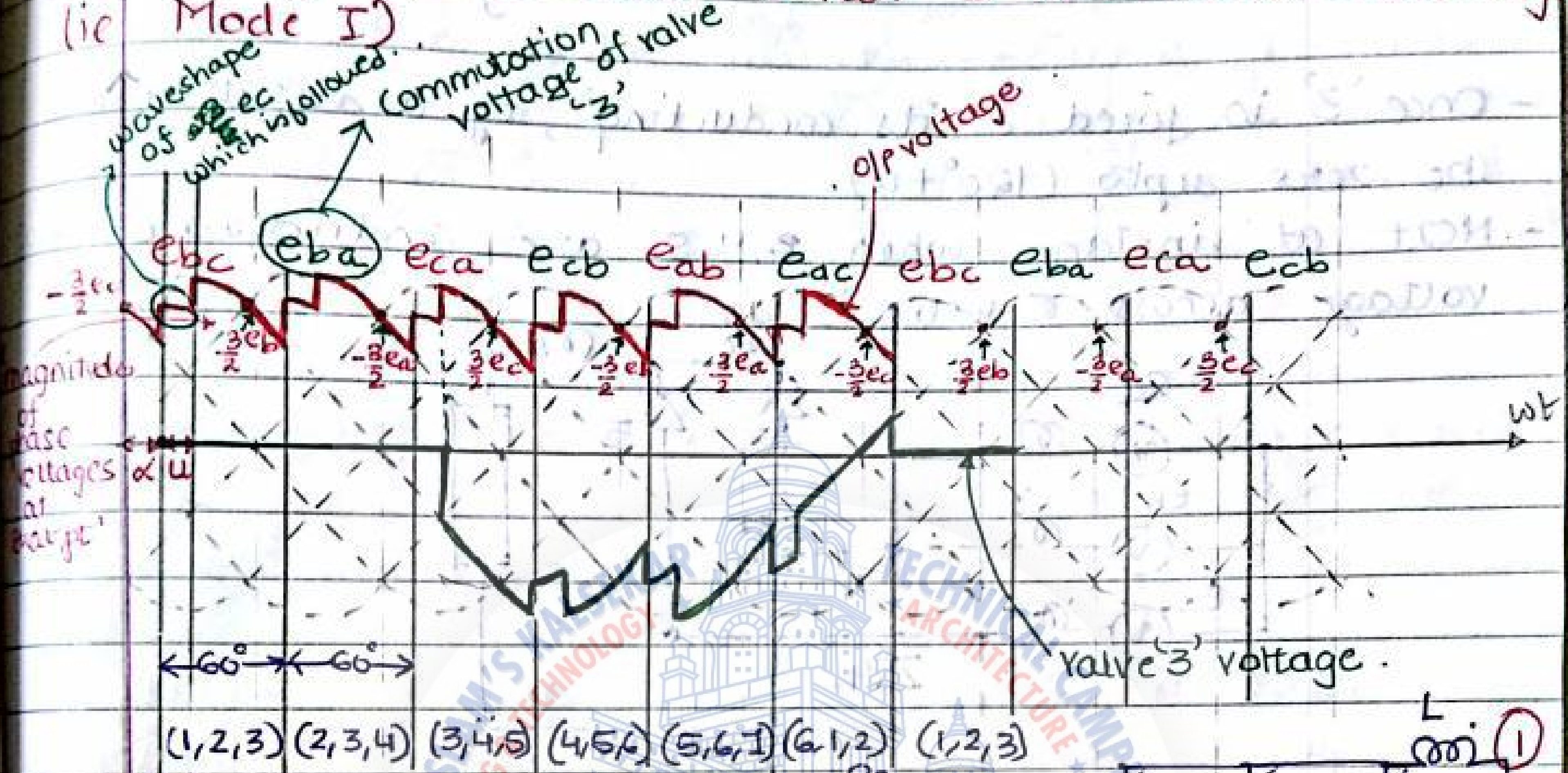
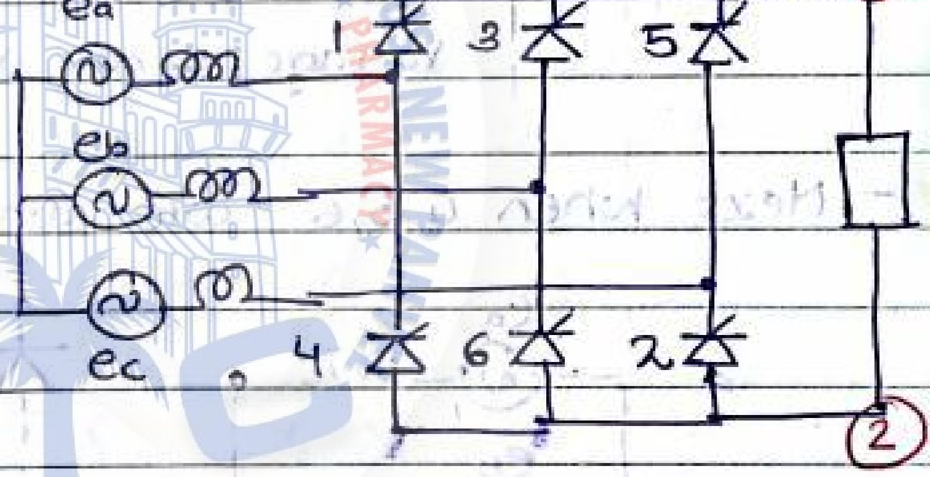
• O/P Voltage of the valve when '2' & '3' valve are conducting (ie Mode I).





For O/P voltage plotting ⟹

• When 1,3,2 are conducting

vg at terminal ① = $\frac{e_a+e_b}{2} = \frac{-e_c}{2}$

& vg at terminal ② = e_c

∴ Net voltage = $-\frac{e_c}{2} - e_c = -\frac{3e_c}{2}$

• When 2,3,4 are conducting

vg at terminal ① = e_b

vg at terminal ② = $\frac{e_a+e_c}{2} = \frac{-e_b}{2}$

∴ Net voltage = $e_b - \left(\frac{-e_b}{2}\right)$

$= \frac{3}{2}e_b$.

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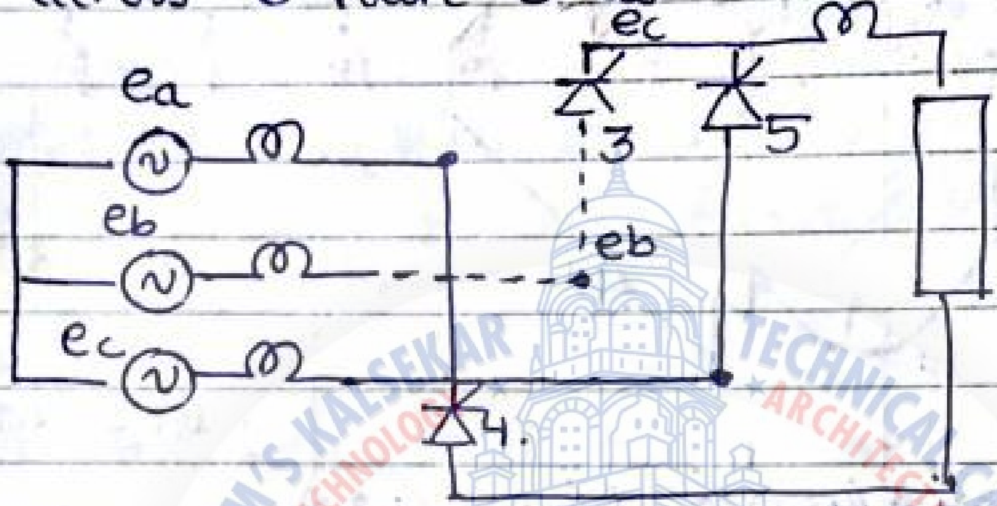
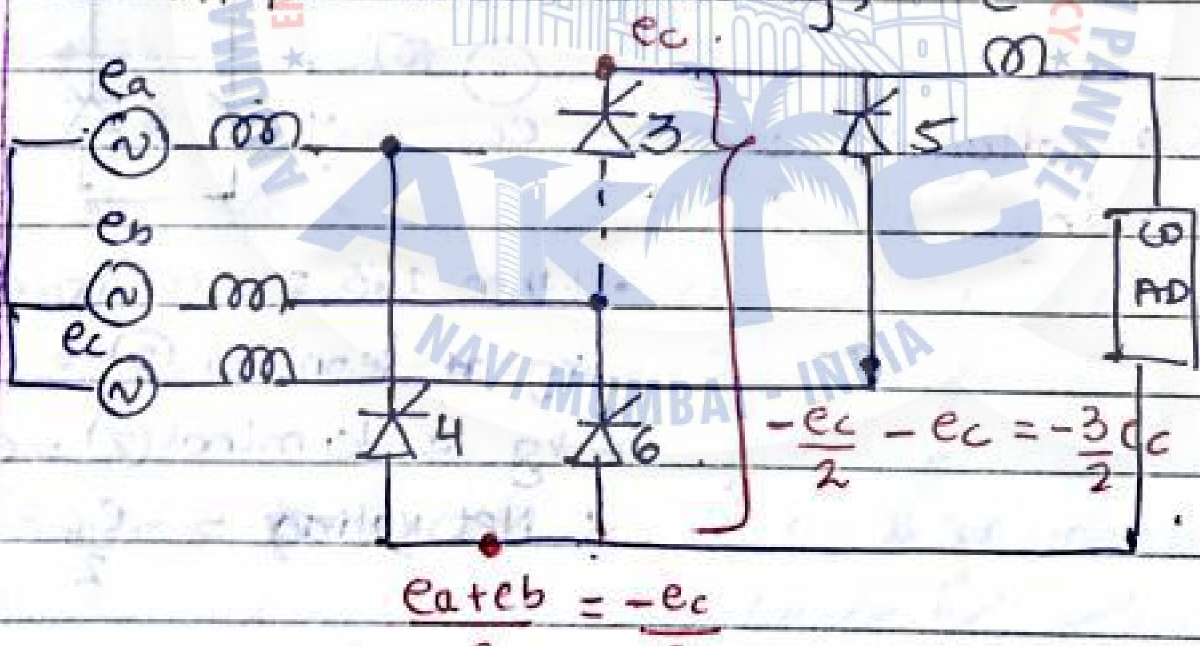
⇒ For Voltage across Valve '3' plotting

- Once '3' is fired & its conducting, Vg across it will be zero upto $(120° + u)$.
- Next at instant when 3,4,5 are conducting & '3' not voltage across valve '3' is



∴ Voltage across valve 3 = $e_b - e_c = e_{bc}$

- Next when 4,5,6 are conducting;



- So when 4,5,6 conducting; voltage across valve '3' $= -\frac{3}{2}e_c$

- When 5,6 conducting (after 'u') $= e_b - e_c = e_{bc}$.

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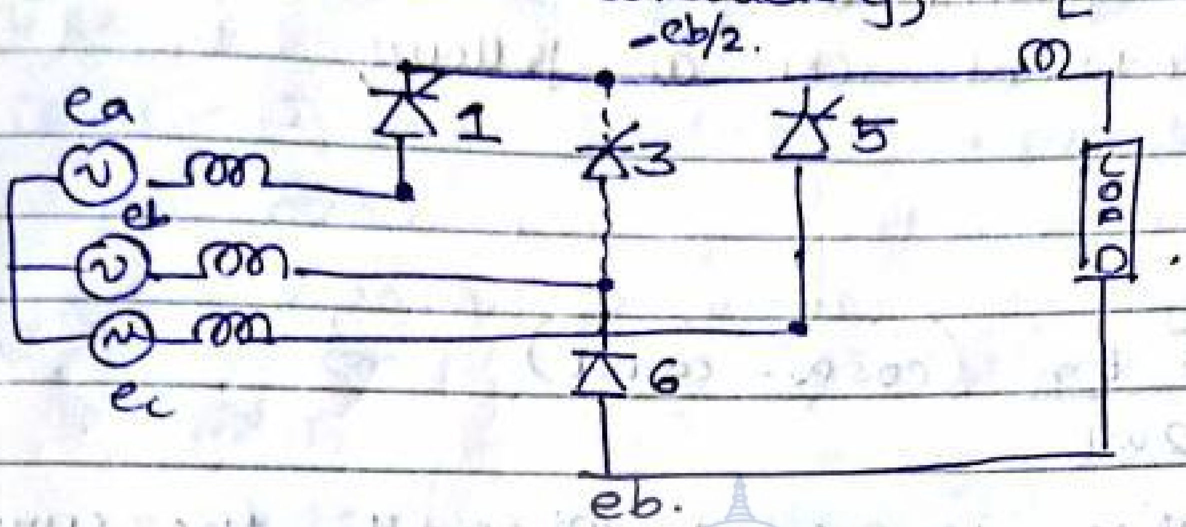
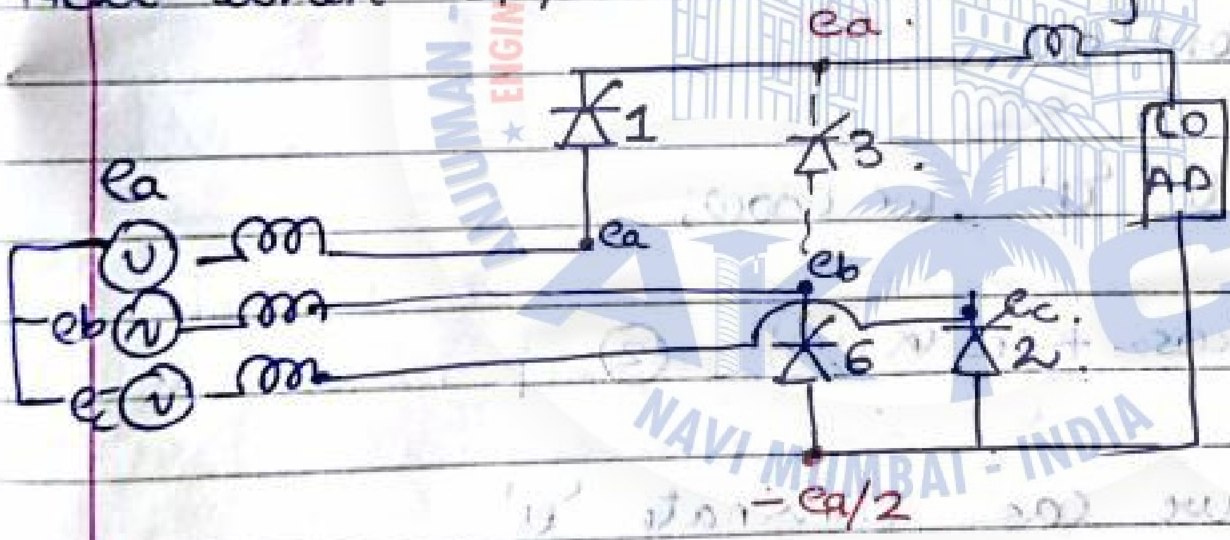
- Next when 5,6,1 are conducting;



$$\therefore\ eb-\left(\frac{-eb}{2}\right) = eb+\frac{eb}{2} = \frac{3}{2}eb$$

- when 6, 1 are conducting ; $eb-ea = eba$

- Next when 6,1,2 are conducting



$-ea/2 - ea$
$= -3/2\, ea$

when 1,2 are conducting ; $eb - ea = eba$

- We can ~~relate~~ represent the power electronic ckt in terms of electrical ckt as follows;

We know;

$$i_s = \frac{\sqrt{3}\,E_m}{2\omega L}(\cos\alpha - \cos\omega t)$$

once commutation is over at $\omega t = \alpha + u$, the current becomes;

$$I_d = \frac{\sqrt{3}\,E_m}{2\omega L}(\cos\alpha - \cos(\alpha+u)) \quad \text{— (1)}$$

- we need the express the above in electrical ckt because the measurement 'u' is difficult because it depends on 'L' on source etc which is not under our control.

- So we eliminate 'u', we know;

$$V_d = \frac{V_{do}}{2}(\cos\alpha + \cos(\alpha+u)) \quad \text{— (2)}$$

from (1) & (2) we can eliminate 'u'

so; $I_d = \frac{\sqrt{3}\,E_m}{2\omega L}[\cos\alpha - \cos(\alpha+u)]$

$$V_d = \frac{3\sqrt{3}\,E_m}{2\pi}(\cos\alpha + \cos(\alpha+u))$$

or;

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$$\frac{I_d}{I_{s2}} = \cos\alpha - \cos(\alpha+u)$$

or $\cos\alpha - \frac{I_d}{I_{s2}} = \cos(\alpha+u)$ → put this in Vd.

$$\therefore V_d = \frac{3\sqrt{3}\,E_m}{2\pi}\left(\cos\alpha + \cos(\alpha+u)\right)$$

$$= \frac{V_{do}}{2}\left(\cos\alpha + \cos\alpha - \frac{I_d}{I_{s2}}\right)$$

$\frac{3}{\pi}\omega L$ → commutation Reactance ie $\frac{3}{\pi} X_L$

$$= V_{do}\cos\alpha - \frac{V_{do}}{2}\cdot\frac{I_d}{I_{s2}}$$

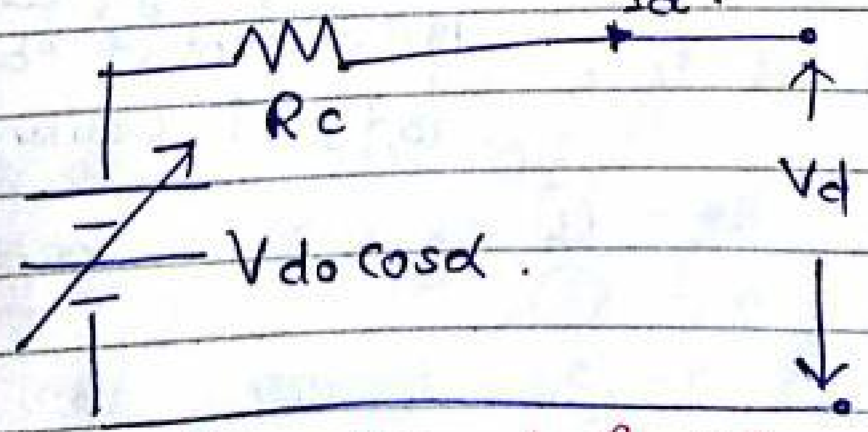
$$= V_{do}\cos\alpha - \left(\frac{3}{\pi}\omega L\right) I_d$$

$$\boxed{V_d = V_{do}\cos\alpha - R_c\, I_d}$$

→ Equivalent commutation resistance.

or $\boxed{R_c = \frac{3}{\pi}\omega L = \frac{3}{\pi} X_L}$

- from above eq it is clear that since 'Id' is constant o/p Voltage Vd can be varied by varying 'α' alone.



Eq ckt of Rectifier

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* Inverter operation. ($u < 60°$).

- The analysis of inverter operation is not different from the rectification. ie (α is delayed $> 90°$).

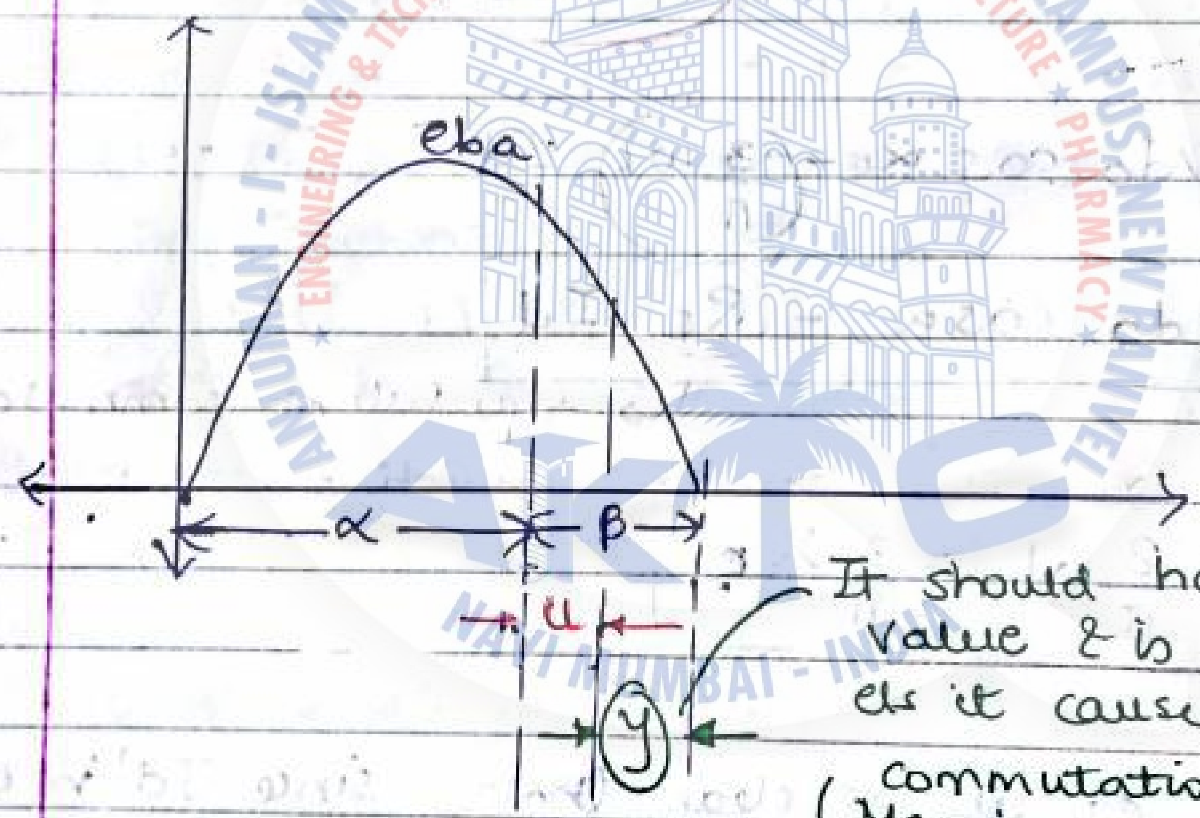
- However the inverter eqs are normally expressed in terms of the angle of advance

$$\boxed{(\beta = \pi - \alpha)}$$

or extinction angle)

$$\boxed{(\gamma = \beta - u)}.$$

- To understand α, γ, u, β consider the following



It should have minimum value & is important. els it causes unsuccessful commutation
(Meaning normally $i_1 \downarrow$ $i_3 \uparrow$, if γ_{min} is not maintained due to voltage reversal at 180° instant of eba, $i_3 \downarrow$, $i_1 \uparrow$ causing malopera[tion]

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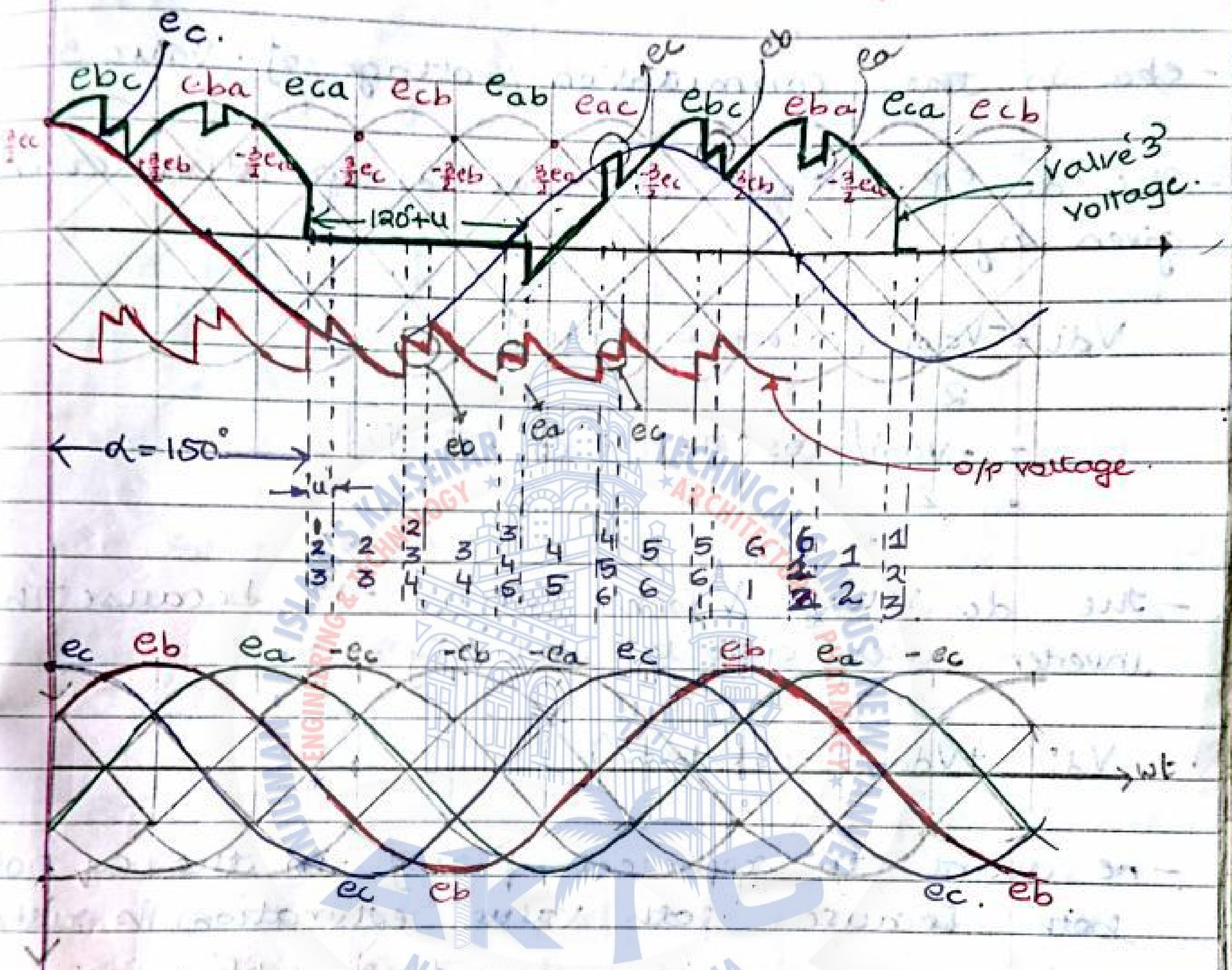
* O/P voltage of inverter / valve voltage for '3' valve.



When 1, 2 are conducting & 3 is given pulse;
O/P voltage at terminal ① $= \frac{e_a + e_b}{2} = \frac{-e_c}{2}$;
Voltage at terminal ② $= e_c$
Net voltage $= \frac{-e_c}{2} - e_c = -1.5e_c$.

When 2, 3 are conducting;
voltage at terminal ① $= e_b$
" ② $= e_c$
Net voltage $= e_b - e_c = e_{bc}$.

for o/p voltage plotting

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- assume $\alpha = 150°$, $\beta = 30°$, $\mu = 15°$, $\gamma = 15°$

- eba is the commutation voltage of valve 3.

- The dc o/p voltage in inverter operation can be given by;

$$V_{di} = \frac{-V_{doi}}{2}\left[\cos\alpha + \cos(\alpha+\mu)\right]$$

$$= \frac{-V_{doi}}{2}\left[\cos(\pi-\beta) + \cos(\pi-\gamma)\right]$$

- The dc voltage V_{doi} is taken -ve because the inverter uses opposite polarity.

$$V_{di} = \frac{+V_{doi}}{2}\left[\cos\beta + \cos\gamma\right]$$

- we want to represent β or γ in the eq not both because both value estimation is risky

- $V_{di} = -\left(V_{doi}\cos\alpha - R_c I_d\right)$ check rectifier part for this derivation

$$= -\left(V_{doi}\cos(\pi-\beta) - R_c I_d\right)$$

$$\boxed{V_{di} = V_{doi}\cos\beta + R_c I_d.}$$

or;

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$\rightarrow \frac{2V_{di}}{V_{doi}} = \cos\beta + \cos\gamma.$

or $-\cos\gamma + \frac{2V_{di}}{V_{doi}} = \cos\beta$

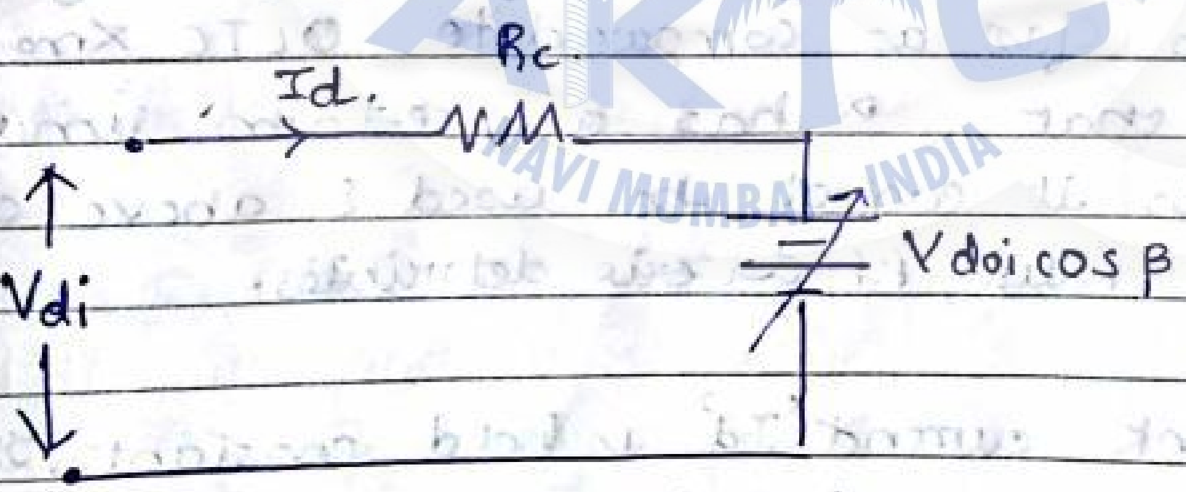
$\rightarrow V_{di} = V_{doi}\cos\beta + R_c I_d.$

$= V_{doi}\left(-\cos\gamma + \frac{2V_{di}}{V_{doi}}\right) + R_c I_d.$

$V_{di} = -V_{doi}\cos\gamma + 2V_{di} + R_c I_d.$

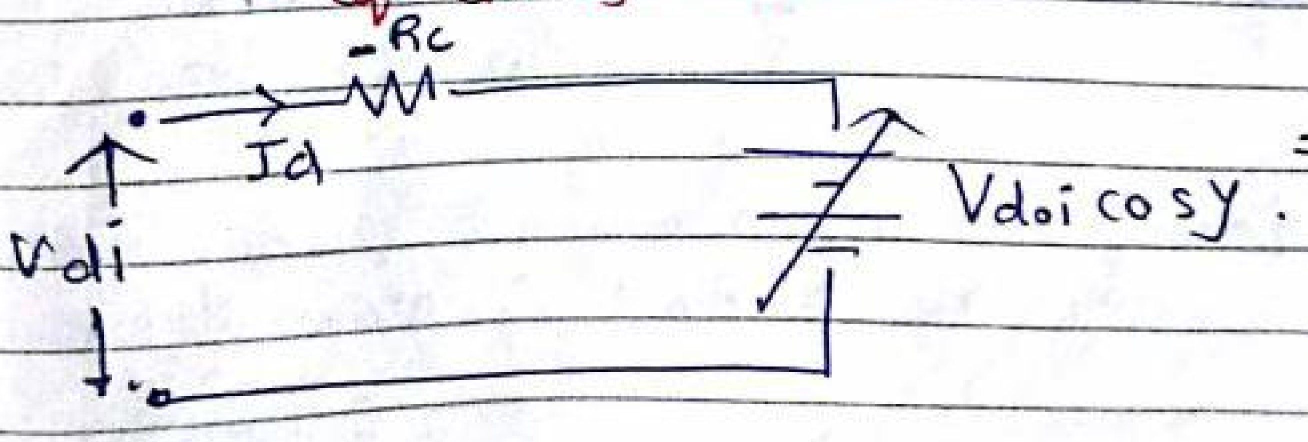
$-2V_{di} + V_{di} = -V_{doi}\cos\gamma + R_c I_d.$

$$\boxed{V_{di} = V_{doi}\cos\gamma - R_c I_d}$$



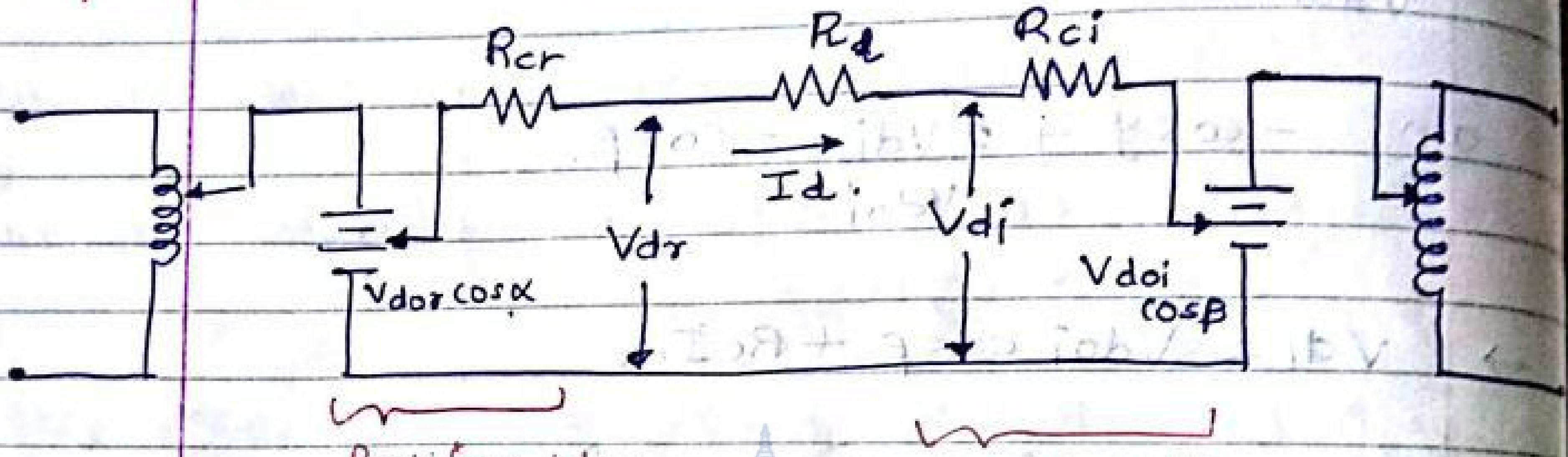
⇒ in terms of β

• Eq. ckt of inverter



⇒ in terms of γ.

* Equivalent ckt of HVDC link.



- The OLTC transformer which acts as controller for varying voltage
- From above it is clear that the voltage at two ends can be varied by
  (1) the OLTC Xmer
  (2) By varying α, β

- But it is prefered voltage is varied by varying 'α' because its fast as compared to OLTC Xmer
- We know that α has a certain limit range below αmin it cannot be used & above a certain α value, P.F ~~deterio~~ detoriates.

- The HVDC link current 'Id' is held constant. The rectifier side ~~xor~~ maintains the voltage whereas the inverter side maintains the current.

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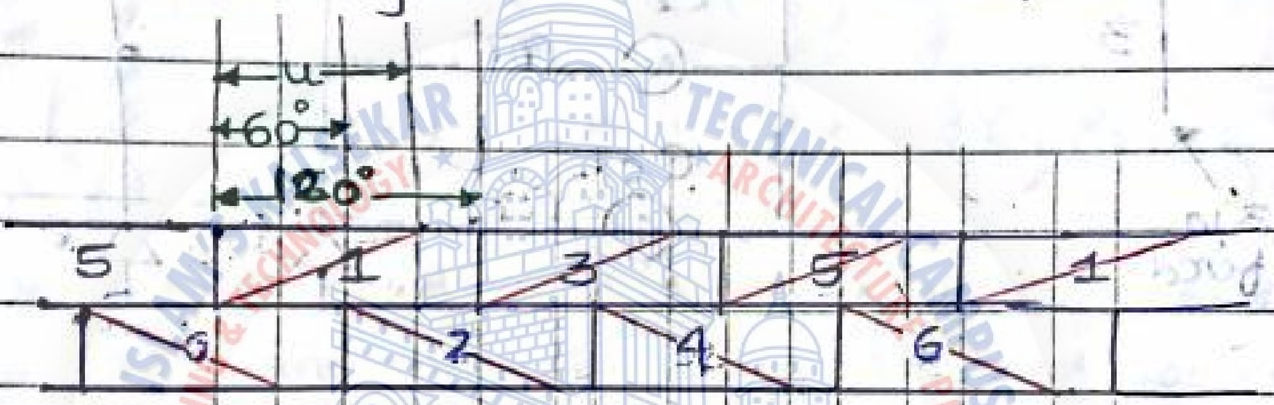
* Mode III (3-4 Valves conduction mode) (abnormal mode of operation)

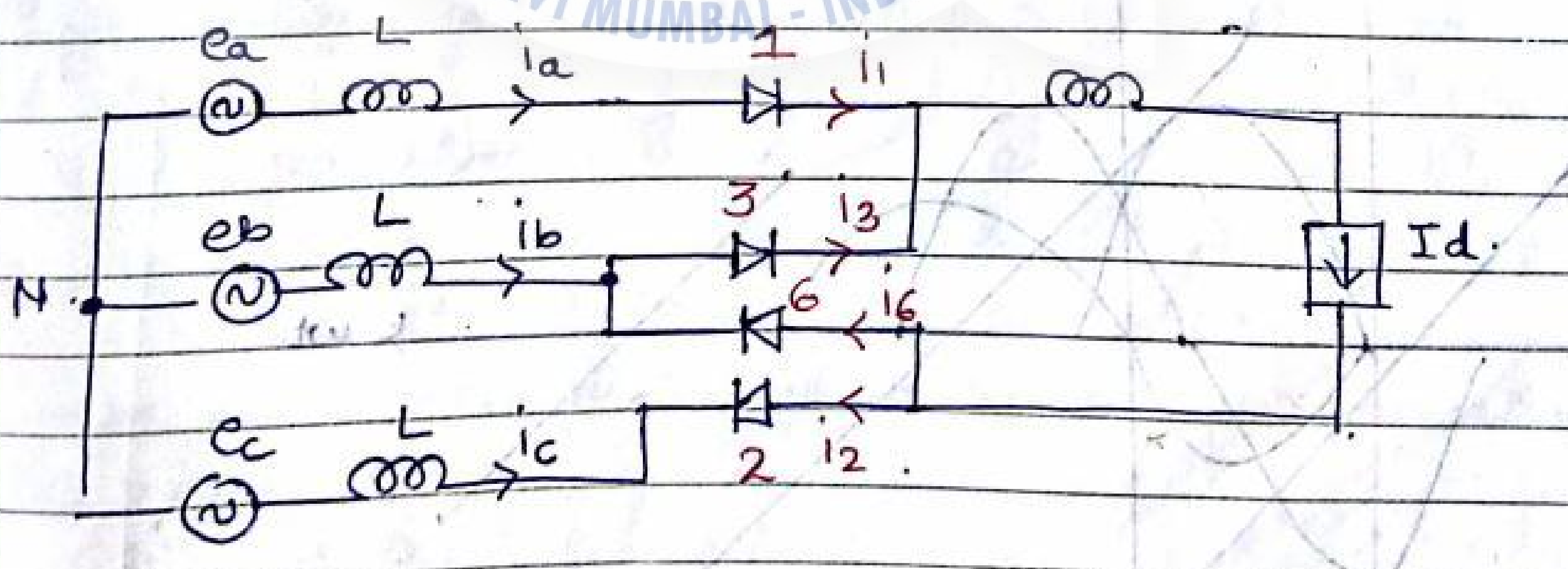
- Operation of bridge converter with overlap angle $60^\circ \leq u \leq 120^\circ$ is abnormal.
- This condition is encountered under overload dc short ckt or low ac voltage.
- However it is self curing i.e after overlap time it starts conducting in 3 valves mode.



4	5	5	6	6	1	1	2	2	3	3	4	4
5	6	6	1	1	2	2	3	3	4	4	5	5
6	1	1	2	2	3	3	4	4	5	5	6	6
1		2		3		4		5		6		1

} valve conduction

- Consider a 4 valve conduction case:



- In this case, 3,6 are conducting, that results in short circuit ∴ No voltage at o/p.

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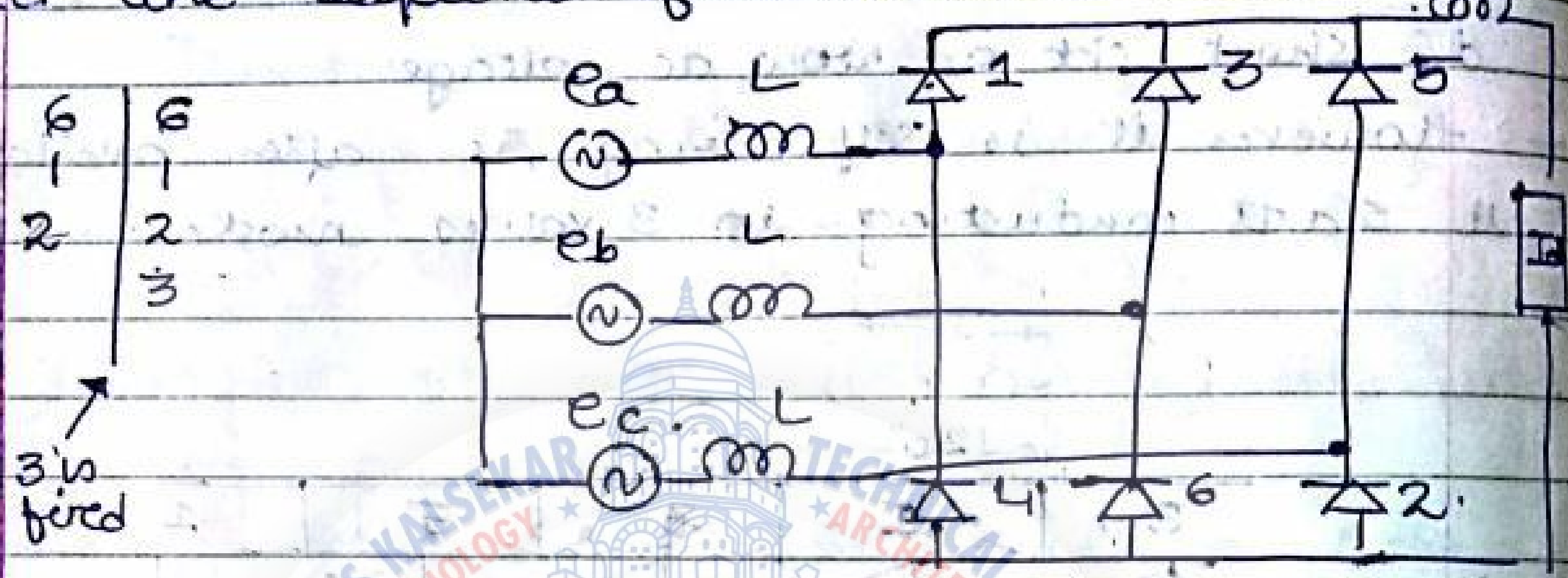
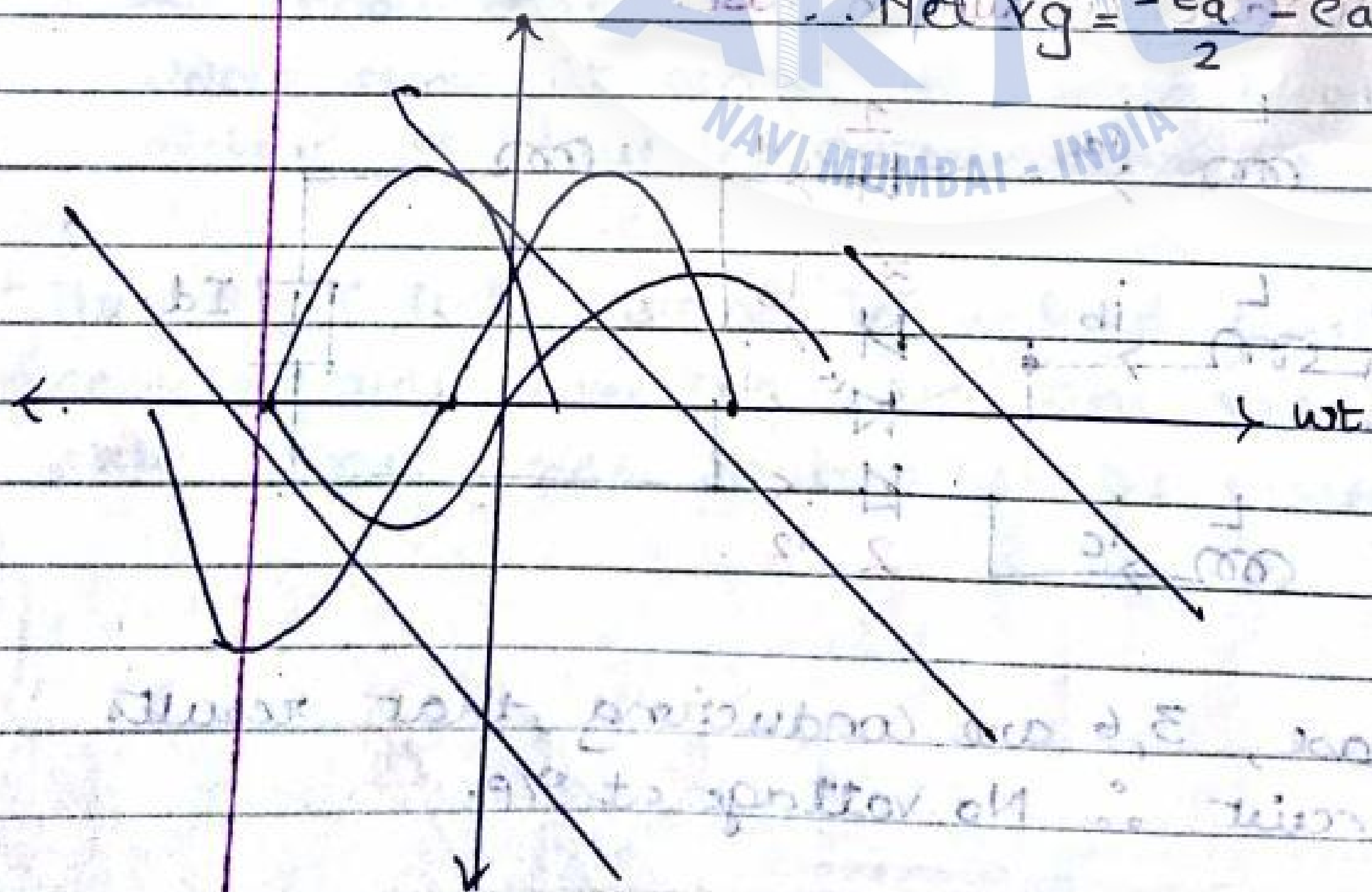
- In this condition 'α' cannot be less than 30°.
In 2-3 valve conduction mode α could be varied between 0-180°.

- Consider the sequence of conduction below.



- When 6, 1, 2 conducts;
~~The o/p vg at terminal 1 = ea.~~
~~Then e/p~~
→ Vg across cathode of '3' = e_a.
— " — Anode of '3' $= -e_a/2$.
∴ Net $V_g = -\frac{e_a}{2} - e_a = -1.5\, e_a$.



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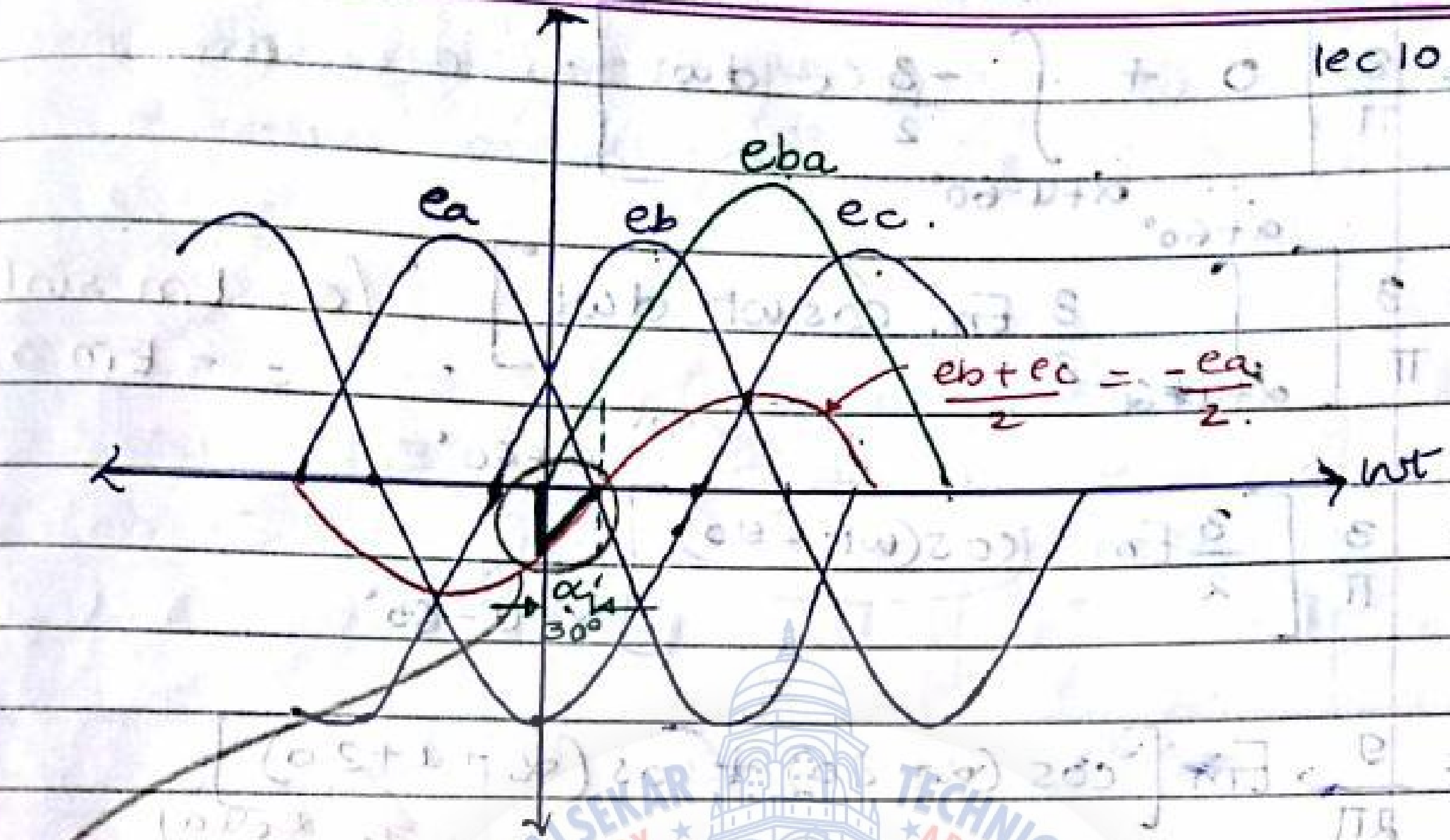
lec 10; 40:



It will not conduct since the voltage across '3' is '-ve' and only after 30° do the voltage becomes '+ve'

check valve conduction fig

$$Vd = \begin{cases} 0 & (\alpha < wt \leq (\alpha + u - 60°)) \\ -1.5\,ea & (\alpha + u - 60°) < wt \leq \alpha + 60° \end{cases}$$

4 valve conduction

3 valve conduction (ie, 1 2, 3 conduction)

o/p voltage is drastically low hence this mode is to be avoided.

$\cancel{\alpha} + 60 - \cancel{\alpha} - u + 60$
$(120 - u)$

$e_c = \cos($

$$-V_d = \frac{8}{\pi}\left[0 + \int_{\alpha+u-60^\circ}^{\alpha+60^\circ} \frac{-3e_c}{2}\, d\omega t\right]$$

$$= \frac{3}{\pi}\left[\int_{\alpha+u-60^\circ}^{\alpha+60^\circ} \frac{3}{2} E_m \cos\omega t \, d\omega t\right] \qquad (e_c = E_m \sin(\omega t - \pi/2) = -E_m \cos\omega t$$

$$= \frac{3}{\pi}\left[\frac{3}{2} E_m \overset{\sin\omega t}{\cos(\omega t - 90)}\right]_{\alpha+u-60^\circ}^{\alpha+60^\circ}$$

$$= \frac{9}{2\pi} E_m \left[\cos(\alpha - 30) + \cos(\alpha + u + 30)\right]$$

$$= \frac{\sqrt{3}}{2} V_{do} \left[\cos(\alpha - 30) + \cos(\underbrace{\alpha + u}_{\delta\,(\text{delta})} + 30^\circ)\right]$$

* O/P voltage waveforms in Rectification mode with 3-4 valve conduction

$u > 60^\circ$

$\alpha > 30^\circ$

Let's take, $\alpha = 45^\circ$, $u = 75^\circ$.

(lec 11 7:06)

Conduction valves	From	To	DC Voltage	Valve voltage '3'
6123	45° (+15) =	60°	0 (DC short ckt).	0
123	60° (+45)	= 105°	$-1.5e_c$.	0
1234	105° (+15)	120°.	0	0
234	120.	165	$1.5e_b$	0
2345	165	180.	0	0
345	180.	225	$-1.5e_a$.	0
3456.	225	240	0	0
456	240	285	$1.5e_c$	$-1.5e_c$.
4561	285	300	0	0 (S.C across all phases)
561.	300	345	$-1.5e_b$	$1.5e_b$.

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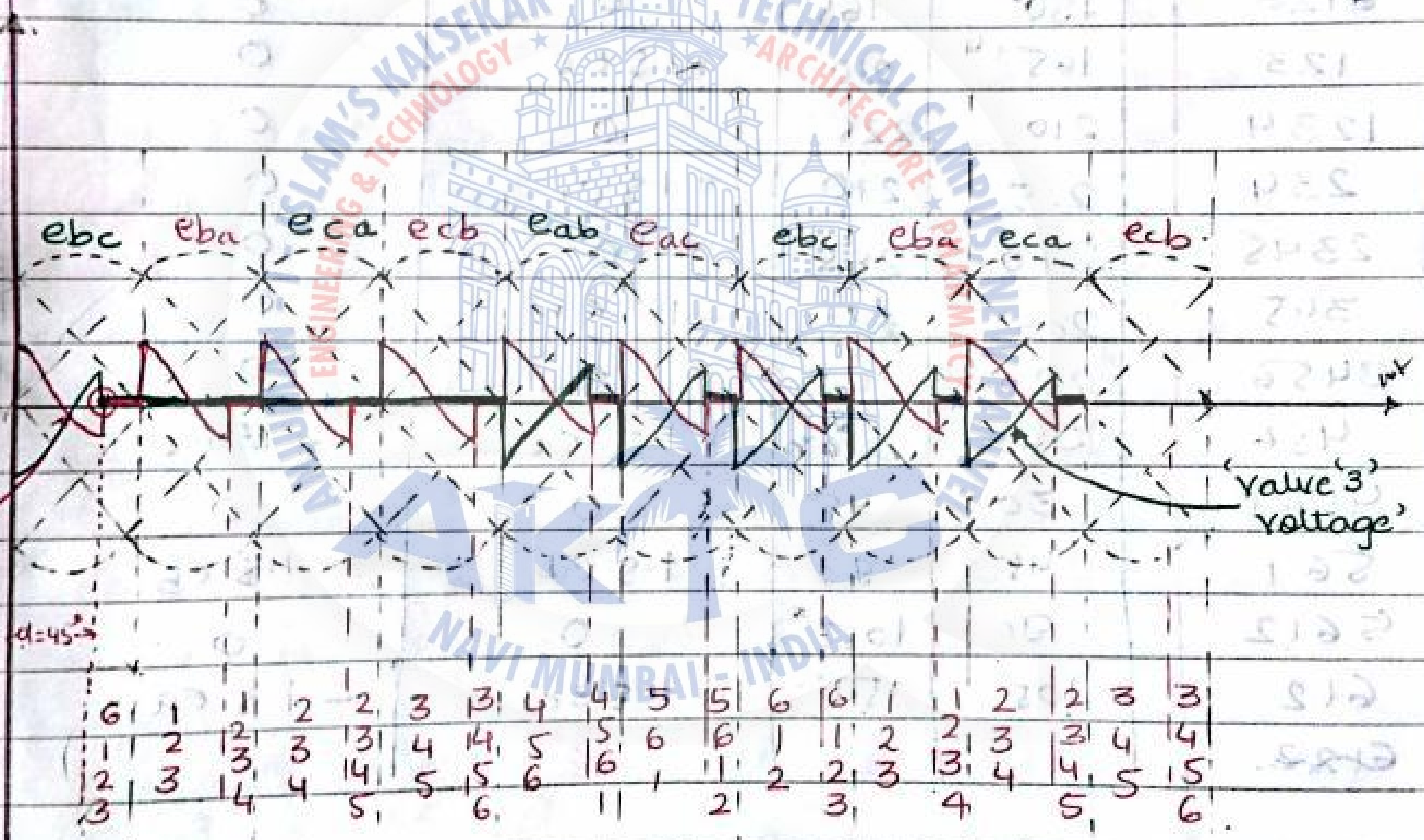
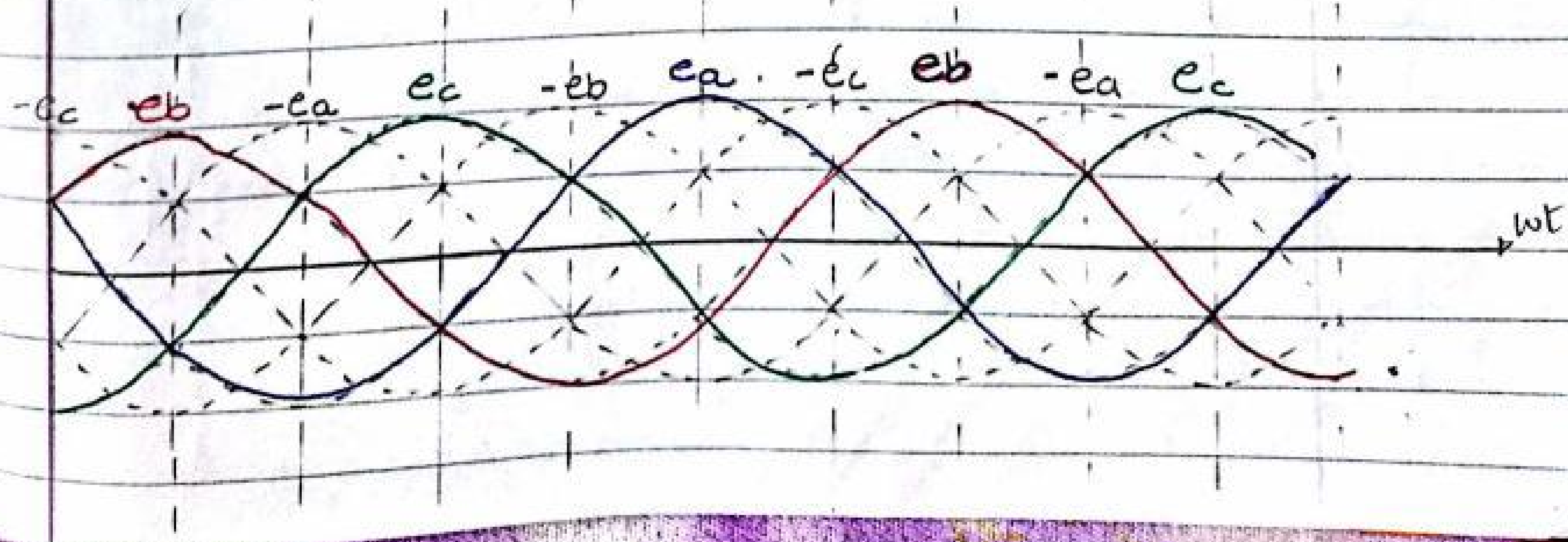
When 3 valves conduct – 45°
4 valves conduct – 15°

5 6 1 2	3 4 5	360	0.	0 (S.C across all phase)
6 1 2	3 6 Ø	45°	·1.5ea.	–1.5 ea.





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* O/P voltage waveforms for inversion mode with 3-4 valve conduction

$\alpha = 150^\circ$ | 3 valves — 45°
$u = 75^\circ$. | 4 valves — 15°.

Conduction Valves	From	To	DC voltage	Valve 3 Voltage
6123	$150^\circ + 15^\circ = $	165	0	0
123	$165 + 45 = $	210	$-1.5e_c$	0
1234	210	225	0	0
234	225	270	$+1.5e_b$	0
2345	270	285	0	0
345	285	330	$-1.5e_a$	0
3456	330	345	0	0
456	345	$360+30 = $ 30	$+1.5e_c$	$-1.5e_c$
4561	30	45	0	0
561	45	90	$-1.5e_b$	$1.5e_b$
5612	90	105	0	0
612	105	150°	$+1.5e_a$	$-1.5e_a$
~~6123~~.				

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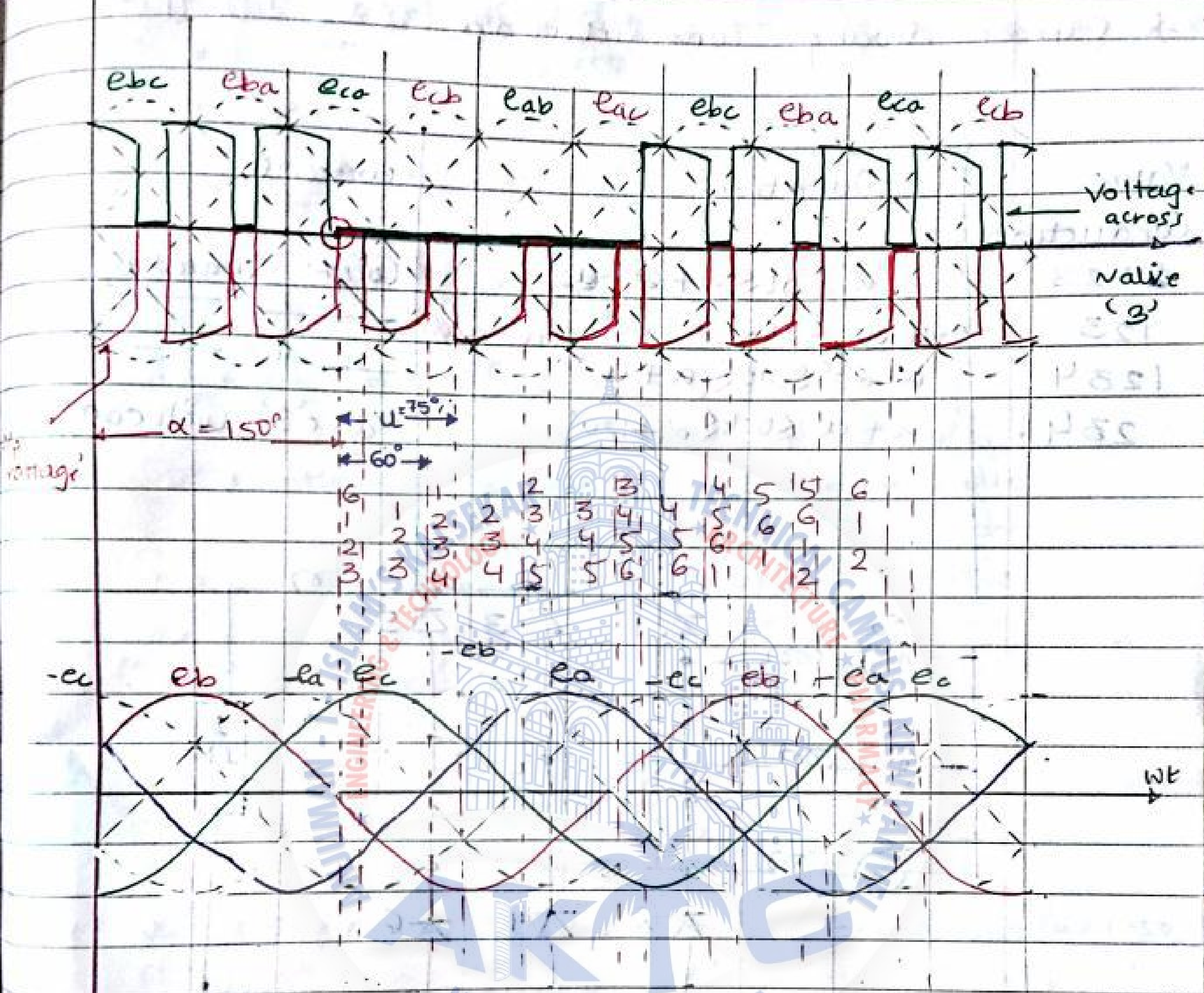
ebc
eba
eca
ecb
eab
eac
ebc
eba
eca
ecb
voltage across valve (3)
voltage
α = 150°
u = 75°
60°
-ec
eb
-ea
ec
-eb
ea
-ec
eb
-ea
ec
wt

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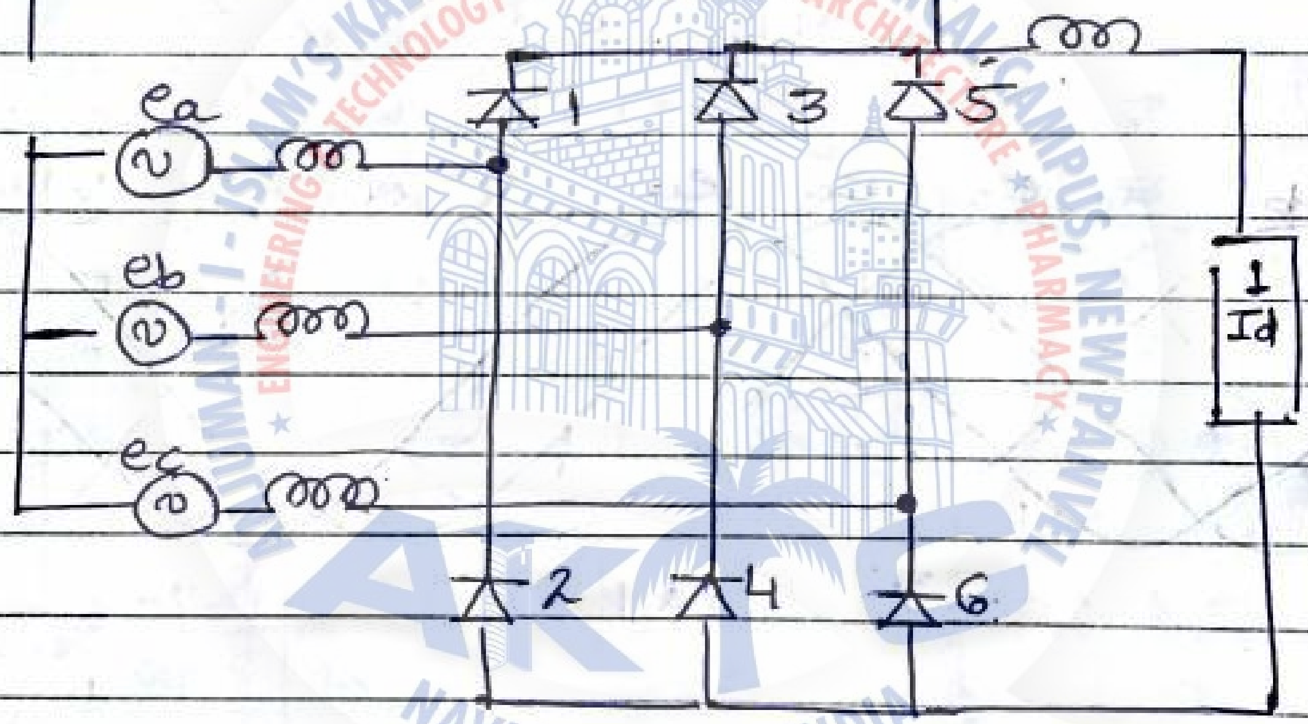
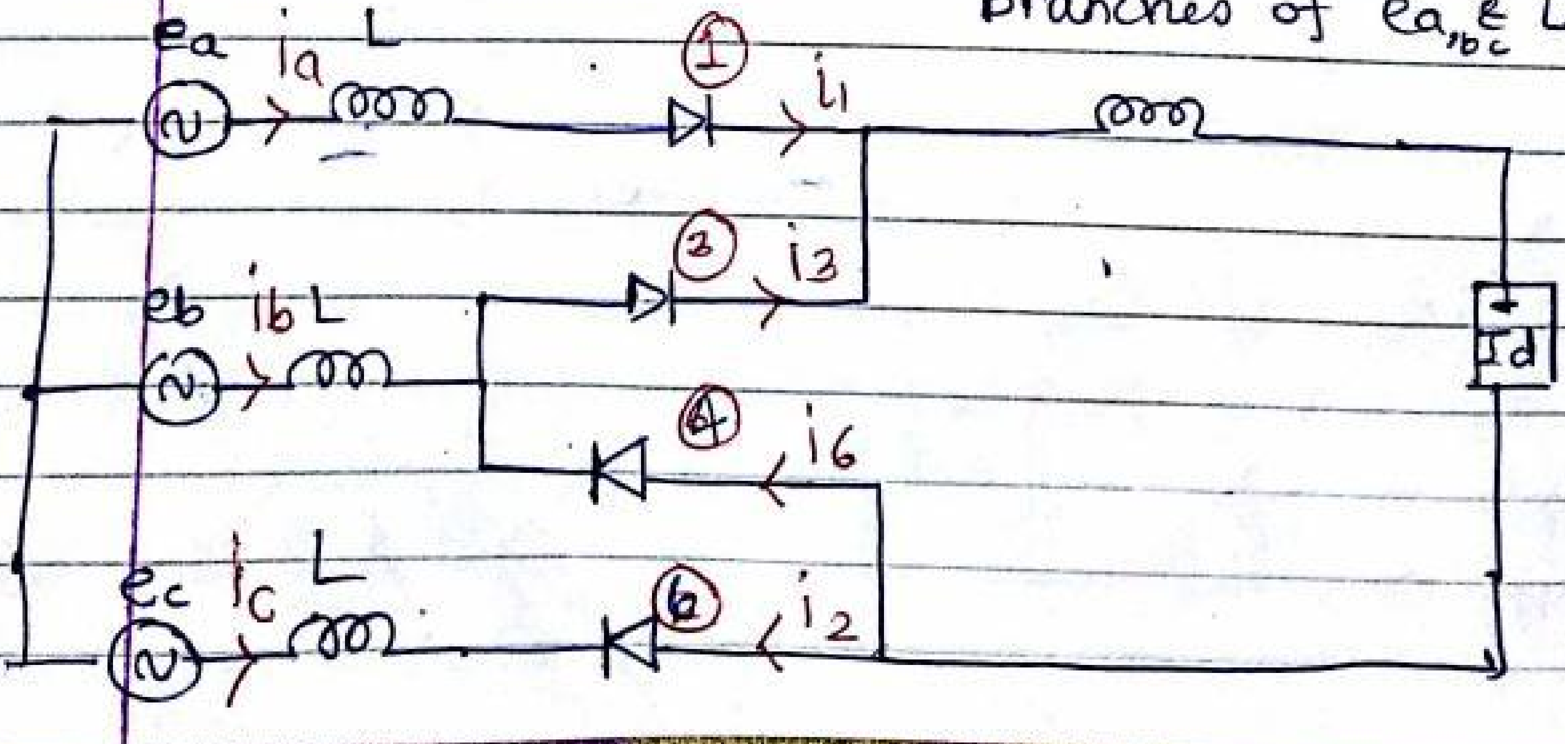
* Current during commutation; (lec11, 45:00)

Valve Conduction	Duration	Current
6123	$\alpha \leq \omega t \leq \alpha + u - 60^\circ$	Current shared by '1' & '3'
123	$\alpha + u - 60 \leq \omega t \leq \alpha + 60^\circ$ $\alpha + 60^\circ + (u - 60^\circ)$	"
1234	$\alpha + 60^\circ \leq \omega t \leq (\alpha + u)$	"
234.	$\alpha + u \leq \omega t \leq \alpha + 120^\circ$	Valve '3' will carry 'I_d'



• Consider conduction of 6123; (S.C will cause 3 parallel Branches of $e_{a,b,c}$ & L).



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$\rightarrow e_a - L\frac{di_a}{dt} = e_b - L\frac{di_b}{dt}$ —— ①

and;

$\rightarrow e_a - L\frac{di_a}{dt} = e_c - L\frac{di_c}{dt}$ —— ②

Adding ① & ②

$$2e_a - 2L\frac{di_a}{dt} = \underbrace{(e_b+e_c)}_{=-e_a} - L\underbrace{\left(\frac{di_b}{dt} + \frac{di_c}{dt}\right)}_{=\frac{d(-i_a)}{dt}}$$

$$2e_a - e_b - e_c = 2L\frac{di_a}{dt} - L\frac{di_b}{dt} - L\frac{di_c}{dt}$$

$$2e_a + e_a = 2L\frac{di_a}{dt} - L\frac{di_b}{dt} - L\frac{di_c}{dt}$$

$$3e_a = 2L\frac{di_a}{dt} + L\frac{di_a}{dt}$$

$$\therefore \cancel{3}e_a = \cancel{3}L\frac{di_a}{dt}$$

In fig $i_a = i_1$

$i_1 + i_3 = I_d$

$\therefore i_1 = -i_3 \quad (I_d = 0)$

$\frac{di_1}{dt} = -\frac{di_3}{dt}$

$$\rightarrow \frac{di_a}{dt} = \frac{e_a}{L} = \frac{di_a}{dt} = \frac{-di_3}{dt} = \frac{E_m \sin(\omega t + 150°)}{L}$$

e_a.

$\left[\because e_{ba} = \sqrt{3}E_m \sin\omega t \right.$

$\therefore$ ~~e_a ...~~

$e_a = E_m \sin(\omega t + 150)$

$$\rightarrow \frac{-di_3}{dt} = \frac{E_m}{L}\sin(\omega t + 150°)$$

(lec 11; 52:00]

$$\therefore i_3 = \int \frac{-E_m}{L}\sin(\omega t + 150).$$

$$= \frac{E_m}{\omega L}\left[\cos(\omega t + 150) - \cos(\alpha + 150)\right]$$

$$= \frac{E_m}{\omega L}\left[-\cos(\omega t - 30) + \cos(\alpha - 30)\right]$$

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$$\rightarrow I_3 (\text{at } \omega t = \alpha + u - 60) = \frac{E_m}{\omega L}\left[-\cos(\alpha + u - 90^\circ) + \cos(\alpha - 30)\right]$$

• When valves 1,2,3 are conducting; $\delta - 60 \leq \omega t \leq \alpha + 60$ ($\delta = \alpha + u$)

$$\rightarrow \frac{di_3}{dt} = \frac{e_b - e_a}{2L} = \frac{\sqrt{3}\,E_m}{2L} \sin \omega t \quad \text{(from previous derivation).}$$

$$\rightarrow i_3 = \frac{\sqrt{3}\,E_m}{2L} \int_{\delta-60}^{\omega t} \sin \omega t \, (d\omega t) + \underbrace{I_3 (\text{at } \delta - 60)}_{\text{initial current}}$$

$$\rightarrow i_3 = \frac{\sqrt{3}\,E_m}{2\omega L}\left[-\cos \omega t + \cos(\delta - 60)\right] + I_3 (\text{at } \delta - 60)$$

$$I_3 (\text{at } \omega t = \alpha + 60) = \frac{\sqrt{3}\,E_m}{2\omega L}\left[\cos(\delta - 90) - \cos(\alpha + 60)\right] + \frac{E_m}{\omega L}\left[-\cos(\delta - 90) + \cos(\alpha - 30)\right].$$

$$= \frac{E_m}{\omega L}\left[\cos(\alpha - 30) + \frac{1}{2}\cos(\delta + 30) - \frac{\sqrt{3}}{2}\cos(\alpha + 60)\right]$$

* when valves 1,2,3,4 are conducting (please note $i_1 \neq i_a$)

$$e_a - L\frac{di_a}{dt} = e_b - L\frac{di_b}{dt} \; ; \; e_b - L\frac{di_b}{dt} = e_c - L\frac{di_c}{dt}$$

$$\therefore 2e_b - e_a - e_c = 2L\frac{di_b}{dt} - L\frac{di_a}{dt} - L\frac{di_c}{dt}$$

$$e_b = L\frac{di_b}{dt} = L\frac{di_3}{dt} = E_m \sin(\omega t + 30^\circ)$$

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3 21

$$i_3 = \frac{E_m}{\omega L}\left[\cos((\alpha+60)+30) - \cos(\omega t + 30)\right] +$$

I_3 (at $\alpha + 60$)

6123 → α
123 → $\alpha + 60$
1234 → ωt
60°

at $\omega t = \delta$

$$I_d = I_3 = \frac{E_m}{2\omega L}\left[\cos(\alpha - 30) - \cos(\delta + 30)\right]$$

For $u > 60$

We want to express V_d & I_d without 'u' term;
We know that for $(u < 60)$ we have the following expression;

$$V_d = \frac{V_{do}}{2}(\cos\alpha + \cos\delta)$$

$$V_{do} = \frac{3\sqrt{3}}{\pi} E_m$$

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• Relation between Current & Voltage:

$$V_d = \frac{9E_m}{2\omega L}\left[\cos(\alpha - 30) + \cos(\delta + 30)\right]$$

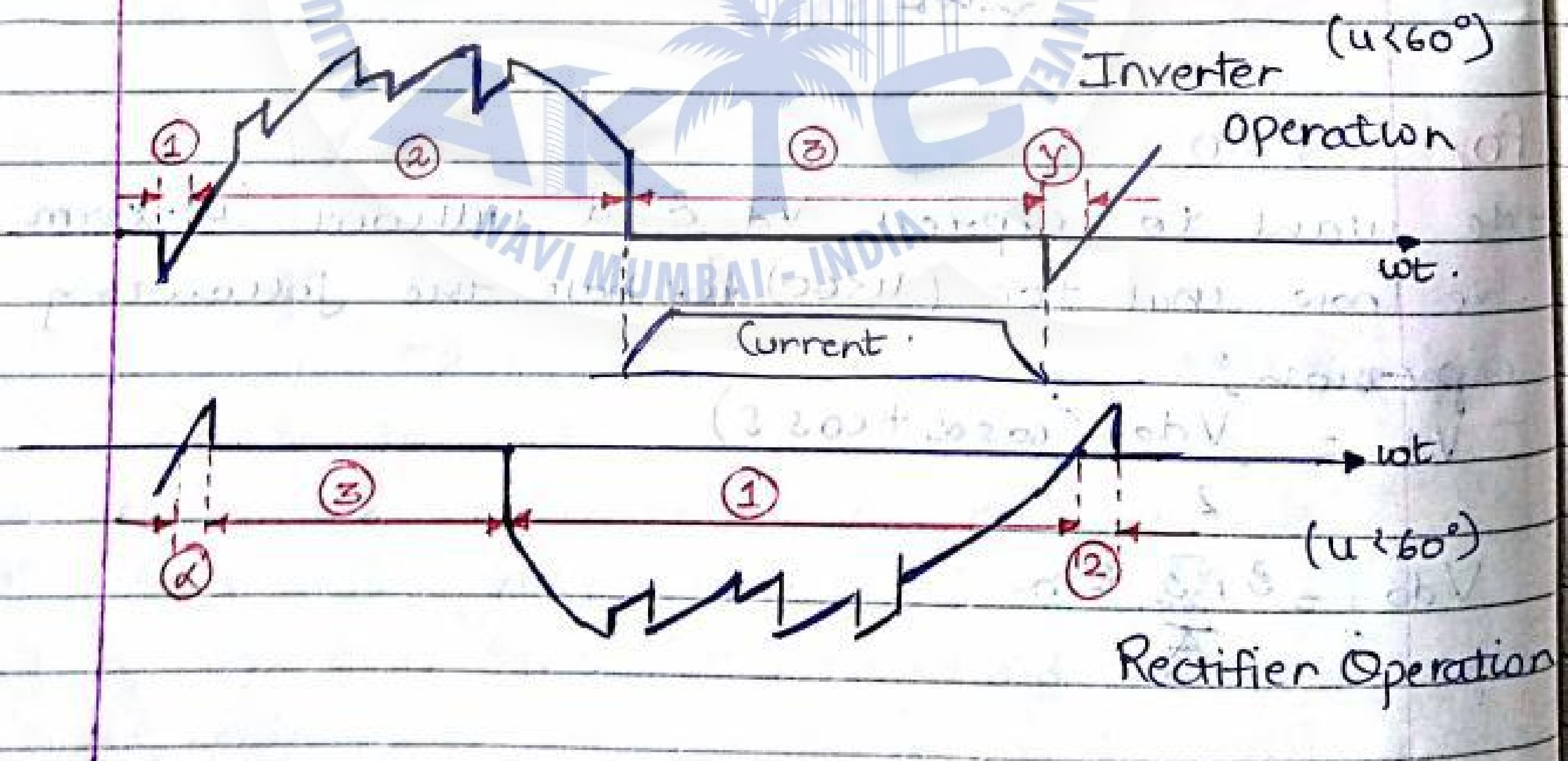
$$I_d = \frac{E_m}{2\omega L}\left[\cos(\alpha - 30) - (\cos(\delta + 30))\right]$$

shift terms of this eq to get in term of 'Id'. Then substitute in above 'Vd' to get below 'Vd'

$$V_d = \sqrt{3}\, V_{do} \cos(\alpha - 30) - 3 R_c I_d$$

$$\because R_c = \frac{3\omega L}{\pi}$$

* Concept of Extinction Advance angle & Commutation margin angle



• Valve Voltages

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- Extinction Advance angle (γ) is the time angle between the end of conduction & the reversal of sign of sinusoidal commutation voltage of the source.

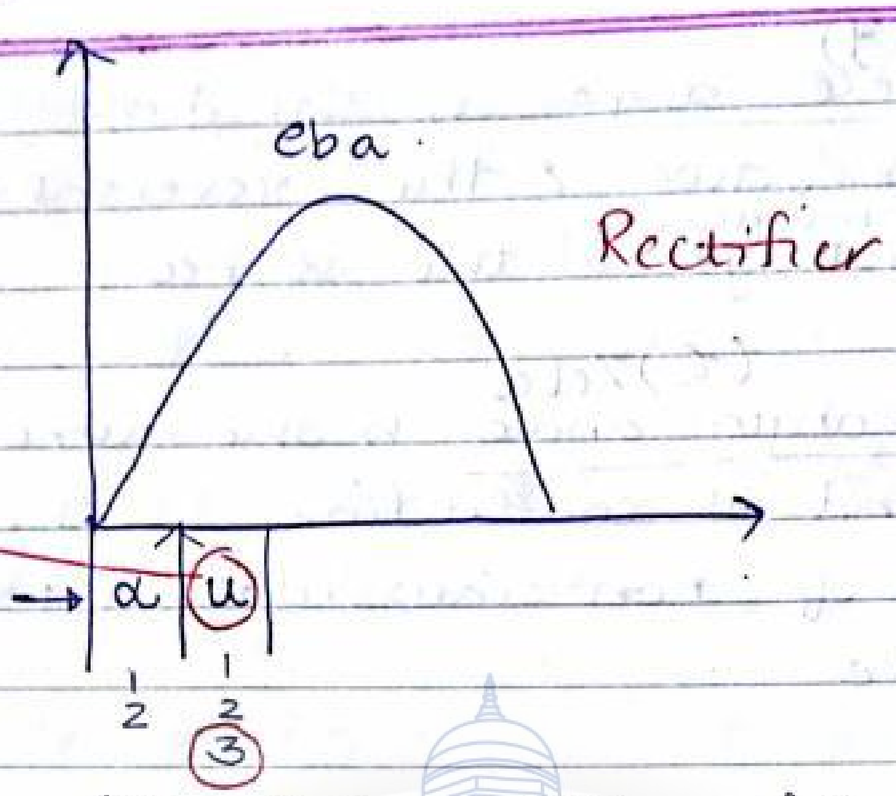
- Commutation Margin angle (ζ) Zeta. is the time angle between the end of conduction & the reversal of the sign of non-sinusoidal voltage across outgoing valve.

- The inverter is very much prone to commutation failure. In rectifier, most of the time the valve voltage is -ve. If there is any spurious signal appearing at the gate of thyristor, it cant conduct. The positive portion of wave is very small.

- The reverse happens in inverter operation. Most of the time valve voltage is positive. So due to improper commutation, the valve which is conducting will continue to conduct. This is the commutation failure.

For eg; commutation between 1 & 3 is taking place. Current in 1 is ↓ and in 3 is ↑ but due to reversal of voltage across 3 the current begins to flow back to 1 & so the commutation may not occur.

(1)

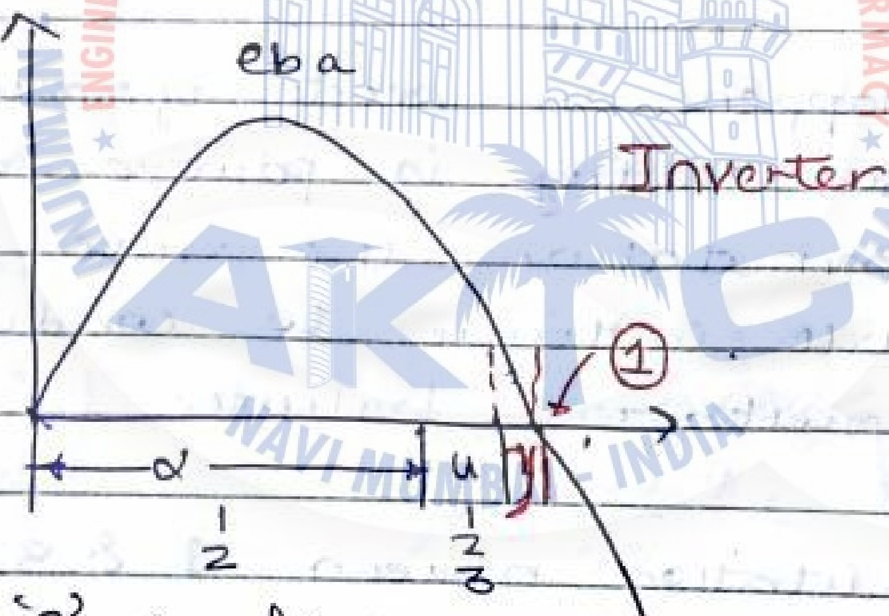


(thus period may enlarge but it comes under eba wave & doesnt go beyond)

Commutation voltage of valve 3.

- If valve '3' is fired at lower value of 'α' as shown above it has enough time to turn off since 'u' period can be more.

(2)



- If valve '3' is fired at higer value of 'α' 'u' period may extend beyond point 1 where voltage gets reversed & current flow between the commutating valves is reversed & results in commutation failure.
- Thus the angle 'γ' is to be maintained constant.