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SCHOOL OF ENGINEERING & TECHNOLOGY
SCHOOL OF PHARMACY
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Double Integration

Prof. Sohail Kasim, Asst. Professor

Department: HAS (FE)

AIKTC
SCHOOL OF ENGINEERING

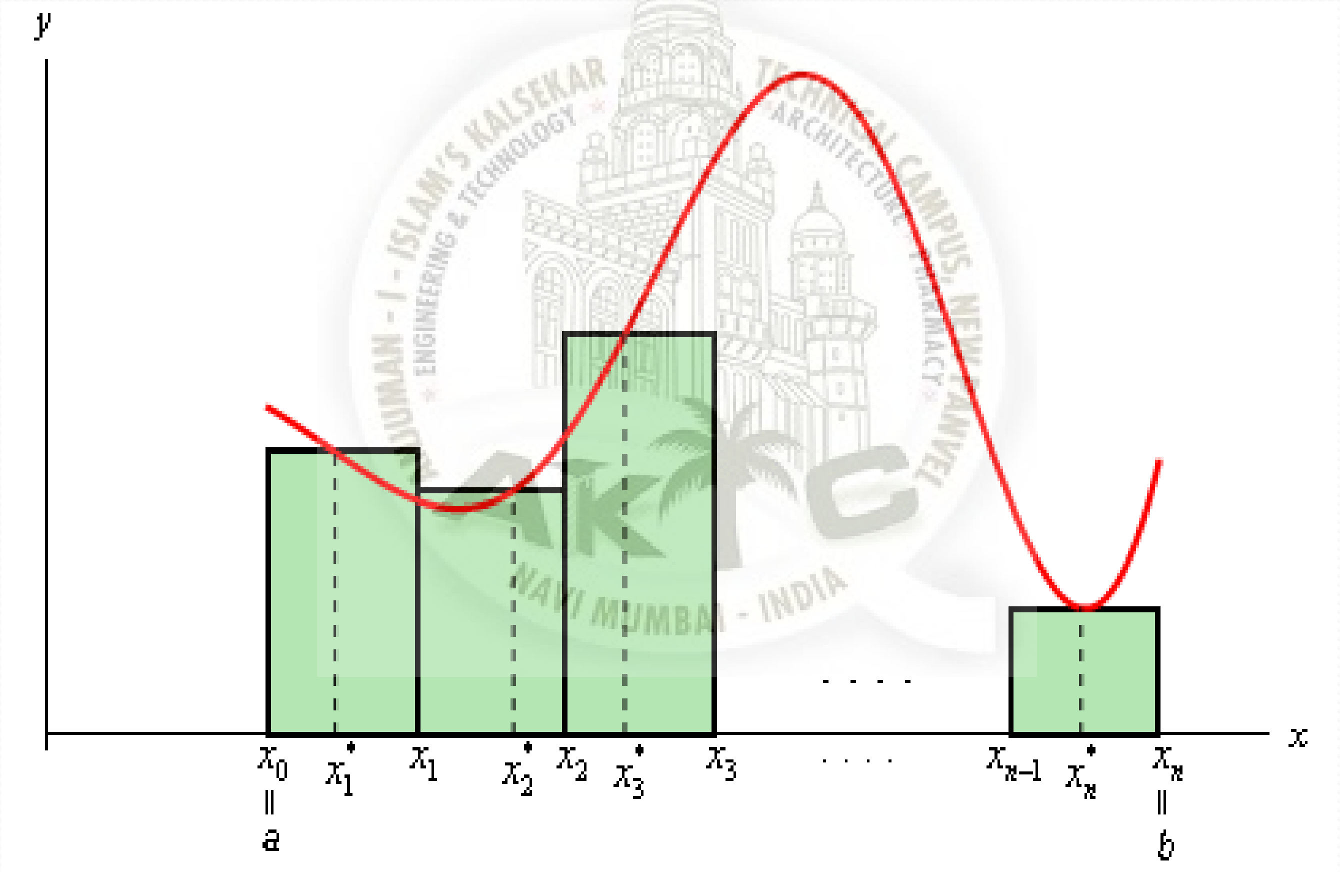
Revision of single Integration

- Before we begin the double integrals let us revise the definition of definite integrals for functions of one real variable

$$\int_a^b f(x)\,dx$$

- We are integrating with respect to x that is we are taking the value of x from the interval $a \leq x \leq b$.

- Now, when we deal with the definite integral we first thought of this as an area problem.
- We first ask what the area under the curve is and to do this we brake up the interval $a \le x \le b$ into n subintervals of length Δx.
- Choose a point, $x*i$, from each interval as shown below.


y
x
x_0
a
x_1^*
x_1
x_2^*
x_2
x_3^*
x_3
x_{n-1}
x_n^*
x_n
b

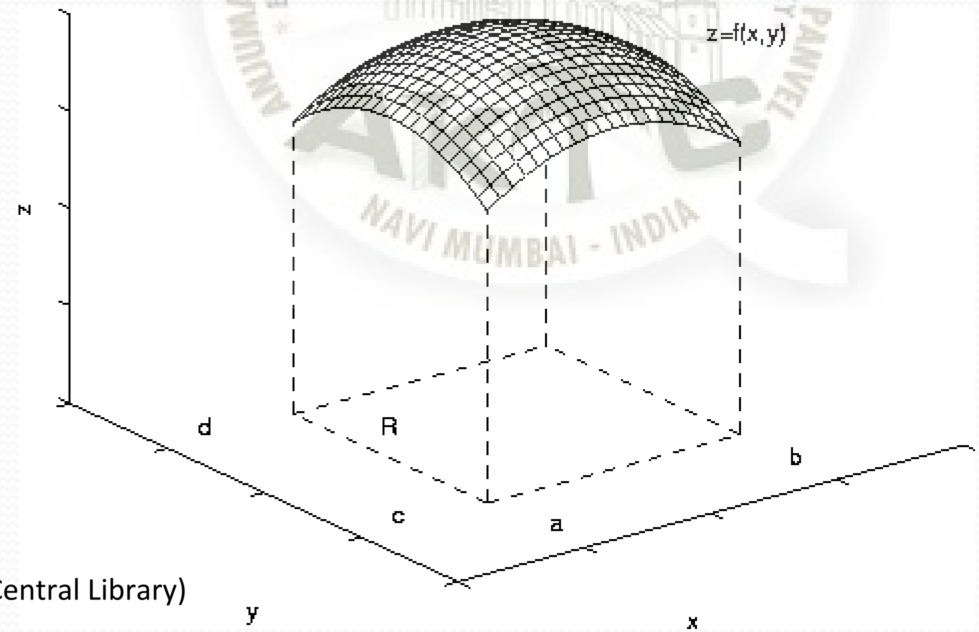
- Each of the rectangles has height of $f(x_i^*)$ and we could then use the area of each of these rectangles to approximate the area as follows.
- $A \approx f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_i^*)\Delta x + \cdots + f(x_n^*)\Delta x$

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\, \Delta x_k$$

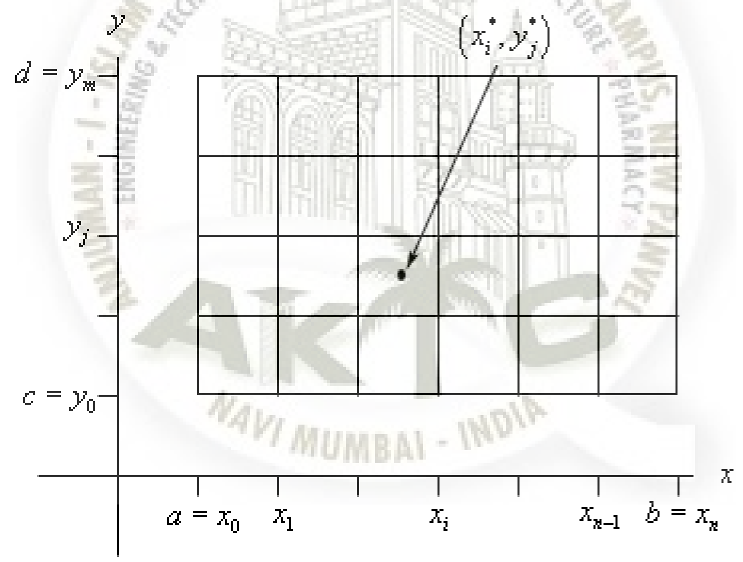
- And we say that $f(x)$ is integrable if the above limit exists.

- The definite integral can be extended to functions of more than one variable. Consider a function of two variables $z=f(x,y)$.
- For defining the we will start out by assuming that the region in R2 is a rectangle which we will denote as follows,
- $R=[a,b]\times[c,d]$ that is $a\le x\le b$ and $c\le y\le d$.

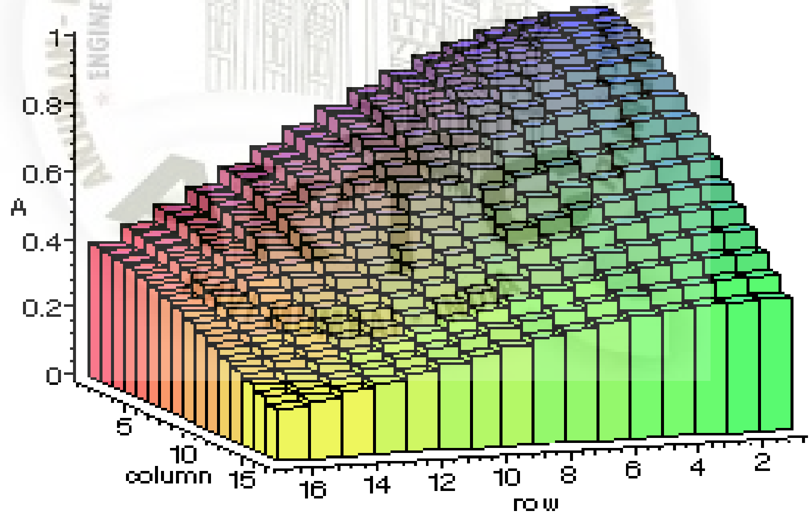
- For positive f(x,y), the definite integral is equal to the volume under the surface z=f(x,y) and above xy-plane for x and y in the region R. This is shown in the figure below.



- Similar to the functions of one variable let's first ask what the volume of the region under R(and above the *xy*-plane of course) is.
- we can subdivide [a,b] into small intervals with a set of numbers $\{x_0,x_1,\ldots,x_m\}$ so that
- $a=x_0<x_1<x_2<\ldots<x_i<\ldots<x_{m-1}<x_m=b$.
- Similarly, a set of numbers $\{y_0,y_1,\ldots,y_n\}$ is said to be a partition of [c,d] along the y-axis, if
- $c=y_0<y_1<y_2<\ldots<y_j<\ldots<y_{n-1}<y_n=d$.


y
$d = y_m$
y_j
$c = y_0$
(x_i^*, y_j^*)
x
$a = x_0$
x_1
x_i
x_{n-1}
$b = x_n$

- Over each of these smaller rectangles we will construct a box whose height is given by $f(x*i, y*j)$.



$$\int\int_R f(x,y)\,dA \approx \sum_{i=1}^{M}\sum_{j=1}^{N} f(x_i,y_j)A_{ij}$$

- In the above sum if the take N and M to infinity then the sum on the right converges to a value which is the double integral over the rectangular region R.

Applications

- Double integral has plenty of applications in science and engineering. Some of them are given below.
- Area of a 2D region
- Volume
- Mass of 2D plates
- Force on a 2D plate
- Average of a function
- Center of Mass and Moment of Inertia
- Surface Area

Refrences

- **Calculus and Analytic Geometry by Thomas and Finney.**
- **Calculus: Early Transcendentals by James Stewart.**