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Eigen Values and Eigen Vectors

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SOME APPLICATIONS OF THE EIGEN VALUES AND EIGENVECTORS OF A SQUARE MATRIX

- Communication systems
- Designing bridges
- Designing car stereo system
- Electrical Engineering
- Mechanical Engineering
- Oil companies frequently use eigenvalue analysis to explore land for oil
- Google search is an eigen value problem

EIGEN VALUE AND EIGENVECTOR

Def. Let A be an nxn matrix. A scalar λ is called an eigen value of A if there exists a nonzero nx1 vector x such that $Ax = \lambda x$

- **Method to find eigen vectors and eigen values of any square matrix A**
  We know that,
- $AX = \lambda X$
  $\Rightarrow AX - \lambda X = 0$
  $\Rightarrow (A - \lambda I)\,X = 0$ .....(1)
- Above condition will be true only if $(A - \lambda I)$ is singular. That means, $|A - \lambda I| = 0$ .....(2)
- (2) is known as characteristic equation of the matrix.

CONTINUED..

- The roots of the characteristic equation are the eigen values of the matrix A.
- Now, to find the eigen vectors, we simply put each eigen value into (1) and solve it by Gaussian elimination, that is, convert the augmented matrix $(A - \lambda I) = 0$ to row echelon form and solve the linear system of equations thus obtained.

Some Important Properties of Eigen Values

- Eigen values of real symmetric and hermitian matrices are real
- Eigen values of real skew symmetric and skew hermitian matrices are either pure imaginary or zero
- Eigen values of unitary and orthogonal matrices are of unit modulus $|\lambda| = 1$
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of A, then $k\lambda_1, k\lambda_2 \ldots\ldots k\lambda_n$ are eigen values of kA
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of A, then $1/\lambda_1, 1/\lambda_2 \ldots\ldots 1/\lambda_n$ are eigen values of A^{-1}
- If $\lambda_1, \lambda_2 \ldots\ldots \lambda_n$ are the eigen values of A, then $\lambda_1^k, \lambda_2^k \ldots\ldots \lambda_n^k$ are eigen values of A^k
- Eigen values of A = Eigen Values of A^T (Transpose)
- Sum of Eigen Values = Trace of A (Sum of diagonal elements of A)

DETERMINANT

- The determinant of the 2 × 2 matrix A =
  [a b
  c d] is
  detA = ad − bc.

- The determinant of the 3 × 3 matrix A =

  [a11 a12 a13
  a21 a22 a23
  a31 a32 a33] is
  det A = a11a22a33 + a12a23a31 + a13a21a32
  − a31a22a13 − a32a23a11 − a33a21a12.

SIMILAR MATRICES

- Definition: The n × n matrices A and B are said to be similar if there is an invertible n × n matrix P such that $A = PBP^{-1}$.

DIAGONALIZATION

- A square matrix A is said to be diagonalizable if it is similar to a diagonal matrix.
- i.e.a diagonal matrix A has the property that there exists an invertible matrix P and a diagonal matrix D such that A = PDP−1.
- If A = PDP−1, then Ak = PDkP−1.

ALGEBRAIC MULTIPLICITY & GEOMETRIC MULTIPLICITY

- Let A be an n×n matrix with eigen value . The algebraic multiplicity of is the number of times is repeated as a root of the characteristic polynomial.
- Let A be an n n matrix with eigen value . The geometric multiplicity of is the dimension of the eigenspace of λ.

REFERENCES

- Linear Algebra- Hoffman & Kunze (Indian editions) 2002 4)
- Linear Algebra- Anton & Torres (2012) 9th Indian Edition.

- THANK YOU…