Con. 6584–13. **LJ–10445**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** from the remaining **six** questions.
(3) Statistical table is **permitted**.

1. (a) Evaluate $\int f(z)\,dz$ along the parabola $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$ where $f(z) = x^2 - 2iy$. **5**

(b) If the mean of the following distribution is 16. Find m, n and variance. **5**

X :	8	12	16	20	24
P (X = x) :	1/8	m	n	1/4	1/12

(c) Evaluate $\int_c \bar{F}\cdot d\bar{r}$ where $\bar{F} = 2x\,i + (xz - y)\,j + 2z\,k$ from 0 (0, 0, 0) to p (3, 1, 2) along the line OP. **5**

(d) A random sample of 50 items gives the mean 6·2 and standard deviation 10·24. Can it be regarded as drawn from a normal population with mean 5·4 at 5% level of significance ? **5**

2. (a) Evaluate $\int_c \frac{\sin^6 z}{\left(z - \pi/6\right)^n}\,dz$. Where 'c' is the circle $|Z| = 1$ for $n = 1$, $n = 3$. **6**

(b) Ten individuals are chosen at random from a population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height of the universe is 65 inches. **6**

(c) Table shows the respective heights x and y of a sample of 12 fathers and their oldest sons. Calculate rank correlation coefficient. **8**

Height of father (inches) x	65	63	67	64	68	62	70	66	68	67	69	71
Height of son (inches) y	68	66	68	65	69	66	68	65	71	67	68	70

3. (a) Evaluate by Green's theorem $\int_c \left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy$, where 'c' is the boundary of the surface in the xy-plane enclosed by the x-axis and the semicircle $y = \sqrt{1 + x^2}$. **6**

(b) For a normal distribution 30% items are below 45 and 8% items are above 64. Find the mean and variance of the normal distribution. **6**

(c) Obtain Taylor's and Laurent's expansions of $f(z)=\frac{z-1}{z^2-2z-3}$ indicating regions of convergence. **8**

4. (a) A car-hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1·5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused. ($e^{-1.5} = 0{\cdot}2231$). **6**

(b) Evaluate $\oint_c \frac{e^z}{\text{Cos } \pi z}\, dz$, where 'c' is the circle $|z| = 1$. **6**

(c) Find the line of regression for the following data and estimate y corresponding to x = 15·5. **8**

X :	10	12	13	16	17	20	25
Y :	19	22	24	27	29	33	37

5. (a) A die was thrown 132 times and the following frequencies were observed. **6**

No. Obtained :	1	2	3	4	5	6	Total
Frequency :	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased.

(b) The equations of the two lines of regression are x = 19·13 – 0·87 y and y = 11·64 – 0·50 x. **6**

Find :–

(i) the means of x & y.

(ii) the coefficient of correlation between x & y.

(c) Verify Stokes theorem for $\overline{F} = (2x - y)\, i - y z^2 j - y^2 zk$ and 's' is the surface of hemisphere $x^2 + y^2 + z^2 = a^2$ lying above the xy-plane. **8**

6. (a) Fit a Binomial distribution to the following data :– **6**

x :	0	1	2	3	4	5	6
F :	5	18	28	12	7	6	4

(b) Write short notes on :– **6**

(i) Null hypothesis and alternative hypothesis

(ii) Type I error and Type II error

(iii) Level of significance and confidence interval.

(c) Calculate the correlation coefficient between x and y from the following data :– **8**
$N = 10$, $\Sigma x = 140$, $\Sigma y = 150$, $\Sigma (x - 10)^2 = 180$ $\Sigma (y - 15)^2 = 215$,
$\Sigma (x - 10)(y - 15) = 60$.

7. (a) If X denotes the out come when a fair die is tossed, find moment generating function (M.G.F.) of X, and hence find the mean and variance of X. **6**

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 + x + 3}{x^4 + 5x^2 + 4} dx$, using contour integration. **6**

(c) Using Gauss's divergence theorem, evaluate $\iint_S (ax^2 + by^2 + cz^2) ds$ over the sphere $x^2 + y^2 + z^2 = 1$. **8**

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