shilpa-2nd half-(b)13-91

Con. 6427–13. **LJ–10240**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Figutes** to the **right** indicate **full** marks.

1\. (a) Find an analytic function f (z) whose imaginary part is e^{-x} (y sin y + x cosy). **5**
(b) Obtain Half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$. **5**
(c) Express the following matrix A as P + iQ, where P and Q are both Hermitian **5**

$$A = \begin{bmatrix} 2 & 3-i & 2+i \\ i & 0 & 1-i \\ 1+2i & 1 & 3i \end{bmatrix}$$

(d) Find the inverse Laplace Transform of $\tan^{-1}\left(\frac{2}{S^2}\right)$. **5**

2\. (a) Find inverse Laplace Transform of the following by using convolution theorem **6**

$$\frac{1}{(s^2+a^2)(s^2+b^2)}$$

(b) Prove that the matrix – **6**

$$\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$$ is unitary.

(c) Find Bilinear Transformation which maps the points $-1, 1, \infty$ of z-plane onto $w = -i, -1, i$. **8**

3\. (a) Prove that $\int_0^\infty e^{-\sqrt{2}t} \sin t \sinh t \, dt = \frac{\pi}{8}$. **6**
(b) Obtain the complex form of Fourier series for $f(x) = e^{ax}$ in (o, a). **6**
(c) Solve by using Laplace Transform – **8**
$(D^2 + 3D + 2)y = 2(t^2 + t + 1)$ with $y(0) = 2$, $y'(0) = 2$.

4\. (a) Show that the following system of equations is consistent if a, b, c are in A.P. **6**
$3x + 4y + 5z = a$, $4x + 5y + 6z = b$, $5x + 6y + 7z = c$.
Find the solution when $a = 1$, $b = 2$, $c = 3$.

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(b) If f(z) is analytic function, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **6**

(c) Evaluate :– **8**

(i) $L^{-1}\left\{\frac{(s+1)\,e^{-\pi s}}{s^2+2s+5}\right\}$

(ii) $L^{-1}\left\{\frac{1}{(s-4)^4\,(s+3)}\right\}$.

5. (a) Show that the function $W = \frac{4}{z}$ transforms the straight lines x = c in the z - plane into circle in the w - plane. **6**

(b) Find Non-singular matrices P and Q such that PAQ is in normal form. Also find rank of A, where A is – **6**

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 8 & 5 & 8 \end{bmatrix}$$

(c) Expand f (x) = x sin x in the interval $0 \le x \le 2\pi$. Deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$. **8**

6. (a) Find Laplace Transform of – **6**

$f(t) = a \sin pt,\ 0 < t < \pi / p$,
$f(t) = 0,\ \pi / p < t < 2\pi / p$ and
$f(t) = f(t + 2\pi / p)$.

(b) Obtain Fourier series of x cosx in $(-\pi, \pi)$. **6**

(c) Verify cayley – Hamilton theorem for the matrix A and ontain A^{-1}. **8**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

7. (a) Show that the functions $f_1(x) = 1$ $f_2(x) = x$ are orthogonal on (–1,1). Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on that interval. **6**

(b) Find the eigen values of adj. A and $A^2 - 2A + I$, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ **6**

(c) Show that the following function satisfies Laplace's equation and find its correponding analytic function and the Harmonic conjugate, **8**

$u = \frac{1}{2} \log \left(x^2 + y^2\right)$

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