Con.6426–13. **GS–6897**

(3 Hours) **[Total Marks :100**

N.B. (1) Question No.1 is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Marks to the **right** indicate **full** marks.

1. (a) Check if the following function is harmonic. $f(\gamma,\theta)=\left(\gamma+\frac{a^2}{\gamma}\right)\cos\theta$ **5**

(b) Integrate function $f(z) = x^2 + iy$ from A(1, 1) to B(2, 4) along the curve $x = t$, $y = t^2$ **5**

(c) Prove that the eigen values of an orthogonal matrix are +1 or –1. **5**

(d) Construct the dual of the followig LPP : **5**

Maximize $z = x_1 + 3x_2 - 2x_3 + 5x_4$

Subject to $3x_1 - x_2 + x_3 - 4x_4 = 6$

$5x_1 + 3x_2 - x_3 - 2x_4 = 4$

$x_1, x_2 \geq 0,$ x_3, x_4 unrestricted.

2. (a) Evaluate $\oint_c \frac{e^z}{\cos \pi z} dz$ c is the circle $|z| = 1$. **6**

(b) Diagonalise the Hermitian matrix $A = \begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$ **6**

(c) Use Simplex method to solve the LPP : **8**

Maximise $z = 1000x_1 + 4000x_2 + 5000x_3$

Subject to $x_1 + 2x_2 + 3x_3 \leq 14$

$3x_1 + 2x_2 \leq 14$

and $x_1, x_2, x_3 \geq 0$

3. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2+x+2}{x^4+10x^2+9} dx$ using contour integration. **6**

(b) State Cayley-Hamilton theorem. Use it to find A^{-1} and A^4 **6**

Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

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(c) Use Penalty method to Minimise $z = x_1 + 2x_2 + x_3$ **8**

Subject to $x_1 + \frac{x_2}{2} + \frac{x_3}{2} \leq 1$

$\frac{3}{2}x_1 + 2x_2 + x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

4. (a) If $A = \begin{bmatrix} 4 & 3 \\ 7 & 8 \end{bmatrix}$ find A^{100} **6**

(b) If f(z) is analytic function, **6**

prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$

(c) Use Dual simplex method to Minimise $z = 3x_1 + 2x_2 + x_3 + 4x_4$ **8**

Subject to

$2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$

$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$

$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$

$x_1, x_2, x_3, x_4 \geq 0$

5. (a) Find the bilinear transformation that maps the points 1, –i, 2 in z-plane onto the points 0, 2, –i in w-plane. **6**

(b) $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ is A^3 derogatory ? **6**

(c) Evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta} d\theta$ where $0 < b < a$ **8**

6. (a) If $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are positive integers, then prove that **6**

(i) a + b + c is an eigen value of A and

(ii) If A is non-singular, one of the eigen values in negative.

(b) Find the image of region bounded by x = 1, y = 1 and x + y = 1 under the transformation $w = z^2$ **6**

(c) Use Lagrangian Multiplier Method to Optimise 8

$$z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

s.t. $x_1 + x_2 + x_3 = 20$

$x_1, x_2, x_3 \geq 0$

7. (a) Find Laurent's series for the function 6

$$f(z) = \frac{1}{(z-1)(z-2)}$$ in the regions

(i) $1 < |z - 1| < 2$ (ii) $1 < |z - 3| < 2$

(b) Find the analytic function f(z) whose imaginary part is 6

$e^{-x}[2xy \cos y + (y^2 - x^2) \sin y]$

(c) Using Kuhn Tucker method, 8

Opimise the function $2x_1 + 3x_2 - \left(x_1^2 + x_2^2 + x_3^2\right)$

s. t. $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 6$

$x_1, x_2 \geq 0$
