Module: 1

LOAD FLOW STUDIES

- Introduction to load flow study
- Network model formulation
- Ybus formation by singular transformation method
- Load flow problem
- Gauss Siedal Method
- Newton Raphson Method (Polar form & Rectangular form)
- Decoupled Load flow method
- Fast decoupled load flow method
- Concepts of DC load flow

1. Introduction to load flow study

- To study the operational features of a composite power system is symmetrical steady state which gives the most important mode of operation of a power system
- Three major problems encountered in this mode of operation are listed below in the hierachial order
  1. Load flow problem
  2. Optimal load scheduling problem
  3. System control problem.
- Load flow study in power system parlance is the steady state solution of the power system network.
- The main information obtained from this study comprises the magnitudes, phase angles of load bus voltages, reactive powers at the generator buses, real and reactive power flow on transmission lines, and other variables being specified.
- This information is essential for the continuous monitoring of the current state of the system for analzing the effictivness of alternative plans for future system expansion to meet the increased load demand.

Power System Operation Corporation Ltd.

- National Load Dispatch Centre (Katwaria New Delhi)
- Five Regional load Dispatch Centre

→ National Regional Load dispatch centre
→ Western Regional load dispatch centre
→ Eastern Regional load dispatch centre
→ Southern Regional load dispatch Centre
→ North-Eastern Regional load dispatch Centre

1.1 Modeling of power system components.

- Basically an AC transmission grid consists of (i) synchronous generators (ii) loads, (iii) transformers & (iv) transmission lines.

(i) Synchronous generators : For the purpose of power flow solution synchronous generators are not represent explicitly, rather than their presence is in implicitly modeled.

(ii) loads : Loads can be classified into three categories (i) constant power (ii) constant impedance and (iii) constant current.
However within the normal operating range of the voltage almost all the loads behave as constant power loads. Hence at any bus the real and reactive power loads are specified respectively.

(iii) Transformer : For power system steady state and fault studies, generally the existing current of the transformer is neglected as it is quite low compare to the normal load current flowing through the transformer.

- A two winding transformer connected between buses 'i' and 'j' is represented by its per unit leakage impedance as shown in fig 1.1 (a)

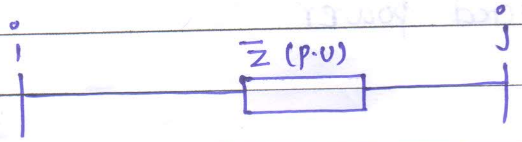


fig 1.1 (a) Equivalent circuit of a two winding transformer.

- The transformer tap ratio for fig 1.1 (a) is 1:1.
- For a regulating transformer with the transformer ratio 1:t the equivalent circuit of the transformer is shown in fig 1.1 (b)

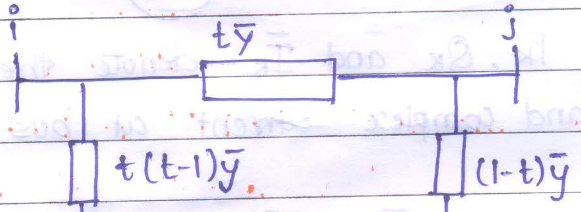


fig 1.1 (b) Equivalent circuit of a regulating transformes with a transformation ratio 1:t

- At a time the transformer ratio is also represented as a:1 the equivalent circuit for this is as shown in fig 1.1 (c)

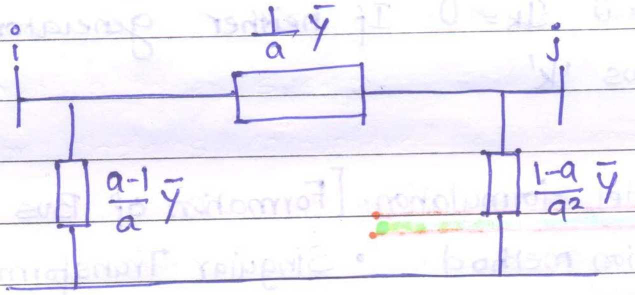
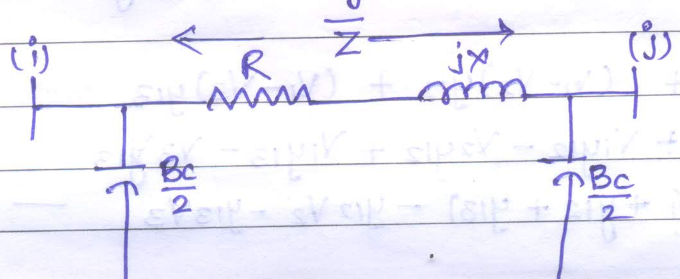


fig 1.1 (c) Equivalent circuit of a regulating transformer with transformation ratio a:1

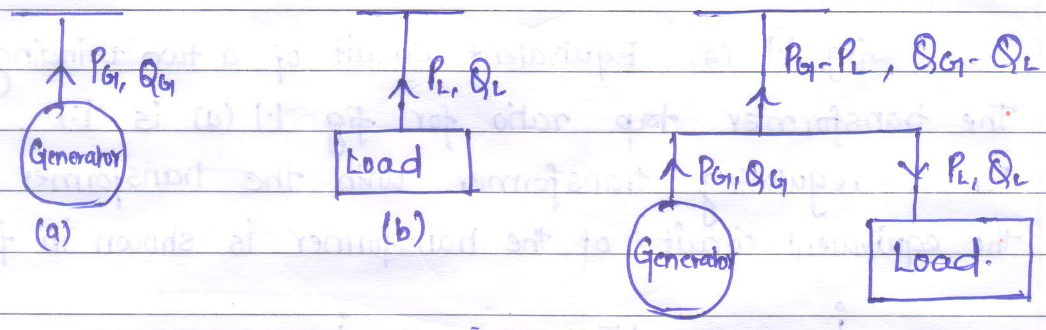
Note: fig 1.1 (b) and fig 1.1 (c) the quantities '(a)' and '(t)' are real (ie. the transformer is changing only the voltage magnitude, not its angle)

(iv) Transmission line: In a transmission grid, the transmission lines are generally of medium length is always represented by the nominal π model as shown in fig 1.1 (d) where Z= total series impedance of the line



Bc= total shunt charging susceptance of the line.

Illustration of injected power



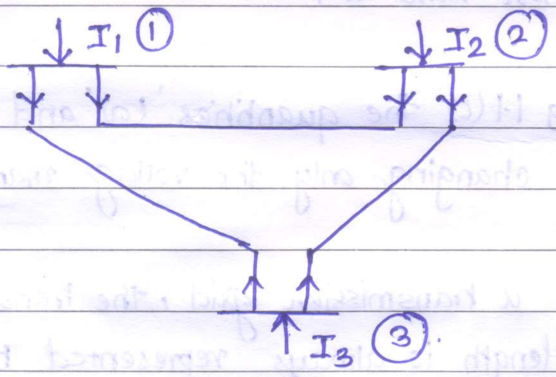
To summarize if P_K, Q_K and $\bar{I}_K$ denote the injected power, reactive power and complex current at bus 'k' respectively

- $P_K = P_G$ $Q_K = Q_G$ and $\bar{I}_K = +\bar{I}_G$ if only a generator is connected to the bus 'k'
- $P_K = -P_L$ $Q_K = -Q_L$ and $I_K = -I_L$ if only a load is connected to the bus 'k'
- $P_K = P_G - P_L$ $Q_K = Q_G - Q_L$ and $\bar{I}_K = \bar{I}_G - \bar{I}_L$ if both generator and load are connected to the bus 'k'
- $P_K = 0$ $Q_K = 0$ $I_K = 0$ If neither generator nor load is connected to the bus 'k'

2. **Network model formulation** [Formation of Bus Admittance Matrix Y_{Bus}]
   - Direct inspection method
   - Singular Transformation Method.

Direct Inspection Method

Let us consider 3 bus system as shown in fig 2.1



let I_1, I_2, I_3 denote the current flowing into the buses.

$$I_1 = y_{11}V_1 + (V_1 - V_2)y_{12} + (V_1 - V_3)y_{13}$$
$$= y_{11}V_1 + V_1 y_{12} - V_2 y_{12} + V_1 y_{13} - V_3 y_{13}$$
$$= V_1(y_{11} + y_{12} + y_{13}) - y_{12}V_2 - y_{13}V_3 \quad \text{—} \quad (1)$$

$$I_2 = V_2 y_{22} + (V_2 - V_1) y_{21} + (V_2 - V_3) y_{23}$$
$$= V_2 y_{22} + V_2 y_{21} - V_1 y_{21} + V_2 y_{23} - V_3 y_{23}$$
$$= -V_1 y_{21} + V_2 (\cancel{y_{22}} + y_{21} + y_{23}) - V_3 y_{23} \quad \text{—— (2)}$$

$$I_3 = V_3 y_{33} + (V_3 - V_1) y_{31} + (V_3 - V_2) y_{32}$$
$$= -V_1 y_{31} - V_2 y_{32} + V_3 (y_{31} + y_{32}) \quad \text{—— (3)}$$

Putting eq (1), (2) and (3) in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} (y_{12}+y_{13}) & -y_{12} & -y_{13} \\ -y_{21} & (y_{21}+y_{23}) & -y_{23} \\ -y_{31} & -y_{32} & (y_{31}+y_{32}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$I_{BUS} = Y_{BUS} \cdot V_{BUS}$$

Mutual Admittance matrix

where $Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$

Mutual Admittance matrix

Self Admittance matrix

$Y_{11} = (y_{12} + y_{13})$ $\quad Y_{12} = -y_{12}$ $\quad Y_{13} = -y_{13}$

$Y_{21} = -y_{21}$ $\quad Y_{22} = (y_{21} + y_{23})$ $\quad Y_{23} = -y_{23}$

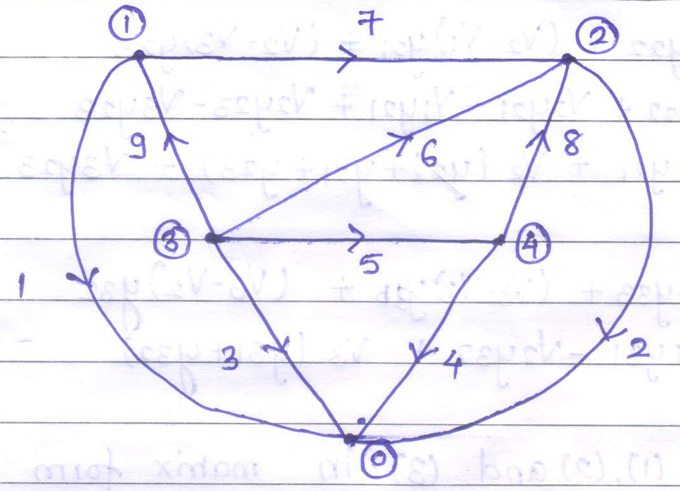
$Y_{31} = -y_{31}$ $\quad Y_{32} = -y_{32}$ $\quad Y_{33} = (y_{31} + y_{32})$

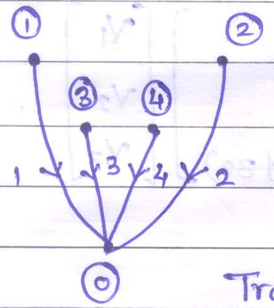
Singular Transformation Method.

Formation of Y_{bus} by singular Transformation method.

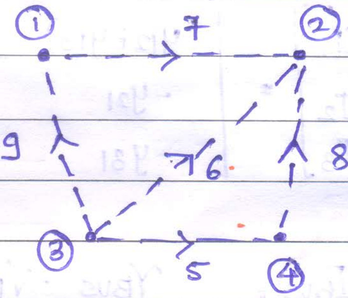
This approach has got great theoretical and practical significance.

Graph : To describe the geometrical features of a network it is replaced by a single segments called elements whose terminals are called nodes.





Tree



Co-tree

——— Branch

- - - - Link

e = element = 9

b = buses = $m-1$ = 4

l = link = $e-b$ = $9-4$ = 5

m = total number of nodes = 5

Primitive network

- A network element may be in general contain active and passive components

The fig 2.2(a) and (b) shows the alternative impedance and admittance form representation of a general network element

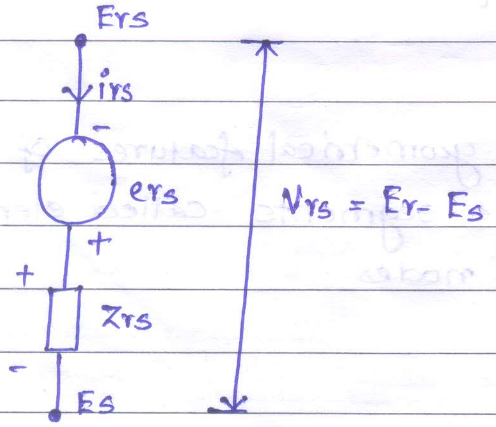


fig 2.2(a) Impedance form

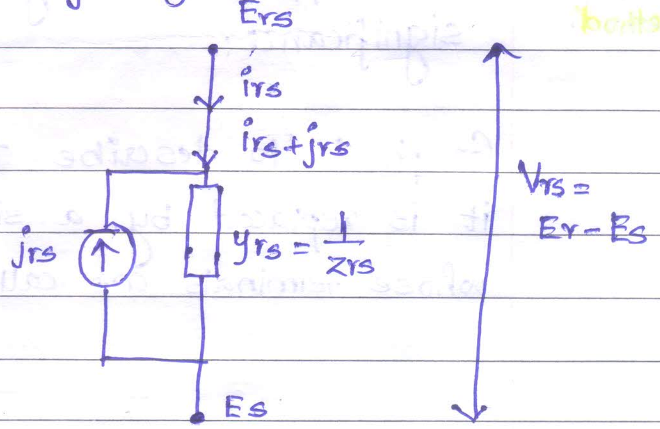


fig 2.2 (b) Admittance form

- The impedance form is a voltage source e_{rs} in series with impedance z_{rs}
- The admittance form is a current source j_{rs} in parallel with admittance y_{rs}
- The element current is i_{rs} and element voltage $v_{rs} = E_r - E_s$ where E_r and E_s are the voltages of the element nodes are r and s respectively.

Voltage relation form fig 2.2 (a)

$$v_{rs} + e_{rs} = z_{rs} \cdot i_{rs} \quad — (1)$$

Current relation form fig 2.2 (b)

$$i_{rs} + j_{rs} = y_{rs} \cdot v_{rs} \quad — (2).$$

$$j_{rs} = -y_{rs}\, e_{rs}$$

Also $y_{rs} = \dfrac{1}{z_{rs}}$

In impedance form

$$V + E = ZI \quad — (3)$$

In admittance form

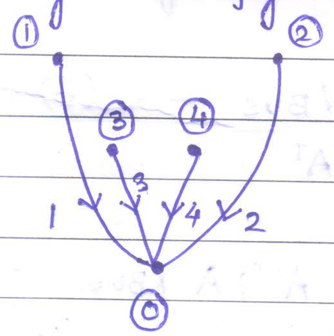
$$I + J = YV \quad — (4)$$

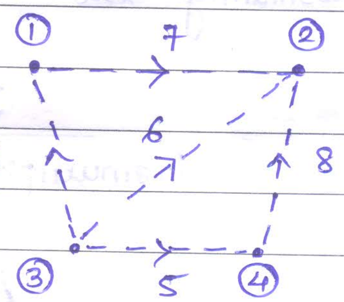
V = voltage element and I = current relation.

J and E source vectors

Z and Y - primitive impedance and admittance matrices.

Considering the fig





$$V_{b1} = V_1$$

$$V_{b2} = V_2$$

$$V_{b3} = V_3$$

$$V_{b4} = V_4$$

$$V_{15} = V_3 - V_4$$

$$V_{16} = V_3 - V_2$$
$$V_{17} = V_1 - V_2$$
$$V_{18} = V_4 - V_2$$
$$V_{19} = V_3 - V_1$$

or in matrix form

$$V = AV_{BUS} \quad \text{--- (5)}$$

where V is the vector of element voltages of order $e \times 1$
(e = numbers of elements)
V_{BUS} is the vector of bus voltages of order $b \times 1$
(b = numbers of branches = numbers of buses = n)
A is the bus incidence matrix of order $e \times b$ given by.

elements ↓ \ Bus →	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	0	0	1	-1
6	0	-1	1	0
7	1	-1	0	0
8	0	-1	0	1
9	-1	0	1	0

a_{ik} = 1 if i^{th} element is incident to and oriented away from k^{th} node (bus)
= −1 if i^{th} element is incident to but oriented towards the k^{th} node (bus)
= 0 if the i^{th} element is not incident to the k^{th} node

Substituting the value of $V = AV_{BUS}$ from eq^n (5) into eq^n (4)

$$I + J = YAV_{BUS} \quad \text{--- (6)}$$

Premultiplying by A^T

$$A^TI + A^TJ = A^TYAV_{BUS} \quad \text{--- (7)}$$

Since $A^TI = 0$ — (8) and $A^TJ = J_{BUS}$ — (9)
Substitute eq (8) and eq (9) into eq^n (7)

$$J_{BUS} = (A^TYA)V_{BUS}.$$

~~Similarly~~ where

$$\boxed{Y_{BUS} = A^TYA}$$

3. Load flow Problem

The complex power injected by the source into the i^{th} bus of a power system is given as

$$\boxed{S_i = P_i + jQ_i = V_i I_i^* \qquad i = 1, 2, 3, \ldots n} \quad -(1)$$

The complex conjugate of eq^n (1) is as follows

$$\boxed{S_i = P_i - jQ_i = V_i^* I_i \qquad i = 1, 2, 3 \ldots n} \quad -(2)$$

We know that

The relationship between node current and voltage in the linear network can be described by the following node equation

$$\boxed{I_i = \sum_{k=1}^{n} Y_{ik} V_k \qquad i = 1, 2, 3 \ldots n} \quad -(3)$$

Substituting the value of I_i in eq^n (2) we get

$$\boxed{P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k} \quad -(4)$$

Hence basically the real power and reactive power will be as follows.

$$\boxed{P_i = \text{real}\left[V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right]} \quad -(5)$$

$$\boxed{Q_i = -\text{Imag}\left[V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right]} \quad -(6)$$

• The voltage at a typical bus i of the system can be given as follows

$$\boxed{V_i = \underbrace{|V_i| e^{j\delta_i}}_{\text{exponential form}} = \underbrace{|V_i| \angle \delta_i}_{\text{polar form}} = \underbrace{|V_i| (\cos\delta_i + j\sin\delta_i)}_{\text{rectangular form}}}$$

Y_{ik} an element of the admittance matrix is given by

$$Y_{ik} = |Y_{ik}| e^{j\theta_{ik}} = |Y_{ik}| \angle \theta_{ik} = |Y_{ik}| \cos\theta_{ik} + j|Y_{ik}| \sin\theta_{ik}$$
$$= G_{ik} + jB_{ik}$$

Note =
★★

→ [where θ = close to $90°$ because resistance and conductance are always negligible in power system]

→ [If they are not then the power system is said to be ill condition power system]

Now the power flow equation can be written as

$$\text{Real power } P_i = |V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$\text{Reactive power } Q_i = -|V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$$

- These above 2 equations are important which are termed as static load flow equations.
- Hence we have basically 2n Power flow equations and each bus has 4 variables resulting into 4n variables P_i, Q_i, V_i & δ_i
- Practical considerations allow a power system analyst to fix a priori two variables
- For this purpose, the buses in a system are classified into three categories and in each category, two different quantities are specified.

- PQ Bus

- At these buses loads are connected and therefore these buses are also termed as load buses.
- Generally the values of loads (real and reactive) connected at these buses are known and hence at these buses P_i and Q_i are specified (or known)
- Consequently V_i and θ_i need to be calculated for these buses.

PV Bus

- Any bus where the voltage magnitude can be held constant at a specified value is called voltage controlled bus
- At generator bus it is possible to maintain the terminal voltage at the specified value by adjusting the excitation of generator.
- At the generator bus the active power generation can be controlled by adjusting the mechanical power input. Hence at a generator bus $|V_i|$ and P_{Gi} are specified.
- The variables to be determined are Q_i and δ_i

Slack bus

- Real and reactive power are not specified but the voltage magnitude and phase angle are specified.
- In a load flow study real and reactive power cannot be fixed in priori at all the buses as the net complex power flow into the network is not known in advance, the system power loss being unknown till the load flow study is complete.
- It is therefore, necessary to have one bus (ie the slack bus) at which the complex power is unspecified so that it supplies the difference in the total system load plus losses and the sum of the complex powers specified at the ~~remaing~~ remaining buses.

Operating Constraints

- Voltage magnitude must satisfy the inequality constraint

$$|V_{min}| \leq |V_i| \leq |V_{max}|$$

- Active power generation must satisfy the following inequality constraint

$$P_{Gi,min} \leq P_{Gi} \leq P_{Gimax}$$

- Limitations of the generator field excitation, constraints the reactive power generation

$$Q_{Gimin} \leq Q_{Gi} \leq Q_{Gimax}.$$

- Constraints on the maximum permissible power angle between two buses i and j connected via a transmission line

$$|\delta_i - \delta_j| \leq |\delta_i - \delta_j|_{max}$$

- The complex power has to be conserved at all buses

$$\sum_i P_{Gi} = \sum_i P_{Di} + P_L$$

$$\sum_i Q_{Gi} = \sum_i Q_{Di} + Q_L$$

where P_L and Q_L are the total system real & reactive power loss respectively.

Load Flow Methods

- Gauss Siedal Method.
- Newton Raphson Method
- Decoupled Load Flow Method.
- Fast decoupled load flow Method.

- Gauss Siedal Method when PQ buses are present

At a PQ bus - P and Q values are specified at this bus the quantities to be measured are determined are $|V_i|$ & δ_i

$$P_i - jQ_i = V_i^* \sum_{K=1}^{n} Y_{iK} V_K$$

$$P_i - jQ_i = V_i^* \left[Y_{ii} V_i + \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K \right]$$

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K$$

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k\right]$$

during the iterative procedure use the latest values of variables on the right hand side.

- The iterative procedure is continued till no further improvement in voltage is achieved ie.

$$|V_i^{r+1} - V_i^{r}|$$

Gauss Siedal Method when PV buses are present:

At this bus the variables to be determined are Q_i and δ_i

$$\boxed{Q_i = -\mathrm{Imag}\left\{V_i^* \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k + (V_i)^* \sum_{k=i} Y_{ik}V_k\right\}}$$

$$\boxed{\delta_i = \text{angle of } \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1\\k\neq i}}^{n} Y_{ik}V_k\right].}$$

Acceleration of Convergence (α)

- Convergence in the Gauss Siedal method can be speeded up the use of acceleration factor.
- For i^{th} bus, the accelerated value of voltage at the $(r+1)^{th}$ iteration is given by

$$V_i^{(r+1)}(\text{accelerated}) = V_i^{(r)} + \alpha(V_i^{(r+1)} - V_i^{(r)})$$

where α is a real number called as acceleration factor

Generally the recommended value for α is 1.6

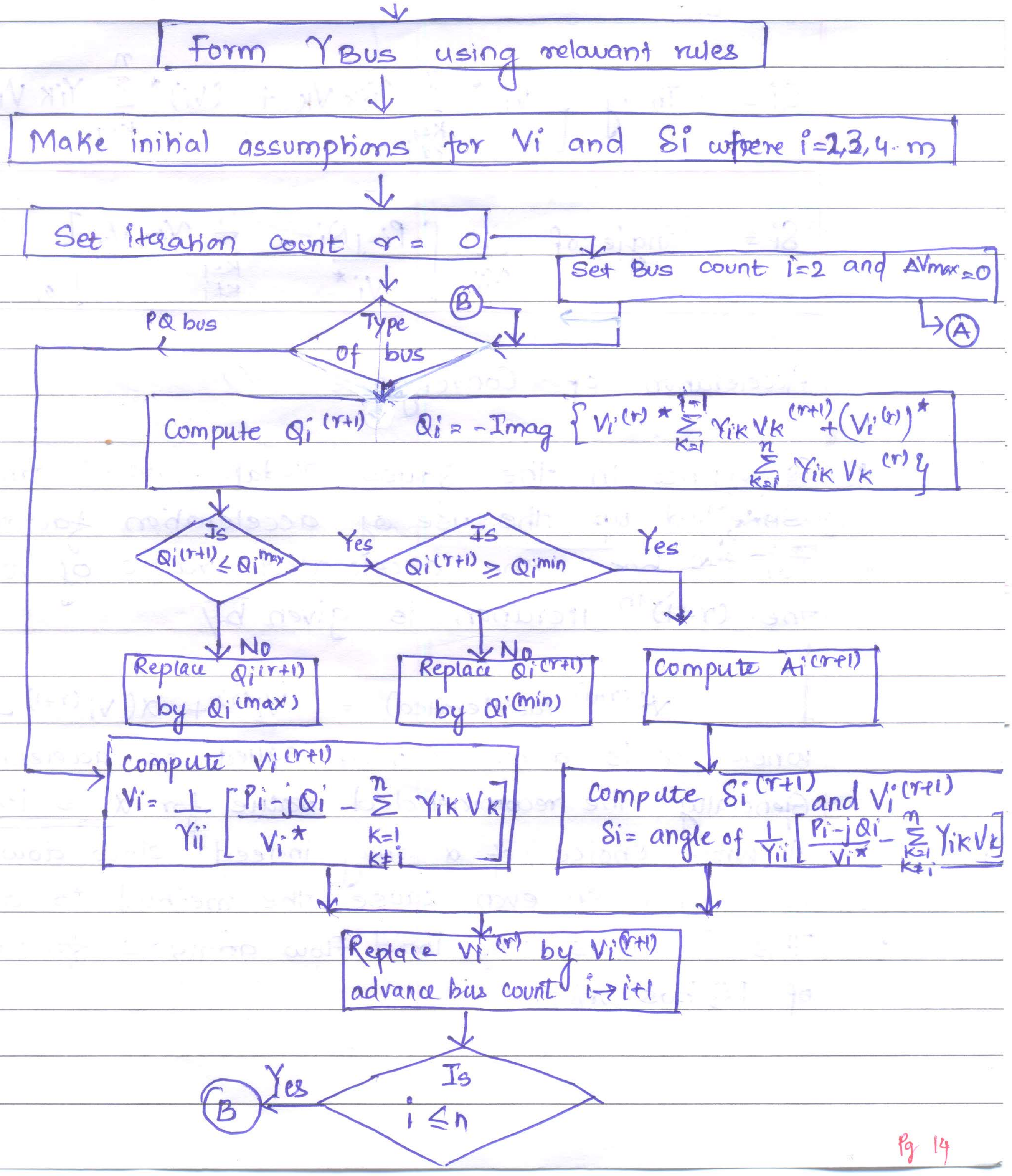
- A wrong choice of α may indeed slow down convergence or even cause the method to diverge.
- This concludes the load flow analysis for the case of PQ bus only.

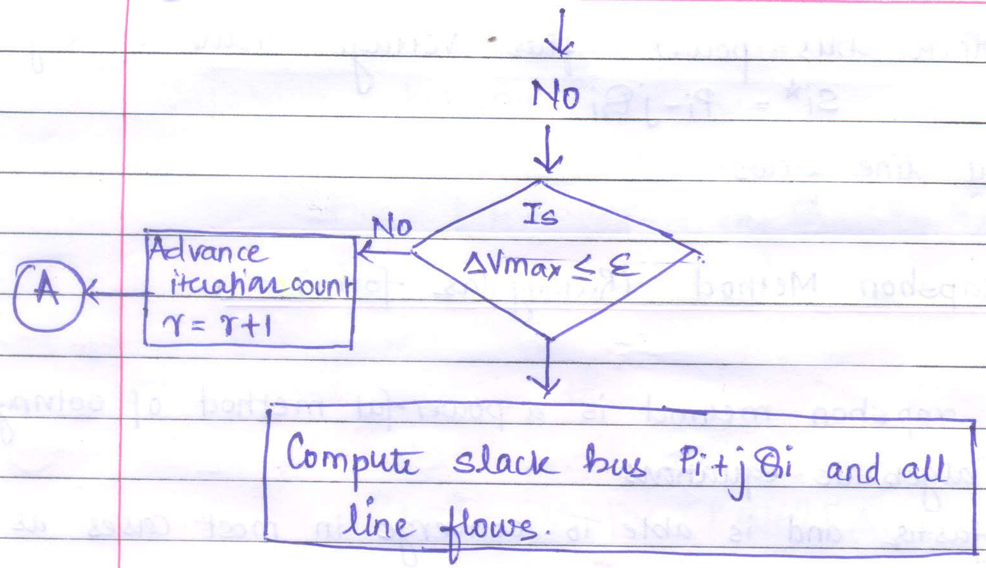
14

Flow chart for load flow solution by the Gauss Siedal iterative using Y_{Bus}.

Read

1. Primitive Y matrix
2. Bus incidence matrix A
3. Slack bus voltage $(|V_1|, \delta_1)$
4. Real bus powers P_i for $i = 2, 3, 4 \cdots n$
5. Reactive bus powers Q_i for $i = m+1 \cdots n$ (PQ Buses)
6. Voltage magnitudes $|V_i^s|$ for $i = 2 \cdots, m$ (PV Bus)
7. Voltage magnitude limits $|V_i|_{min}$ and $|V_i|_{max}$ for PQ buses.
8. Reactive power limits $Q_{i\,min}$ and $Q_{i\,max}$ for PV buses.


Form Y Bus using relavant rules
Make initial assumptions for Vi and δi where i=2,3,4..n
Set iteration count r = 0
Set Bus count i=2 and ΔVmax=0
A
B
Type of bus
PQ bus
Compute Qi (r+1)   Qi = -Imag { Vi(r)* Σ Yik Vk (r+1) + (Vi(r))* Σ Yik Vk (r) }
Is Qi(r+1) ≤ Qi max
Yes
Is Qi(r+1) ≥ Qi min
Yes
No
Replace Qi(r+1) by Qi (max)
No
Replace Qi(r+1) by Qi (min)
Compute Ai(r+1)
compute Vi(r+1)
Vi = 1/Yii [(Pi - jQi)/Vi* - Σ Yik Vk]
Compute δi(r+1) and Vi(r+1)
δi = angle of 1/Yii [(Pi - jQi)/Vi* - Σ Yik Vk]
Replace Vi(r) by Vi(r+1)
advance bus count i→i+1
Is i ≤ n
Yes
B



Algorithm for Gauss Siedal Method.

1. Prepare data which includes number of buses, number of loads slack bus, generator bus data, transmission line data and transformer data

2. Formulate the bus admittance matrix Ybus which is generally done by inspection method.

3. Assume initial voltages for all buses, 2, .. n. In practical power systems, the magnitude of the bus voltage is close to 1.0 pu. Hence the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\angle 0^\circ$. This is normally called as flat start

4. Update the voltages. In any $(r+1)^{th}$ iteration the voltages are given by.

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{\substack{k=1 \\ k\neq i}}^{i-1} Y_{ik} V_k^{(r+1)}\right] \qquad i = 2, 3, \dots n$$

Here note that when computations is carried out for i^{th} bus updated values are already available buses 2, 3, .. (i-1) in the current $(r+1)^{th}$ iteration.

5. Continue iterations till

$$|\Delta V_i^{(r+1)}| = |V_i^{(r+1)}| - |V_i^{(r)}| < \alpha \qquad i = 1, 2, 3, n.$$

α = tolerance value.

6. Compute slack bus power after voltages have converged.

$$S_i^* = P_i - jQ_i$$

7. Compute all line flows.

Newton Rapshon Method (Rectangular form)

- The newton rapshon method is a powerful method of solving non-linear algebraic equations.
- It works faster and is able to converge in most cases as compared to Gauss Seidal method
- The only drawback is the large requirement of computer memory which has been overcome through a compact storage scheme.
- NR method is applied to solve the load flow problem.

Consider a set of n- non-linear algebraic equations

$$f_i(x_1, x_2, \dots, x_n) = 0 \; ; \quad i = 1, 2, \dots, n \qquad -(1)$$

To solve these equations a set of initial values are assumed let the initial values of the unknown be $x_1^0, x_2^0, x_3^0, \dots, x_n^0$ let $\Delta x_1^0, \Delta x_2^0, \Delta x_3^0 \dots \Delta x_n^0$ be the corrections to be added to the initial bus which gives the actual solution. Therefore

$$f_i(x_1^0 + \Delta x_1^0, \; x_2^0 + \Delta x_2^0, \dots, \Delta x_n^0) = 0 \qquad -(2)$$

$$i = 1, 2, \dots, n$$

Expanding this equations in Taylor series around the initial values.

$$f_i(x_1^0, x_2^0, \dots, x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)^0 \Delta x_2^0 + \dots + \left(\frac{\partial f_i}{\partial x_n}\right)^0 \Delta x_n^0\right] + \text{higher order terms} = 0$$

$$-(3)$$

where $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \dots, \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the derivatives of f_i with respect to $x_1, x_2, \dots, x_n$

Neglecting higher order terms we can write eq^n (3) in matrix form

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & & \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \simeq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{— (4)}$$

vector matrix form

$$f^0 + J^0 \Delta x^0 \simeq 0 \quad \text{—— (5)}$$

where J^0 is known as the Jacobian matrix (obtained by differentiating the function vector f with respect to x and evaluating it at x^0)

Eq^n (5) can be written as follows

$$f^0 \simeq [-J^0]\, \Delta x^0 \quad \text{—— (6)}$$

Updated values of x are then

$$x^1 = x^0 + \Delta x^0$$

or in general if we required $(r+1)^{th}$ iteration

$$x^{(r+1)} = x^{(r)} + \Delta x^{(r)}$$

Newton Rapshon Method (Polar Form)

Complex power at bus i

$$S_i = P_i + jQ_i$$

$$S_i = V_i I_i^* \quad \text{— ①}$$

But we know that

$$I_i = \sum_{K=1}^{n} Y_{ik} V_k$$

$$S_i = V_i \sum_{K=1}^{n} Y_{ik}^* V_k^* \quad \text{— ②}$$

$$P_i = \text{real part of } \left[V_i \sum_{K=1}^{n} Y_{ik}^* V_k^* \right] \quad \text{— ③}$$

for $i = 1, 2, \cdots, n$

$$Q_i = \text{imaginary part of } \left[V_i \sum_{K=1}^{n} Y_{ik}^* V_k^* \right] \quad \text{— ④}$$

Since we know that $V_i = |V_i| \angle \delta_i$ — ⑤.

$Y_{ik} = G_{ik} + jB_{ik}$ — ⑥.

$$V_K = |V_K| \angle \delta_K \quad \text{—} (7)$$

$$Y_{iK}^* = G_{iK} - jB_{iK} \quad \text{—} (8)$$

$$V_K^* = |V_K| \angle -\delta_K \quad \text{—} (9)$$

Substituting value of eq^n (5), (8) and (9) in eq^n (2)

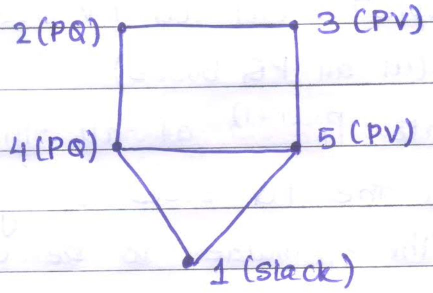
$$P_i + jQ_i = |V_i| \angle \delta_i \sum_{K=1}^{n} (G_{iK} - jB_{iK}) \, |V_K| \angle -\delta_K \quad \text{—} (10)$$

$$P_i + jQ_i = V_i (\cos\delta_i + j\sin\delta_i) \sum_{K=1}^{n} (G_{iK} - jB_{iK}) \, |V_K| (\cos\delta_K - j\sin\delta_K) \quad \text{—} (11)$$

Seperating real and imaginary part

Newton Raphson Method

Form Jacobian for a sample s/m



5-bus network.

$$
\begin{array}{c} \\ \end{array}
\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_2 \\ \Delta Q_4 \end{bmatrix}
\begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 2 \\ 4 \end{array}
\begin{array}{cccc|cc}
2 & 3 & 4 & 5 & 2 & 4 \\
\end{array}
$$

$$
\begin{array}{c|c} & \begin{array}{cccccc} 2 & 3 & 4 & 5 & 2 & 4 \end{array} \end{array}
$$

$$
\begin{array}{c}
\end{array}
\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta P_5 \\ \Delta Q_2 \\ \Delta Q_4 \end{bmatrix}
\begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 2 \\ 4 \end{array}
\left[\begin{array}{cccc|cc}
H_{22} & H_{23} & H_{24} & 0 & N_{22} & N_{24} \\
H_{32} & H_{33} & 0 & H_{35} & N_{32} & 0 \\
H_{42} & H_{43} & H_{44} & H_{45} & N_{42} & N_{44} \\
0 & H_{53} & H_{54} & H_{55} & 0 & N_{54} \\
\hline
J_{22} & J_{23} & J_{24} & 0 & L_{22} & L_{24} \\
J_{42} & 0 & J_{44} & J_{45} & L_{42} & L_{44}
\end{array}\right]
\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta \delta_4 \\ \Delta \delta_5 \\ \frac{\Delta |V_2|}{|V_2|} \\ \frac{\Delta |V_4|}{|V_4|} \end{bmatrix}
$$

Equations for finding the diagonal elements & off-diagonal elements

$$
\left.\begin{aligned}
H_{ii} &= -Q_i - B_{ii}|V_i|^2 \\
N_{ii} &= P_i + G_{ii}|V_i|^2 \\
J_{ii} &= P_i - G_{ii}|V_i|^2 \\
L_{ii} &= Q_i - B_{ii}|V_i|^2
\end{aligned}\right\} \text{diagonal elements.}
$$

Off-diagonal elements

$$H_{ik} = |V_i||V_k| \left(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}\right)$$

$$N_{ik} = |V_i||V_k| \left(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}\right)$$

$$J_{ik} = -|V_i||V_k| \left(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}\right) = -N_{ik}$$

$$L_{ik} = +|V_i||V_k| \left(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik}\right) = H_{ik}$$

Algorithm for NR Method in Polar Co-ordinates.

1. Formulate the Ybus
2. Assume initial voltages as follows

$V_i = |V_{i,sp}| \angle 0°$ (at all PV buses) [sp = specified].

$V_i = 1\angle 0°$ (at all PQ buses)

3. At $(r+1)^{th}$ iteration calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the PQ buses, using voltages from previous iterations $V_i^{(r)}$. The formulae to be used are.

$$P_{i,cal} = P_i = G_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k| (G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i,cal} = Q_i = -B_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k| (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

4. Calculate the power mis-match vector [which includes real & imaginary parts].

$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)}$ (at PV and PQ buses)

$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)}$ (at PQ buses)

5. Calculate the Jacobian matrix $[J^{(r)}]$ using $V_i^{(r)}$

6. Compute

$$\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V|} \end{bmatrix} = |J^{(r)}|^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows.

$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta\delta_i^{(r)}$ (at all buses)

$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta|V_i|^{(r)}$

8. Go to step 3 and iterate till the power value is within acceptable tolerance.

Algorithm for NR method in Rectangular form.

1. Formulate the Ybus.
2. Assume initial voltages as follows

$$e_i + jf_i = |V_{i,sp}| + j0.0 \quad \text{(at all PV buses)}$$
$$e_i + jf_i = 1 + j0.0 \quad \text{(at all PQ buses)}$$

3. At $(r+1)^{th}$ iteration, calculate $P_{i,cal}^{(r+1)}$ at all the PV and PQ bus $Q_{i,cal}^{(r+1)}$ at all the PQ buses and $|V_i|^2$ at PV buses using voltages from previous iteration $V_i^{(r)}$
4. Calculate the power mismatch version [which includes real & imaginary]

$$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)} \quad \text{(at PV and PQ buses)}$$
$$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)} \quad \text{(at PQ buses)}$$
$$\Delta |V_i^{r}|^2 = |V_{i,sp}|^2 - |V_i|^2 \quad \text{(at PV buses)}$$

5. Calculate the Jacobian matrix $[J^{(r)}]$ using $V_i^{(r)}$
6. Compute

$$\begin{bmatrix} \Delta e_i^{(r)} \\ \Delta f_i^{(r)} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \\ \Delta |V_i^{(r)}|^2 \end{bmatrix}$$

7. Update the variables as follows

$$e_i^{(r+1)} = e_i^{(r)} + \Delta e_i^{(r)} \quad \text{(at all buses)}$$
$$f_i^{(r+1)} = f_i^{(r)} + \Delta f_i^{(r)} \quad \text{(at all buses)}$$

8. At PV buses the voltage magnitude is maintained at the specified value. Hence, $e_i^2 + f_i^2 = |V_{i,sp}|^2$ at the PV buses. Therefore we have to adjust the voltage estimates to be used in the next iteration.

The voltage angle is given by

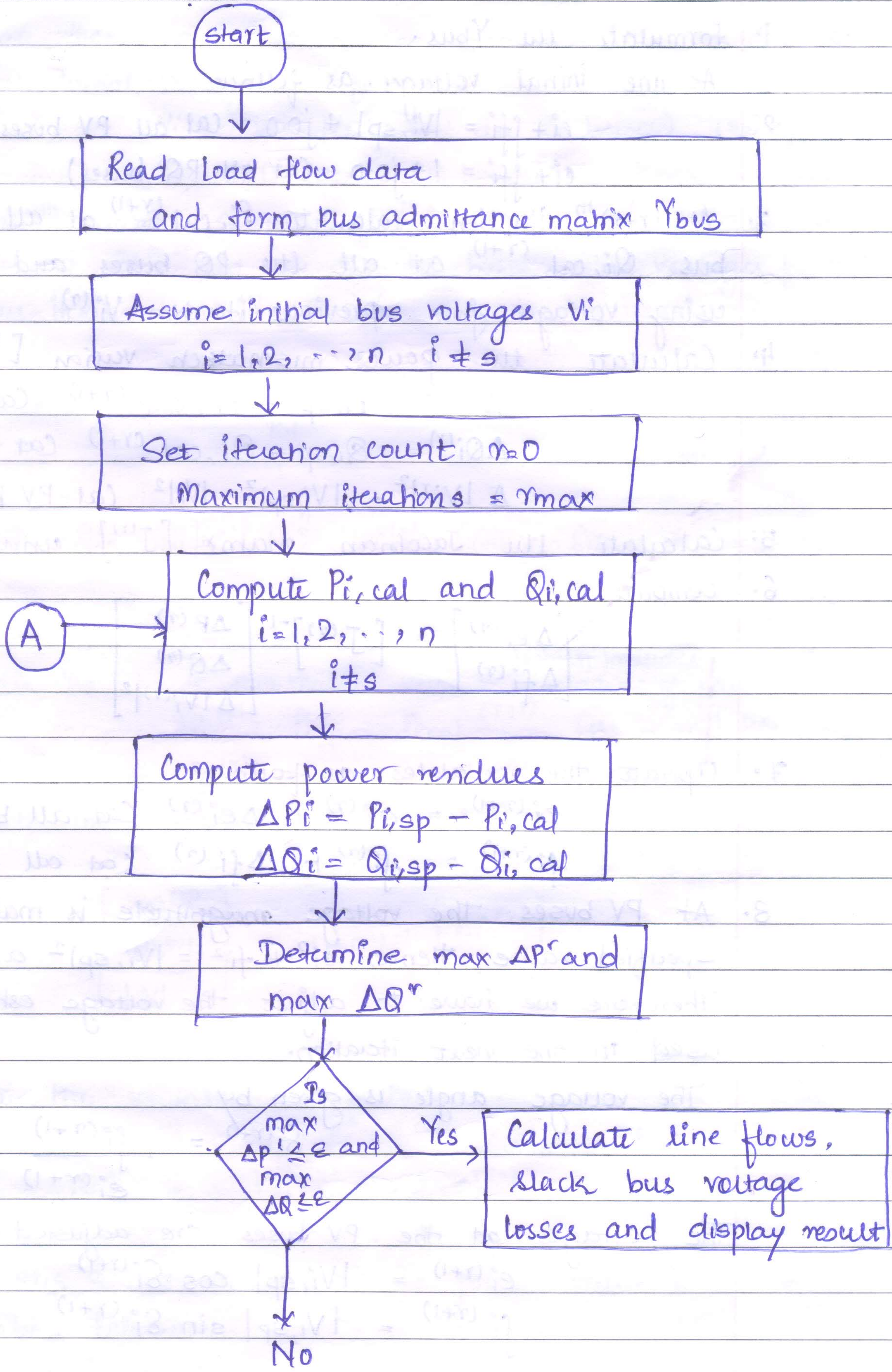
$$\delta_i^{(r+1)} = \frac{f_i^{(r+1)}}{e_i^{(r+1)}}$$

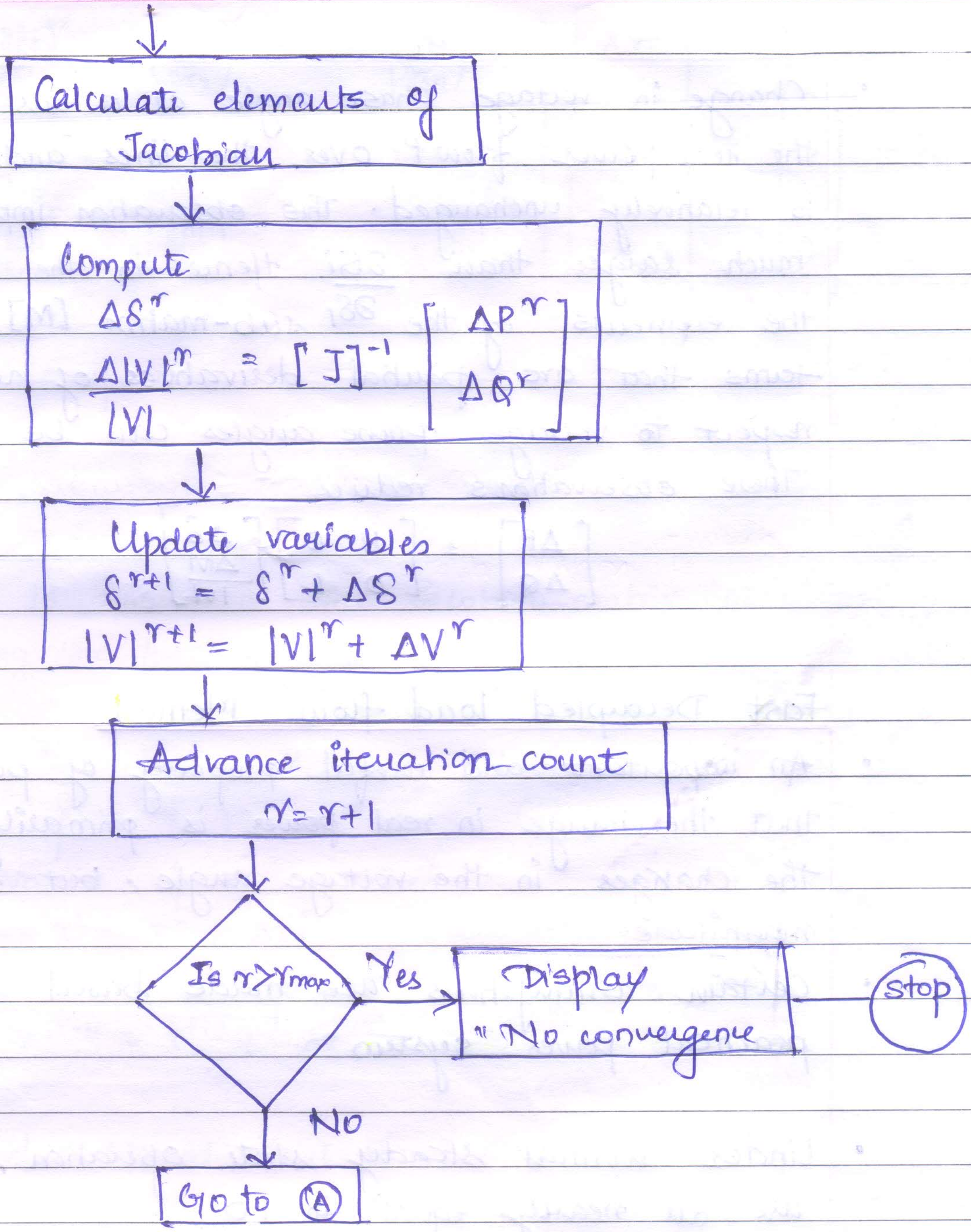
The voltages at the PV buses are adjusted as follows

$$e_i^{(r+1)} = |V_{i,sp}| \cos \delta_i^{(r+1)}$$
$$f_i^{(r+1)} = |V_{i,sp}| \sin \delta_i^{(r+1)}$$

9. Go to step 3 and iterate till the power value is within acceptable tolerance.

Flow Chart for NR method of load flow solution.





Decoupled Load Flow Methods

- When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are.

- Change in voltage magnitude $|V_i|$ at bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that ∂Q is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence in the Jacobian, the $\partial |V_j|$ element of the sub matrix [N] which contains terms that are partial derivatives of real power with respect to voltage magnitude can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_i}$ Hence in the Jacobian the elements of the sub-matrix [M], which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero

These observations reduce

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix}$$

Fast Decoupled load flow Method.

- An important and useful property of power system is that the change in real power is primarily governed by the changes in the voltage angle, but not in voltage magnitude.
- Certain assumptions are made based on observations of practical power system

- Under normal steady state operation, the voltage magnitudes are all nearly equal to 1.0

- As the transmission lines are mostly reactive, the conductances are quite small as compared to the susceptance ($G_{ij} << B_{ij}$)

- Under normal steady state operation the angular differences among the bus voltages are quite small

- The injected reactive power at any bus is always much less than the reactive power consumed by the elements connected to this bus when these elements are shorted to the ground ($Q_{ii} << B_{ii} V_i^2$)

- With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k| B_{ik} \; (i \neq k) \quad — (1)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2 \quad — \quad (2)$$

The matrix reduces to

$$[\Delta P] = [V_i][V_j][B_{ij}'][\Delta \delta] \quad — (3)$$

$$[\Delta Q] = [V_i][V_j][B_{ij}''] \left[\frac{\Delta V}{|V|}\right] \quad — (4)$$

where B_{ij}' and B_{ij}' are negative of the susceptances of respective elements of the bus admittance matrix

In eq^n (3) and (4) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$

We get

$$\left[\frac{\Delta P}{|V|}\right] = [B_{ij}'][\Delta \delta] \quad — (5)$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B_{ij}''] \left[\frac{\Delta V}{|V|}\right] \quad — (6)$$

Eq (5) and Eq (6) constitute the Fast Decoupled Load flow equations.

Algorithm for Fast Decoupled Load flow Method.

Step 1 : Create the bus admittance matrix [Y_{Bus}]

Step 2 : Detect all kinds and numbers of buses and setting all bus voltages to an initial value of 1 pu all voltage angles to 0 and the iteration counter iteration to 0.

Step 3 : Create the matrices B' and B'' ~~according~~

Step 4 : If max $(\Delta P, \Delta Q) \leq$ accuracy

$$\Delta P_i^{[k]} = P_{i,sch} - P_i^{[k]}$$

$$\Delta Q_i = Q_{i,sch} - Q_i^{[k]}$$

then go to step 6

else

(i) Calculate J_1 and J_4 elements using the following equations

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \qquad \frac{\partial P_i}{\partial \delta_j} = -|V_i| B_{ij}$$

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} \qquad \frac{\partial Q_i}{\partial |V_j|} = -|V_i| B_{ij}$$

(ii) Calculate the real and reactive power at each bus and check if MVAR of generator buses are within the limits otherwise update the voltage magnitude at these buses by ± 2 %

(iii) Calculate the power residuals ΔP and ΔQ.

$$\Delta P_i^{(k)} = P_{isch} - P_i^{[k]}$$

$$\Delta Q_i^{(k)} = Q_{i,sch} - Q_i^{[k]}$$

(iv) Calculate the bus voltage and voltage angle updates ΔV and $\Delta \delta$

$$[\Delta \delta_i]^{(k)} = -[B']^{-1} \frac{\Delta P^{[k]}}{|V_i|}$$

$$[\Delta V_i]^{(k)} = -[B'']^{-1} \frac{\Delta Q^{[k]}}{|V_i|}$$

(V) Update the voltage magnitude V and the voltage angle δ at each bus

$$\delta^{[k+1]} = \delta^{[k]} + \Delta \delta_i^{[k]}$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta |V_i^{[k]}|$$

(vi) Increment of the iteration counter iter = iter + 1

Step 5: If iter ≤ maximum number of iteration

$$|\Delta P_i^{(k)}| \leq \varepsilon$$

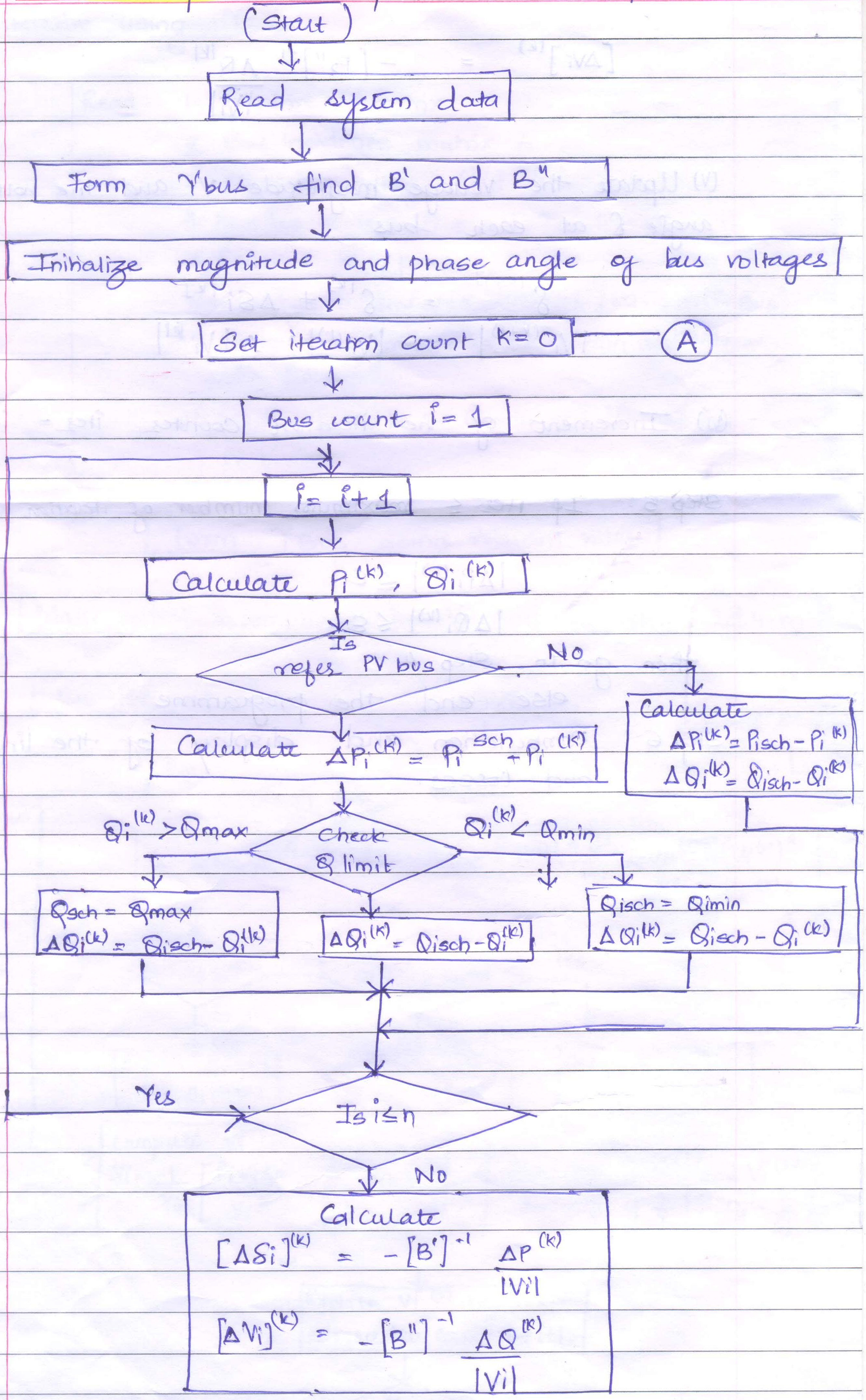
$$|\Delta Q_i^{(k)}| \leq \varepsilon$$

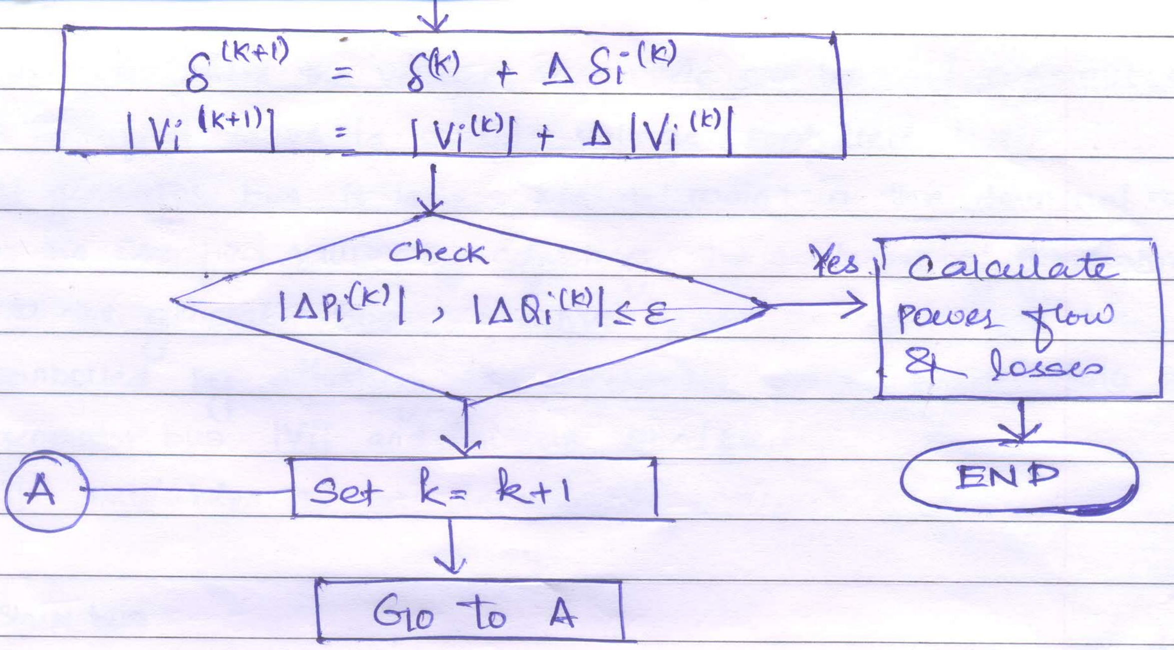
then go to Step 4

else end the programme

Step 6: Computation and display of the line flow and losses.

Flow chart of Fast Decoupled Load Flow Method.





Comparision of Gauss Siedal, NR Method and FDLF methods of Load flow study.

	Gauss Siedal	NR Method	FDLF Method.
1.	Requires a large number of Iterations to reach convergence	1. Requires a less number of iterations to reach convergence	1. Requires a more number of iterations than NR method.
2.	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
3.	It has linear convergence characteristics	It has quadratic Convergence characteristics	—
4.	The number of iterations required for convergence increases with the size of the system	The number of iterations are independent of the size of the system	The number of iterations does not depend on the size of the system.
5.	Less memory required	More memory required	Less memory is required compare to NR method.