26-10-2013-DTP-P-7-MU-20

Con. 5395-13. (OLD COURSE) **LJ - 10246**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of **remaining** six questions.
(3) **Figures** to right indicate **full** marks.

1. (a) Find $L^{-1}\left\{\dfrac{e^{-\pi s}}{s^2-2s+2}\right\}$ **5**

(b) Find the sum g the $f(z)=\dfrac{\sin z}{z\cos z}$ at its pole inside the circle $|z|=2$ **5**

(c) Find the constants a, b, c, d, e, if
$f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - zy^2) + i\,(4x^3y - exy^3 + 4xy)$ is analytic. **5**

(d) Find fourier series for $f(x) = |x|$ in (-1, 1). **5**

2. (a) Using Laplace transform, evaluate **6**

$$\int_0^{\infty} e^{-t}\left(\int_0^t u^2 \sin u \cosh u\,du\right)dt$$

(b) Show that the function $u = \frac{1}{2}\log(x^2 + y^2)$ satisfies Laplace equation, hence find its corresponding analytic function and harmonic conjugate. **7**

(c) Find Fourier expansion of $f(x) = x^2$ in (0,a) **7**
Hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots\dots$

3. (a) Evaluate $\int_c |z|\,dz$, where c is the left half of unit circle from z = -i to z = i **6**

(b) Find i) $L^{-1}\left\{\dfrac{s-29}{(s+4)(s^2+9)}\right\}$ **7**

ii) $L^{-1}\left\{\log\left(\dfrac{s^2+a^2}{\sqrt{s+b}}\right)\right\}$

(c) Find half range cosine series for $f(x) = x$ in (0,2), using Parseval's identity deduce that $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\dots$ **7**

[TURN OVER

4. (a) Find the orthogonal trajectory of the family of curves $2x - x^3 + 3xy^2 = 0$ **7**

(b) Find the fouries expansion of **6**

$$F(x)=\begin{cases}1+\dfrac{2x}{\pi} & -\pi \le x \le 0\\ 1-\dfrac{2x}{\pi} & 0 \le x \le \pi\end{cases}$$

Deduce that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dfrac{1}{7^2}+\ldots\ldots$

(c) Find Laplace transform of **7**

(i) $ey(\sqrt{t})$ (ii) $\dfrac{1}{t}(\cos at-\cos bt)$

5. (a) Evaluate $\oint_c \dfrac{z+1}{z^3-2z^2}dz$ where c is **7**

(ii) $|z|=1$ (ii) $|z-2-i|=2$

(b) Find the bilienar tranformation which maps the points 0, i, -2i of 2-plane on to the points -4i, ∞, 0 respectively of W-plane. Also obtain fixed points of transformation. **6**

(c) Express the function $f(x)=\begin{cases}\sin x & 0<x<\pi\\ 0 & x<0, x>\pi\end{cases}$ **7**

as fourier since Integeal and evaluate $\displaystyle\int_0^\infty \frac{\sin wx.\sin \pi w}{1-w^2}dw$

6. (a) Find $L^{-1}\left[\dfrac{s^2}{s^4+4s+16}\right]$ using convolution theom. **6**

(b) Find the map of the real axis in the z plane under the transformation $W=\dfrac{1}{z+i}$ **7**

(c) Obtain two distinct Laurents series expansions of $f(z)=\dfrac{2z-3}{z^2-4z+3}$ in powers of (z-4) indicating region of convergence. **7**

7. (a) If F (z) is analytic function, prove that **6**

$$\left(\frac{\partial^2}{ox^2}+\frac{\partial^2}{\partial y^2}\right)|F(z)|^2=|F^1(z)|^2$$

(b) Using Residuce theorem the evaluate 7

(i) $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ ii) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)^2} dx, a > 0$

(c) Using Laplace transform solve: 7
$(D^2 - 3D + 2)y = 4e^{2t}$, y (0) = 3, y' (0) = 5
