shilpa-2nd half-(c)13-19

Con. 6301–13. **LJ–10460**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find the characteristic equation of the maric A given below and hence; find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. **5**

$$\text{Where } A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}.$$

(b) Find the orthogonal trajectory of the family of curves $x^3y - xy^3 = c$. **5**

(c) Evaluate $\int_c \frac{\sin^6 z}{\left(z - \pi/6\right)^3} dz$ where c is $|z| = 1$. **5**

(d) Use the dual simplex method to solve the following L.P.P. **5**

Minimise $Z = x_1 + x_2$
Subject to $2x_1 + x_2 \geq 2$
$-x_1 - x_2 \geq 1$
$x_1, x_2 \geq 0$.

2. (a) Find the eigen values and eigen vectors of the matrix. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **6**

(b) Find the imaginary part of the analytic function whose real part is – $e^{2x}(x \cos 2y - \sin 2y)$. Also verify that v is harmonic. **6**

(c) Use Penalty method to solve the following L.P.P. **8**

Minimise $Z = 2x_1 + 3x_2$
Subject to $x_1 + x_2 \geq 5$
$x_1 + 2x_2 \geq 6$
$x_1, x_2 \geq 0$.

3. (a) Using the method of Lagrange's multipliers, solve the follwoing N.L.P.P. **8**

Optimise $Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$
Subject to $x_1 + x_2 + x_3 = 20$ and
$x_1, x_2, x_3 \geq 0$.

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(b) Evaluate $\int_c \frac{z^2}{(z-1)^2 (z-2)} dz$ where c is the circle $|z| = 2 \cdot 5$. **6**

(c) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. **6**

4. (a) Show that the matrix – $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonisable. Find the transforming matrix and the diagonal matrix. **6**

(b) Show that $f(z) = \sqrt{|xy|}$ is not analytic at the origin although Cauchy - Riemann equations are satisfied at that point. **6**

(c) Using Duality solve the following L.P.P **8**

Minimize $Z = 430\, x_1 + 460\, x_2 + 420\, x_3$

Subject to $x_1 + 3x_2 + 4x_3 \geq 3$

$2x_1 + 4x_3 \geq 2$

$x_1 + 2x_2 \geq 5$ and

$x_1, x_2, x_3 \geq 0.$

5. (a) Consider the following problem – **6**

Maximize $Z = x_1 + 3x_2 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

Determine :–

(i) all basic solutions,

(ii) all feasible basic solutions,

(iii) optimal feasible basic solution.

(b) Obtain Taylor's and Laurent's expansions of $f(z) = \frac{z-1}{z^2 - 2z - 3}$ indicating regions of convergences. **6**

(c) Verify Cayley - Hamilton theorem for the matrix A and hance, find A^{-1} and A^4 **8**

where – $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

6. (a) If $u = -r^3 \sin 3\theta$, find the analytic function f (z) whose real part is u. **6**

(b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that 3 tan A = A tan 3. **6**

(c) Solve the following L.P.P. by simplex method. **8**

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0.$

7. (a) Find the bilinear transformation which maps the points $z = \infty$, i, o on to the points o, i, ∞. **6**

(b) Find Laurent's series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ **6**

when (i) $|z| < |$

(ii) $1 < |z| < 2$,

(iii) $|z| > 2$.

(c) Use the Kuhn - Tucker conditions to solve the following N.L.P.P. **8**

Minimise $Z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to $x_1 + 3x_2 \leq 6$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

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