Con. 6397–13. **GS–6063**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Solve any **four** questions from Question Nos. **2** to **7**.

1. (a) Find Laplace transform of $t\sqrt{1-\sin t}$. **20**
   (b) Find the value of z for which the function is not analytic $z = \sin hu \cos v + i \cos hu \sin v$.
   (c) Obtain complex form of Fourier series for $f(x) = e^{ax}$ in $(-l, l)$.
   (d) Find the matrix A, if $\text{adj } A = \begin{bmatrix} -2 & 1 & 3 \\ -2 & -3 & 11 \\ 2 & 1 & -5 \end{bmatrix}$.

2. (a) Reduce the following matrix to Normal form and find its rank : **6**

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 6 & 12 & 18 & 24 \end{bmatrix}.$$

   (b) Find an analytic function whose imaginary part is $e^{-x}(y \sin y + x \cos y)$. **6**
   (c) Expand $f(x) = x \sin x$ in the interval $0 \le x \le 2\pi$. **8**

3. (a) Consider the transformation $w = (1 + i) z + (2 - i)$ and determine the region in the w plane into which the rectangular region bounded by $x = 0, y = 0, x = 1, y = 2$ in the z plane is mapped under this transformation. **6**
   (b) Prove that the matrix is unitary hence find A^{-1} **6**

$$A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

   (c) Find the inverse Laplace transformation : **8**

   (i) $f(s) = \dfrac{1}{s^2 (s+a)^2}$

   (ii) $f(s) = \dfrac{11s^2 - 2s + 5}{2s^3 - 3s^2 - 3s + 2}$.

4. (a) Find Laplace transform of $f(t) = a \sin pt$ **6**

$$0 < t < \pi/p, \quad f(t) = 0, \quad \pi/p < t < 2\pi/p \quad \text{and} \quad f(t) = f\left(t + 2\pi/p\right).$$

   (b) Find the Bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. **6**

(c) Obtain the half range sine series for f(x) : **8**

$$\text{when } f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$$

hence find the sum of $\sum_{2n-1}^{\infty} 1/n^4$.

5. (a) Obtain the Fourier expansion of— **6**

$$f(x) = \begin{cases} \cos x, & -\pi < x < 0 \\ -\cos x, & 0 < x < \pi \end{cases} \quad \text{and}$$
$$f(x) = f(x+2\pi)$$

(b) Show that $u = y^3 - 3x^2 y$ is a harmonic function. Find its harmonic conjugate and the corresponding analytic function. **6**

(c) For what value of λ the equations $3x - 2y + \lambda z = 1$, $2x + y + z = 2$, $x + 2y - \lambda z = -1$ will have no unique solution. Will the equations have any solution for this value of λ ? **8**

6. (a) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ find the eign values of $A^3 + 5A + 8I$. **6**

(b) Show that the set of functions cos x, cos 2x, cos 3x .... is a set of orthogonal functions over $(-\pi, \pi)$. Hence construct a set of orthonormal functions. **6**

(c) Use of Laplace transform to solve :– **8**

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dt} + 8y = 1, \; y(0) = 0, \; y'(0) = 1 .$$

7. (a) Define the fixed points of Bilinear transformation and hence find the same for **6**

$$w = \frac{2z - 2 + iz}{i + z}.$$

(b) Find the inverse Laplace transform of $f(s) = \dfrac{(s+1)\, e^{-s}}{s^2 + s + 1}$. **6**

(c) Find the characteristic equation of the matrix A given below and hence find the matrix represented by, $A^6 - 6A^5 + 9A^4 + 4A^3 - 124A^2 + 2A - I$. **8**

$$\text{where } A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$