Con. 6427-13.

GS–6846

(3 Hours)

[Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.

    (2) Answer any **four** out of remaining **six** questions.

    (3) Use of **statistical** table is **allowed**.

1. (a) Evaluate $\int_c \dfrac{dz}{z^3(z+4)}$, where c is the circle $|z| = 2$.   5

  (b) Evaluate by Green's theorem $\int_c \left(x^2 - y\right) dx + \left(2y^2 + x\right) dy$, where c is the boundry of   5

    the region defined by $y = x^2$ and $y = 4$.

  (c) If the tangent of the angle made by the line of regression of y on x is 0·6 and   5

    $\sigma y = 2\sigma x$, find the correlation coefficient between x and y.

  (d) If the probability density of a random variable is given by –   5

$$f(x) = \begin{cases} kx & , \quad 0 \le x \le 2 \\ 2k & , \quad 2 \le x \le 4 \\ 6k - kx & , \quad 4 \le x \le 6 \end{cases}$$

    Find – (i) k   and   (ii) $p\,(1 \le x \le 3)$.

2. (a) Marks obtained by students in an examination follow a normal distribution. If 30% of   7

    students got below 35 marks and 10% got above 60 marks, find the mean and the

    percentage of students who got marks between 40 and 50.

  (b) Nine items of a sample had the following values –   7

      45, 47, 50, 52, 48, 47, 49, 53, 51

    Does the mean of 9 items differ significantly from the assumed population mean

    47·5 ?

  (c) State and prove Cauchy's integral formula.   6

3. (a) Using Gauss's divergence theorem evaluate, $\iint_s \left(ax^2 + by^2 + cz^2\right) ds$, over the sphere   7

    $x^2 + y^2 + z^2 = 1$.

  (b) Obtain the rank correlation coefficient for the following data :   7

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

  (c) Fit a Binomial distribution to the following data :   6

X	0	1	2	3	4
Frequency	12	66	109	59	10

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4. (a) The regression lines of a sample are x + 6y = 6 and 3x + 2y = 10, Find –   7
  (i) Sample means $\bar{x}$ and $\bar{y}$
  (ii) Coefficient of correlation between x and y
  (iii) Also estimate y when x = 12.

(b) Evaluate $\int\limits_{-\infty}^{\infty} \dfrac{x^2}{\left(x^2+9\right)\left(x^2+4\right)}\,dx$, using Contour integration.   7

(c) Records taken of the number of male and female births in 800 families having four   6
children are as follows :–

No. of male births	:	0	1	2	3	4
No. of female births	:	4	3	2	1	0
No. of families	:	32	178	290	236	94

Test whether the data are consistent with the hypothesis that the Binomial law holds
and the chance of male birth is equal to that of female birth, namely p = q = ½.

5. (a) Find Laurent's series which represents the function $f(z) = \dfrac{2}{(z-1)\,(z-2)}$ when   7

  (i) | z | < 1   (ii) 1 < | z | < 2   (iii) | z | > 2.

(b) Two samples were drawn from two populations and the results were as given below :   7

Population	A	B
Sample size	21	17
Sample S. D.	45	40

Test the hypothesis that variance of A is less than or equal to variance of B using
F-test.

(c) By using Stoke's theorem, evaluate $\int\limits_{c}\left[\left(x^2+y^2\right)i+\left(x^2-y^2\right)j\right].d\bar{r}$, where c is the   6

boundry of the region enclosed by circles x² + y² = 4 and x² + y² = 16.

6. (a) For the following data :   7

X	1	2	3	4	5	6	7	8	9
Y	9	8	10	12	11	13	14	16	15

Find the lines of regression. Show that for x = 6·2 the estimated value of Y = 13·14.
Estimate the value of X for Y = 13·14. Explain why this value of X differs from 6·2.

(b) Evaluate $\oint\limits_{c} \dfrac{e^{z}}{\cos \pi z}\,dz$, where c is the circle | z | = 1   7

(c) A random variable X has the following probability distribution :   6

X	0	1	2	3
P(X = x)	1/6	1/3	1/3	1/6

Find - (i) First four raw moments

(ii) First four central moments.

7. (a) Prove that $\overline{F} = \left(y^2 \cos x + z^3\right)i + \left(2y \sin x - 4\right)j + \left(3xz^2 + 2\right)k$ is a conservative field.   7

Find –

    (i) Scalar potential for $\overline{F}$

    (ii) The work done in moving an object in this field from (0, 1, -1) to ($\pi/2$, –1, 2).

(b) Before an increase in excise duty on tea, 800 people out of a sample of 1000 persons   7
were found to be tea drinkers. After an increase in the duty, 800 persons were known
to be tea drinkers in a sample of 1200 people. Do you think that there has been a
significant dicrease in the consumption of tea after the increase in the excise duty ?

(c) Show that in a poisson distribution with unit mean the mean deviation about mean is   6
(2/e) times standard deviation.

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