2:30 pm ET-R-16 21/11/22

Sem – III - CBGS-16 – KT

(Time: 3 Hours) Max. Marks: 80)

N.B. : 1. Q1 is compulsory

2. Attempt any three questions from Q2 to Q6.

3. Figures to the right indicate full marks.

Q1. (a) Find the Laplace transform of $\int_0^t t \sinh 3t \, dt$. 5

(b) If the position vector of a point is denoted by $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$ then show that $\text{div}(r^n \bar{r}) = (n+3) r^n$. 5

(c) Find the Fourier series expansion of $f(x) = x^2, \ -1 < x < 1$. 5

(d) Find the analytic function $u + iv$ where $u = \frac{1}{2}\log(x^2 + y^2)$ 5

Q2. (a) Find the Laplace inverse of $\frac{s+1}{(s+4)(s^2+1)}$. 6

(b) Find the Bilinear transformation which maps the points $z_1 = 1, z_2 = i, z_3 = -1$ onto $w_1 = 0, w_2 = 1, w_3 = \infty$. 6

(c) Prove that $\bar{f} = (4xy + 3x^2z)i + (2x^2 + 2z)j + (x^3 + 2y)k$ is conservative. Determine the work done by $\bar{f}$ on displacing a particle from $(1,0,1)$ to $(2,1,1)$. 8

Q3. (a) Using Laplace transform, evaluate the integral $\int_0^\infty e^{-t} \frac{\sin 3t + \sin 2t}{t} dt$. 6

(b) Find the Fourier series of $f(x) = x^3, \ -\pi < x < \pi$. 6

(c) Using Gauss divergence theorem, evaluate $\iint_s 4xi + 3yj - 2zk \,\square\, d\bar{s}$ where S is the closed surface bounded by $x =, y = 0, z = 0, 2x + 2y + z = 4$. 8

Q4. (a) Find the Laplace inverse of $\frac{1}{(s^2+9)^2}$ using the convolution property. 6

(b) If $\phi = xyz$ and $\psi = xy + yz + xz$, then find $\nabla \square (\nabla\phi \times \nabla\psi)$. 6

(c) Apply Laplace transform and solve the equation 8

$y'' + 9y = t\,;\ y(0) = -1,\ y'(0) = 0.$

Q5. (a) Find the analytic function $u + iv$ if $u - v = e^x(\cos y - \sin y)$ 6

(b) Find the Laplace transform of $\left(1 + t + t^2\right) H(t-3)$ 6

(c) Obtain the Fourier series of $f(x) = \dfrac{(\pi - x)^2}{4},\ 0 \le x \le 2\pi$ and evaluate the 8

value of $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

Q6. (a) Find the complex form of Fourier series of e^{2x} on $(-\pi, \pi)$. 6

(b) For a scalar function $\Phi = 2xy - y^2 + z^2$, find the directional derivative 6

of Φ at the point $(1, 0, 1)$ in the direction of vector $v = 2i + j + 2k$.

(c) Show that $\int J_3(x)\,dx = J_0(x) - \frac{4}{x} J_1(x)$. 8

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